Analysis of Wave Force on a Large and Thin Floating Platform

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1 Introduction

Regarding the interaction of waves and a very large floating body of zero draft, whose configuration represents a design concept of floating airports, a question was raised at 14th IWWWFB on how to evaluate the side force acting on the zero draft body. This issue will be crucial if we are concerned with computation of the steady drift force and slow oscillating drift force in multi-frequecy waves; the latter in particular is not computed from the far field behavior of the waves scatterd and radiated by the body but to be evaluated with integrating the pressure on the body's side surface. We will need more careful analysis of the flow close to the edges of the thin body to study this problem. In this report we present a result of an attempt to this direction.

2 Solution of the problem

We assume a floating thin plate is very long and the flow when it is in beam seas is taken to be two dimensional with generator parallel to the z axis. The x and z axes are on the mean free surface and the y axis is toward vertically upward. We take the width of the plate is 2 and the draft d, ie the thin plate occupies the region of the mean free surface of $-1 \leq x \leq 1$ and $-\infty < z < +\infty$.

We suppose the regular waves being incident at beam on the plate whose velocity potential is

$$\phi_0(x,y)e^{i\omega t} = e^{ikx+ky+i\omega t} \tag{1}$$

Assumptions for our analysis is that the wave length is very small compared with the width of the plate and the draft is much smaller than the wave length ie.

$$1 >> k^{-1} >> d \tag{2}$$

This assumption will be reasonable when we consider the dimensions of a prototype model of floating airport constructed recently in Japan.

Here our analysis is not with a method proposed, for example, by Ohkusu (1998) but it is rather along a traditional line of decomposing the flow into radiation and scattering ones and expanding the vibration of the plate into a series. We introduce an expression for the vertical displacement $\zeta(x)e^{i\omega t}$ of the plate due to the bending vibration

$$\zeta e^{i\omega t} = \sum_n A_n U_n(x) e^{i\omega t} \quad (-1 < x < 1) \tag{3}$$

$U_n(x)$ is Chevyshev polynominals of the second kind. A_n are to be determined by solving the equation of the plaete vibration with the edge conditions. Reason for this expression is not that Chevyshevs are mode functions but that they lead to the explicit expression of the hydrodynamic pressure on the plate.

The velocity potential $\phi e^{i\omega t}$ of the flow is decomposed into the radiation potential ϕ_R by the vibration of ζ and the scattering potential ϕ_D by ϕ_0

$$\phi = \phi_R + \phi_D \tag{4}$$

Here ϕ_D contains the effect of ϕ_0. Hereafter we suppress $e^{i\omega t}$ unless it is needed.

The linear free surface condition for ϕ is

$$-k\phi + \phi_y = 0 \quad (y = 0, \quad x > 1 \ \text{ or } \ x < -1) \tag{5}$$

Subscript y denotes the differentiation into its direction. Hereafter this notation will be used.

The free surface condition is further simplified in the outer region of $|x \pm 1| = O(1), y = O(1)$ with the assumption of the fore-part of the assumption (2)

$$\phi = 0 \quad (y = 0, \quad x > 1 \ \text{ or } \ x < -1) \tag{6}$$

A solution of ϕ_R in the outer domain is straightforward. In the outer region the draft is too small to be felt and the body boundary condition is imposed on $y = 0$

$$\phi_{Ry} = i\omega \sum_n A_n U_n(x) \qquad (y = 0, \quad -1 < x < 1) \tag{7}$$

The problem is a fmiliar one in the linear wing theory (we impose the condition $\phi_{Rx,y} \to 0$ at infinity). Integral equation for ϕ_{Rx} is given by

$$i\omega \sum_n A_n U_n(x) = \frac{1}{\pi} \oint_1^1 \frac{\phi_{Rx}(\xi, 0)}{\xi - x} d\xi \tag{8}$$

Here a circle on the integral symbol denotes Cauchy's principal value.

Solution is given by inverse Hilbert transform (Tricomi 1957). ϕ_{Rx} and $-\phi_{Ry}$ except the eigensolution is given by th real part and the imaginary part respectively of

$$-\frac{1}{\pi} \frac{i\omega}{\sqrt{1 - z^2}} \int_{-1}^1 \frac{\sqrt{1 - \xi^2}}{\xi - z} \sum_n A_n U_n(\xi) d\xi \tag{9}$$

In this equation z denotes a complex coordinate $z = x + iy$ and the principal value is taken if $z = x, |x| < 1$. We notice that we should add a progressing wave train solution in very thin layer close to the free surface ($|x \pm 1| = 1, ky = O(1)$)

After some algebra and discarding the eigensolution with the condition of continuity of $\phi_R(x, y)$ at $x = \pm 1, y = 0$, we have ϕ_R on the plate surface

$$\phi_R(x, 0) = -i\frac{\omega}{\pi} \sum_n \frac{A_n}{n + 1} U_n(x) \sqrt{1 - x^2} \quad (|x| < 1) \tag{10}$$

Reason for choice of Chevyshev polynomnals is clear. Obviously the pressure on the plate is in-phase of the motion and no damping force acts.

We consider the local flow in the region of wave length size $k|x \pm 1| = O(1), ky = O(1)$ around both edges of the plate. We notice this region is yet large enough, the latter part of the assumption (2) says, to let us assume the plate draft is zero. ϕ_R in this region close to $x = 1$ for example has to satisfy the original free surface condion (5) and the body condition

$$\phi_{Ry} = O(k^{-1}) \qquad (y = 0, \quad -\infty < x < 1) \tag{11}$$

since in this region other edge $x = -1$ is very far away and its existence is not felt. We may take the right hand side is zero in the lowest order solution.

The problem is a dock problem of semi-infinite length which has been well studied (for example Friedrichs and Lewy (1948), Holford (1964)). In our problem the boundary conditions are all homogeneous; the forcing comes from the matching condition with the solution (10) at $x \sim \pm 1$.

A local solution ϕ_R, for example, in the region at $kz \sim 1$ will be

$$\phi_R = -\sqrt{2} i\omega \sum_n A_n \left(Re[\sqrt{1 - z}] + k^{-\frac{1}{2}} F(k(x - 1), ky) \right) \tag{12}$$

where the function F is derived from the result of Holford (1964). When we are concerened with the pressure beneath the plate we may define as

$$F(kx,ky) = \frac{1}{2\pi}\int_0^\infty ds \int_0^\infty d\tau \frac{\Lambda(s)\Lambda(\tau)\cos(\tau ky)e^{-\tau k|x|}}{\tau(\tau - i)(s-i)(\tau+s)} \tag{13}$$

The details of the function $\Lambda(x)$ is not reproduced here for the sake of brevity but it is of $O(x)$ at $x=\infty$ and of $O(\sqrt{x})$ at $x=0$ (see Holford (1964)).

A local solution of ϕ_R at $kz \sim -1$ is obtained in similar manner. We add two local solutions at $x \sim \pm 1$ and the solution (10) to obtain a composite expression of ϕ_R valid everywhere on the plate

$$\phi_R(x,0) = \frac{-i\omega}{\pi}\sum_n \frac{A_n U_n(x)}{n+1}\sqrt{1-x^2} - \sqrt{\frac{2}{k}} i\omega \sum_n A_n \Big[F(k(x-1),0) - (-1)^{n+1} F(k(x+1),0) \Big] \tag{14}$$

We should notice that F to produce the out-of-phase part of the pressure is localized to very small region close to the plate edges. It means the damping force due to the vibration acts only at the edges of the plate.

ϕ_D is obtained almost the same way as ϕ_R. In the outer region at $x,y = O(1)$ the solution is an eigensolution satisfying the homogeneous conditions both on the body and the free surfaces: $\phi_{Dy} = 0$ on the plate ($y=0$) and $\phi_D = 0$ on the free surface. The coefficients are to be determined by the matching with the local solution valid $k(z\pm 1) = O(1)$. The local solution is a solution that satisfies the free surface condition (5) and the condition ensuring the incident and the out-going waves at infinity. It is a dock problem and the solution is found in Holford (1964).

A composite expression of ϕ_D valid everywhere on the plate is

$$\phi_D(x,0) = \sqrt{\frac{1}{k\pi}}\left[\sqrt{\frac{1+x}{1-x}} - \sqrt{\frac{2}{1-x}}\right] + \frac{\sqrt{2}}{\pi}\int_0^\infty \frac{\Lambda(\tau)e^{\tau k(x-1)}}{\tau(1+\tau^2)} d\tau \qquad (-1 < x < 1) \tag{15}$$

We see from this expression that the lowest order part of the wave force is limited in the small area very close to the windward edge.

Equation of the plate bending vibration is obtained immediately with the above expressions for hydrodynamic force:

$$\sum_n A_n \left\{ U_n^{(4)}(x) - \omega^2\left(m - \rho\frac{\sqrt{x^2-1}}{n+1} - \frac{\rho g}{\omega^2}\right) U_n(x) - i\sqrt{2g}\Big(F_- + (-1)^n F_+\Big)\right\} = \phi_D(x,0) \tag{16}$$

where m denotes mass per unit area of the plate and $F_\pm = F(k(x\pm 1),0)$ in (14).

3 Steady force

We are able to evaluate steady drift force on the plate by the farfield computation without relying on the pressure integration. Yet the pressure integration will be useful for estimating the local distribution of it. Furthermore the idea proposed here will be extended to the computation of the slowly varying force in random seas whose evaluation needs the pressure integration.

We investigate the asymptotic behavior of the local solution ϕ_D as $k(z-1)$ approches zero to study the force acting on the side surface of the plate, which extends from $y=0$ to $y=-d$. For the sake of brief description here we focus on ϕ_D, though alsmost similar method is applied to ϕ_R.

As $k(z-1)$ goes to zero, the second term of ϕ_D given by (15) will become

$$\phi_D = \sqrt{2} - \frac{\sqrt{2}}{\pi} Re\Big[k(z-1)\log k(z-1)\Big] + O(k(z-1)) \tag{17}$$

Obviously the first term corresponds to the wave elevation at the edge of the plate $x=1, y=0$. In our case, very large body and small wave length, the incident waves are reflected completely. The second term is caused by the regularity condition imposed on the edge flow. When a infinite wave elevation is admitted there, other solution will appear (Stoker (1957)).

It is an accepted method in engineering to account only for the steady pressure acting above the free surface due to the wave elevation with very shallow draft body. Here we consider the second term to confirm its legitimacy.

Rescale the coordinate by d as $Z = k(z-1)/d$ and rewrite (16) with the first two terms retained, we have

$$\phi_D = \sqrt{2} - \frac{\sqrt{2}}{\pi} X \cdot d \log d + O(d) \tag{18}$$

Φ is a solution of the flow around a semi-infinite dock of finite draft 1 to satisfy $\partial\Phi/\partial Y = 0$ on the mean free surface. Φ must have an asymptotic behavior of (18) as Z goes to infinity. The second term of (18) represents the uniform flow along the X axis; it is a simple problem that is solved by conformal mapping method. Φ will be in the form

$$\Phi = \sqrt{2} - \frac{\sqrt{2}}{\pi} d \log d (X + \Psi(X,Y)) \tag{19}$$

where

$$\Psi(X,Y) = -\frac{1}{\pi} Re(\cosh\zeta), \qquad X - iY = \frac{1}{\pi}(\zeta + \sinh\zeta)$$

We compute the steady force f_S acting on the area projected into the x axis with this solution.

$$f_S = -\frac{\rho g}{2} + \frac{\rho}{2}(d \log d)^2 \int_{-1}^{0} \left(\frac{\partial \Psi(0,Y)}{\partial Y} \right)^2 dY \tag{20}$$

In this computation we need not consider the force at $x=-1$ becuse ϕ_D is zero there (comple reflection). Obviously the second term representing the force acting on the mean wetted side surface is of higher order than as much as d.

The steady force f_B acting on the bottom of the plate which is vibrating is given by

$$f_B = \frac{\rho\omega}{4} \sum_n A_n \int_{-1}^{1} U_n'(x) dx \left[\sqrt{\frac{1}{k\pi}} \left(\sqrt{\frac{1+x}{1-x}} - \sqrt{\frac{2}{1-x}} \right) + \frac{\sqrt{2}}{\pi} \int_0^\infty \frac{\Lambda(\tau) e^{\tau k(x-1)}}{\tau(1+\tau^2)} d\tau \right] \tag{21}$$

4 Remarks

Numerical results are presented at the Workshop.

A future problem is the case of $kd = O(1)$. Analytical solution of the semi-infinite dock with finite draft is not available as far as the present author knows. Maybe numerical technique must be introduced in this case.

5 References

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