

PRESSURE-IMPULSE THEORY FOR PLATE IMPACT ON WATER SURFACE

Weidong Peng & D. H. Peregrine (d.h.peregrine@bris.ac.uk)
School of Mathematics, Bristol University, University Walk Bristol BS8 1TW, England

Abstract

For plate impact on water we present have studied the use of the pressure impulse theory, which appears to give sensible results for the pressure peak. Negative pressures occur after impact but efforts to use pressure-impulse theory have been unsuccessful. On the other hand work by Korobkin & Peregrine (2000) show that negative pressures are likely due to compressibility effects.

Introduction

The problem of impact on water can be traced back to the study of sea plane landing and work by von Karman (1929) & Wagner (1932). Their work has been extended by Cointe and Armand(1987) and Cointe (1989): surveys include Lesser & Field (1983) and Korobkin & Puknachov (1988). More recently, pressure impulse theory has been developed for studying waves breaking against coastal structures Cooker & Peregrine (1995) and Wood, Peregrine & Bruce (2000). Here we investigate the value of the pressure-impulse approach for the impact of a rigid plate falling flat onto a level water surface. As the plate strikes on the water surface, high pressures act for a brief time and decay shortly thereafter.

As has been discussed by Cooker (1996) the pressure after impact can also be determined from pressure-impulse theory since the post impact velocities can be determined. We have noted that the pressure after impact is greater than the initial pressure in all cases for which we have seen experimental data, which is in agreement with Cooker's results, except for the case of impact of a horizontal plate on flat water where the post impact pressure is less than the initial pressure. There are similarities between this problem and the slamming of ships, Lewison (1970) gives a good discussion motivated by that application, and notes the pressure reduction, but we are not convinced by his theoretical analysis.

Pressure-impulse theory

In impact problems, the deceleration of plates and impact loads on fluid boundaries are of large magnitude and short duration so that the dominant terms in the equation of motion for the fluid reduce to inertia and pressure. Further the time scales are so short that the convective terms in the fluid's acceleration are negligible. Thus by integrating over the short duration of the impact the pressure impulse is found and satisfies Laplace's equation if fluid compressibility is negligible. Pressure impulse is defined as

$$P(\mathbf{x}) = \int_{t_b}^{t_a} p(\mathbf{x}, t) dt \quad \text{and} \quad \mathbf{u}_a - \mathbf{u}_b = \nabla P / \rho.$$

where subscript b and a denote before and after the impact respectively, and ρ is the density of the water. More details of boundary conditions etc. may be found in Cooker & Peregrine (1995).

In considering plate motion we consider a two-dimensional model, with m the mass of the plate per unit spanwise length, $2L$ its length and V its downward velocity. The equation governing plate motion is

$$m \frac{dV}{dt} - mg = - \int_{-L}^L p(x,0) dx,$$

where pressure, p , is measured relative to atmospheric. Integrating this equation over the impact gives the change in velocity.

The result is that in terms of $z=x+iy$ where the origin of axes is at the centre of the plate with Oy vertically upwards,

$$P(x, y) = -\rho V_a \Re \left\{ i \left[z + \sqrt{z^2 - L^2} \right] \right\}$$

and the plate velocity after impact is

$$V_a = mV_b / (m + M)$$

where the added mass, M , is $\frac{1}{2}\pi\rho L^2$ per unit spanwise length. On the plate, the pressure-impulse is $\rho V_a L \sqrt{L^2 - x^2}$, and the total impulse is $\frac{1}{2}\pi\rho V_a L^2$.

An experimental result provided by Lloyd, Stansby & Wright (1999) for pressure impulse at the center of the plate is about 1.052 (kPa ms) with $m = 118$ kg, $L = 0.5$ m, $V_b = 5.4$ m/s with width across the flume of 0.5 m.. The recorded peak pressure is about 255 kPa and $\Delta t = 0.085$ s. Our theoretical result of the pressure impulse is about 1.014 kPa ms. If the gravity is also considered and $V_b + g\Delta t$ replaces V_b , the theoretical result is 1.029 kPa ms, which agrees almost too well with the experimental result. The peak pressure can be estimated using approximate formula $p = 2P / \Delta t$.

Angular motion

Unless the impact is perfectly symmetrical the impact also changes the angular motion of the plate. Again we consider only two-dimensional motion for simplicity, and suppose the symmetry of the above problem is broken by an initial angular velocity, ω_b , but the plate is still horizontal at impact. Then integration of the angular momentum equation over the impact interval gives the change in angular momentum as

$$I_0(\omega_a - \omega_b) = \int_{-L}^L xP(x,0) dx,$$

where I_0 is the momentum of inertia of the plate about its centre. However, we now have a different value for the pressure impulse since there is the change in velocity on the plate surface due to its rotation to be accounted for. This gives an extra term, P_ω , to be added to the pressure impulse of direct impact, that is

$$P_\omega = \frac{1}{4} \rho \omega_a L \Re \left\{ i \left(z - \sqrt{z^2 - L^2} \right)^2 \right\}.$$

Post-impact pressure

In studies of wave impact on walls the pressure after the impulsive peak is always greater than the initial pressure, with a few exceptions when especially large air pockets are trapped between wave and wall. Similarly in the experiments of Lloyd et al. (1999) when a plate lands on a wave crest the pressure after the impact is greater than the initial atmospheric pressure. However, Lloyd et al. also give pressures from a plate falling onto flat water, and these show post-impact pressures that are significantly below atmospheric pressure. This is an aspect of considerable interest since should this be the case in a slam impact of a vessel the resulting change in direction of pressure forces on the structure could be especially dangerous if the structure is weakened by a severe slam.

This negative pressure has been measured and remarked upon before. Lewison (1969) gives a discussion of the plate, air and water motion and attributes the negative pressure to the volume available to the trapped air increasing due to the water being accelerated to a velocity larger than the plate's final velocity. However, in comparable wave impact problems there has to be an unusually large volume of air trapped to give such an effect. Indeed in experiments to study the effect of trapped air at a wall by comparing a modified pressure-impulse theory with measured pressures there are generally positive pressures (Wood, Peregrine & Bruce, 2000). We see no reason to suppose that an exceptionally large air pocket is trapped beneath a falling plate.

The pressure immediately after the impact might be obtained from the pressure-impulse theory, from the values of velocities just after the impulse. This has been discussed in Cooker (1996) which indicates that the pressure is increased by $\frac{1}{2}\rho V_b^2$ for impact on a wall. Cooker's approach leads to difficulties when the velocities after impact have a singularity, as they do in this case. Some effort has been put into overcoming this problem with no success to date. It seems likely that a local solution for jet formation at the edges of the plate may be needed to exclude the singularities from the problem.

A further source of pressure variation is due to compressibility of the water. For a rather special case, a hemispherical body sitting at rest in a semi-infinite sea and given an impact, is studied in Korobkin & Peregrine (2000). This is a case where, if the nonlinear terms are neglected because of the short duration of an impulsive movement, the resulting linear equations can be solved explicitly, giving a solution for low Mach numbers with pressure changes small compared with atmospheric pressure, that holds until displacements are no longer small or gravity becomes important. The availability of such a solution is partly due to the vertical sides at the water line giving a solution without any jet formation. The solution shows that a significant fraction of energy is radiated in acoustic waves with the pressure on the body after the initial peak becoming negative, and for some values if the body mass is larger than the added mass the pressure oscillates. However, in most experimental situations the negative pressure is likely to be strongly modified by the effect of

acoustic reflections from the bed beneath the body. Many experiments including those reported by Lewison (1969) fail to record the depth of water.

Conclusions

The value of the pressure-impulse results is not clear. There are simple solutions for the plate impact, which appear to give sensible results that can be useful in some situations where overall properties of an impact are needed. However, the solutions give a singular velocity field at the edge of the plate. This leads to problems in making further use of the approach to find pressures after the impact.

The consideration of the effects of compressibility in a simpler problem by Korobkin & Peregrine (2000) show that pressures after impact can be negative. The duration of such pressures is on an acoustic time scale.

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