SIMPLIFIED MODEL OF THE FREAK WAVE FORMATION FROM THE RANDOM WAVE FIELD

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1. Introduction

The freak wave appearance is a phenomenon characteristic of many areas of the World Ocean. Mallory (1974) collected data of eleven documented events of catastrophic ship collisions with freak waves in the vicinity of the Indian Coast of South Africa. Probably, last such an event occurred in this area with the ship «Taganrogsky Zaliv» in 1985 (Lavrenov, 1985). Lavrenov (1998) suggested that the main reason of the frequent appearance of the freak waves near the south-eastern shore of the South Africa is the wave amplification on the opposite Agulhas current. Sand et al (1990) analysed prototype records of extreme single waves recorded on the Danish Continental Shelf, and pointed out that such waves are too high, too asymmetric and too steep. They also reported the freak wave records on deep water in the Gulf of Mexico. The statistical methods can predict the probability of the appearance of the freak waves (exactly, the averaged number of the possible events during the storm), but can not answer the question, when and where will be next extreme waves. The physical mechanisms of the formation of the freak waves are wave focusing and superposition of waves of different scales. An interesting example of the freak wave formation in the laboratory tank using the spatial - temporal focusing of the dispersive train was demonstrated by Kjeldsen (1990). Considering this mechanism very important, we analyse herein its effect on 2D and 3D water waves on the basis of the linear theory in the presence of a random wave field.

2. Focusing of dispersive trains

To describe 2D water waves with narrow spectrum we will use the linearised version of the parabolic equation for the complex wave envelope *A(x,t)* (Mei, 1993):

$$\frac{\partial A}{\partial t} = \frac{i}{2}\frac{\partial^2 A}{\partial x^2} \tag{1}$$

with the dimmensionless coordinate and time

$$x' = k_0(x - c_g t), \qquad t' = k_0^2 \left|\frac{dc_g}{dk}\right| t\ , \tag{2}$$

where k_o is the carrier wavenumber, and c_g is the group velocity on the carrier wavenumber, calculated from the dispersion wave relation

$$\omega = \sqrt{g \tanh(kh)}\ . \tag{3}$$

The water depth *h* is assumed to be arbitrary. Equation (1) can be solved exactly with initial condition in the form of the Gaussian wave packet (see, for instance, Clauss & Bergmann, 1986)

$$A(x,0) = A_0 \exp\left(-K_0^2 x^2\right), \tag{4}$$

where A_o is the wave height and K_o is the inverse wave impulse length. The solution of equation (1) for either time moment is

$$A(x,t) = \frac{A_0}{\sqrt[4]{1+4t^2K_0^4}} \exp\left(-\frac{K_0^2 x^2}{1+4t^2K_0^4}\right) \cos\left[\frac{2tx^2K_0^4}{1+4t^2K_0^4} - \frac{a\tan(2tK_0^2)}{2}\right]. \tag{5}$$

This solution describes well-known dispersion effect: its wave amplitude decreases with time, its length increases, and the Gaussian impulse transforms into a wave train. But if we consider expression (5) for any negative time as an initial condition, the wave dynamics will have an another behaviour: the wave train will transform into the Gaussian impulse at moment t = 0 ("collapse" time), and then the wave packet will disperse again. This focusing of the wave train on the first stage can be very significant, and it depends from parameters, A_o and K_o.

Real wind waves are random waves which are usually considered as an ensemble of different spectral components with random phases. Due to nonlinearity of the wave filed the correlation between phases is not zero, also the variable wind leads to the appearance of coherent components in the wind wave spectrum. We may consider the solution (5) as an example of such a coherent component. Let us consider the superposition of this coherent component with the random wave field. The latter is the sum of the Fourier-component with different amplitudes and phases, each of them, of course, should be the solution of the equation (1),

$$A = \sum_{n=1}^{\infty} a_n \exp i\left(K_n x - \frac{K_n^2}{2} t + \phi_n\right). \tag{6}$$

For illustration, parameters in (6) are chosen as follows

$$a_n = \frac{(-1)^n}{1+0.25n}, \qquad K_n = \frac{(2n)^{1.8}}{n+1.3}, \qquad \phi_n = \frac{\pi n^3}{6}, \tag{7}$$

and, qualitatively, the wave field is varied as the random field with the significant wave height about 4 (dimensionless units).

The result of the superposition of the coherent wave component (5) with the random field (6) is displayed in Fig. 1 (numbers - dimensionless time). In fact, except in the vicinity of the "collapse" time ($t = 0$), the coherent component is not identified from the resulting records, but during a short period of time, the impulse wave of an extreme amplitude (in our calculations about 10) is generated and then disappears. This simplified model demonstrates all features of the freak wave phenomenon: its "invisibility" in the random field of the wind waves on the sea surface and occurrence during short time in confined part of the sea surface.

3. Conclusion

The mechanism of the spatial - temporal focusing of the coherent component in the wind wave field is probably the key point in the freak wave phenomenon. Calculations were performed also for 3D water waves within 2D version of the parabolic equation for the wave amplitude (these results are not described here in detail): the resulting wave field is shown in Fig. 2. Three-dimensional effects lead to more narrow domain of appearance of the freak wave and shortened period of time for its "visibility". Next step is to study the nonlinear mechanism of the wave focusing and wave-noise interaction, and also to study of the conditions for generation of coherent component in wind wave field.

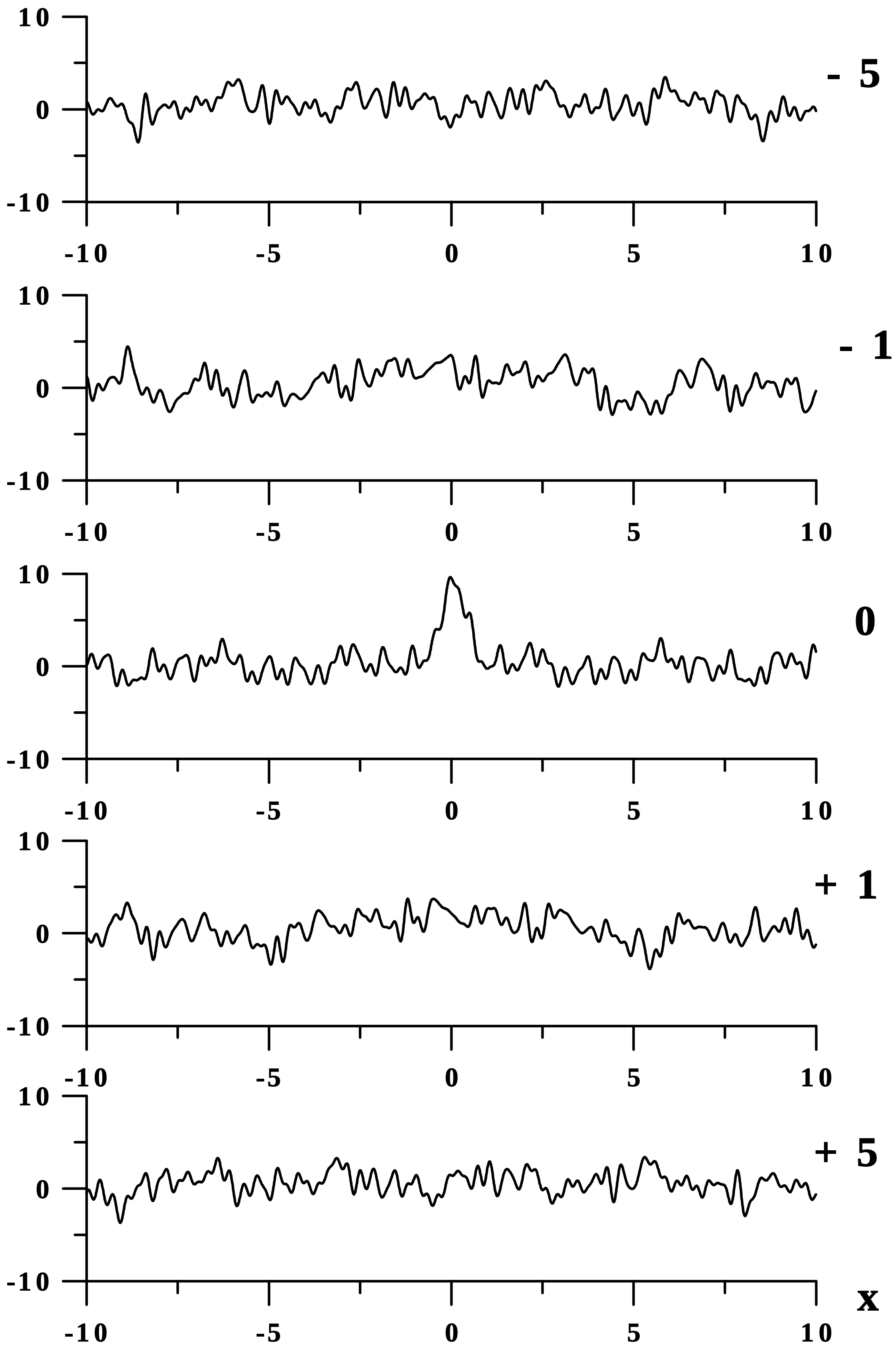

Fig. 1. Evolution of the wave field with the coherent component.

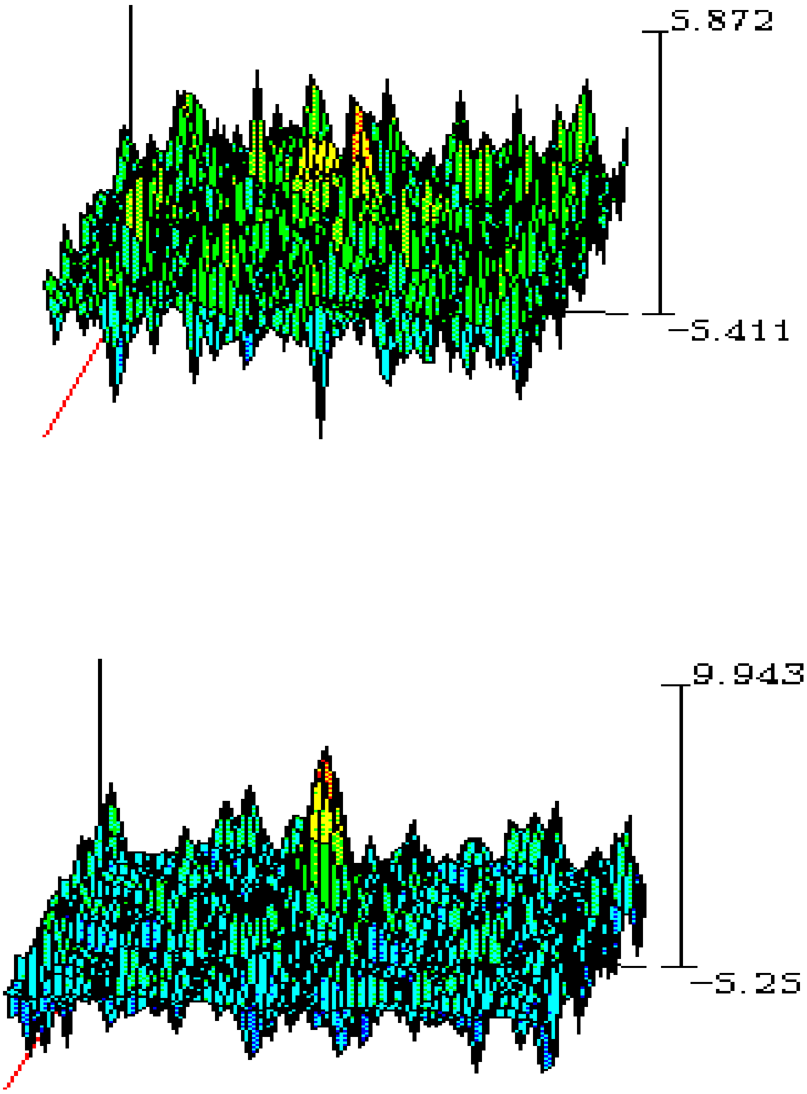

Fig. 2. Focusing of 3D dispersive waves (upper for *t* = - *1*, bellow for the “collapse” time).

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