On the sloshing modes in moonpools, or the dispersion equation for progressive waves in a channel through the ice sheet

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Barges with large rectangular moonpools housing the upper ends of the risers are now being considered as floating production systems in mild areas like Western Africa. Typical dimensions are 180x60 m for the barge and 80x20 m for the moonpool (the 'well-bay'), the draft ranging from 5 up to 25 m.
An hydrodynamic problem encountered with this concept is how much water motion takes place in the moonpool, under wave induced pressures and barge motion.
At the previous workshop (Molin, 1999) consideration was given to the piston mode, when the bulk of water inside the moonpool heaves up and down nearly like a rigid body. Results were given for the natural frequency and associated shape of the free surface, under the simplifying assumption that the beam and length of the barge be much larger than the breadth and length of the moonpool.
In this sequel, with the same simplifying assumption, we analyze the sloshing modes that take place in the longitudinal direction. An eigen value problem is formulated and natural frequencies and modal shapes are obtained. A striking result is that the natural frequencies of the longest sloshing modes are much larger than could be expected, and that they are quite sensitive to the 'aspect ratio' b/l of the moonpool. Similar effects are obtained for progressive waves in a moonpool of infinite length, tantamount to a channel through the icepack.

The barge is assumed to be a rectangular box of length L, beam B and draft h. The moonpool is located at mid-deck, and is also rectangular with length l and width b. The waterdepth is taken to be infinite. Use is made of a cartesian coordinate system xyz, with its origin located at one of the lower corners of the moonpool. The z axis is upward, $z = 0$ is the bottom of the moonpool, $z = h$ the free surface.
The problem is tackled via potential flow theory. Be $\Phi(x, y, z, t)$ the velocity potential inside the moonpool. When the beam and length of the barge are assumed to be infinite, Φ obeys the boundary condition at $z = 0$

$$\Phi(x, y, 0, t) = \frac{1}{2\,\pi} \int_0^b \int_0^l \frac{\Phi_z(x', y', 0, t)}{\sqrt{(x-x')^2 + (y-y')^2}}\, dx'\, dy'. \tag{1}$$

Φ is sought for under the form

$$\Phi(x, y, z, t) = \Re\left\{\varphi(x, y, z)\, e^{-i\,\omega\,t}\right\}$$

$$\varphi = \sum_{n=0}^{N} \sum_{q=0}^{Q} \cos\lambda_n\, x\ \cos\mu_q\, y\ (A_{nq}\ \cosh\nu_{nq}\, z + B_{nq}\ \sinh\nu_{nq}\, z) \tag{2}$$

with $\lambda_n = n\,\pi/l,\ \ \mu_q = q\,\pi/b,\ \ \nu_{nq}^2 = \lambda_n^2 + \mu_q^2$
and, when $n = q = 0$, the hyperbolic functions are to be replaced with $A_{00} + B_{00}\ z/h$.
The Laplace equation and the no-flow condition at the vertical walls are fulfilled. Only the boundary conditions at $z = 0$ (equation 1) and $z = h$ ($g\ \varphi_z - \omega^2\ \varphi = 0$) remain.
Inserting (2) in (1), multiplying both sides with $\cos\lambda_m x\ \ \cos\mu_p y$ and integrating in x and y gives

$$A_{mp} = \frac{2}{\pi\ b\ l\ (1+\delta_{m0})\ (1+\delta_{p0})} \sum_{n=0}^{\infty} \sum_{q=0}^{\infty} \nu_{nq}\ I_{mnpq}\ B_{nq}, \tag{3}$$

where

$$I_{mnpq} = \int_0^l dx \int_0^l dx' \int_0^b dy \int_0^b dy' \; \frac{\cos\lambda_m x \; \cos\lambda_n x' \; \cos\mu_p y \; \cos\mu_q y'}{\sqrt{(x-x')^2+(y-y')^2}}. \tag{4}$$

I_{mnpq} is non zero only when both $m+n$ and $p+q$ are even. For the longitudinal sloshing modes, $(p,\ q)$ are even, and all $(m,\ n)$ are either odd or even. Advantage is taken of this to reduce the size of the eigen value problem, which is obtained by combining (3) with the free surface equation and eliminating one set of the A_{nq} or B_{nq} coefficients, like in the 2D case (Molin, 1999).

Figure 1 shows a plot of the free surface obtained for the first mode, for a 80x20x5 m moonpool. The maximum elevation is not obtained at the end walls, but at about 10 m. The associated period is 7.36 s, which for a progressive wave in an unbounded ocean corresponds to a wavelength of 84.5 m, quite far off twice the length of the moonpool (160 m)!
For these calculations the double series was truncated at $n = 25$ and $q = 10$. It was found that, when only one term is retained, $n = 1$ (or 2, 3, etc.), $q = 0$, the natural frequency is obtained with a good accuracy, even though the modal shape be not strictly sinusoïdal. It is found to be given by

$$\omega_{n0}^2 = g\,\lambda_n \; \frac{1 + J_n \; \tanh\lambda_n h}{J_n + \tanh\lambda_n h} \tag{5}$$

where

$$J_n = \frac{n}{b\,l^2} \int_0^b dy \int_0^b dy' \int_0^l dx \int_0^l dx' \; \frac{\cos\lambda_n x \; \cos\lambda_n x'}{\sqrt{(x-x')^2+(y-y')^2}} = \frac{n}{b\,l^2}\; I_{nn00}. \tag{6}$$

J_n can be expressed as

$$J_n = \frac{2}{n\,\pi^2\, r} \left\{ \int_0^1 \frac{r^2}{u^2\,\sqrt{u^2+r^2}} \left[1 + (u-1)\;\cos(n\,\pi\,u) - \frac{\sin(n\,\pi\,u)}{n\,\pi}\right] du + \frac{1}{\sin\theta_0} - 1 \right\} \tag{7}$$

where $r = b/l$ and $\tan\theta_0 = r^{-1}$.

Figure 2 shows J_n, versus $\lambda_n\, b = n\,\pi\, b/l$, for $n = 1,\ 2,\ 10$. It appears that J_n is always less than 1. Hence one can introduce β_n such that

$$J_n = \tanh\beta_n \tag{8}$$

which leads to the dispersion equation

$$\omega_{n0}^2 = g\,\lambda_n \;\coth(\lambda_n\, h + \beta_n). \tag{9}$$

When $\lambda_n b$ goes to zero, so do J_n and β_n. Hence very large values of ω_{n0} can be obtained when both the draft h and the breadth b are small, as compared to the length l of the moonpool. In the case considered here equation (9) gives 7.41 s for the first natural period, to be compared with the exact value of 7.36 s.

At this stage one may conjecture that similar effects should be observed for progressive waves in an infinite length moonpool, that is a cut through a rigid ice sheet. In such case one starts from the expression, for the velocity potential inside the channel

$$\varphi(x,y,z) = e^{i\,k_0\,x} \sum_{n=0}^{\infty} \cos\mu_n y \; (A_n \;\cosh\nu_n z + B_n \;\sinh\nu_n z) \tag{10}$$

where $\mu_n = n\,\pi/b$, $\nu_n^2 = k_0^2 + \mu_n^2$, and only even values of n are retained. The same boundary condition (1) applies at $z = 0$, giving

$$e^{i\,k_0\,x} \sum_n \cos\mu_n y\ A_n = \frac{1}{2\,\pi} \int_{-\infty}^{\infty} dx' \int_0^b e^{i\,k_0\,x'} \frac{\sum_n \nu_n\ \cos\mu_n y'\ B_n}{\sqrt{(x-x')^2 + (y-y')^2}}\, dy' \tag{11}$$

or, setting $x' = x + u$:

$$\sum_n \cos\mu_n y\ A_n = \frac{1}{2\,\pi} \int_{-\infty}^{\infty} du \int_0^b e^{i\,k_0\,u} \frac{\sum_n \nu_n\ \cos\mu_n y'\ B_n}{\sqrt{u^2 + (y-y')^2}}\, dy'. \tag{12}$$

Again, multiplying both sides with $\cos\mu_m\,y$ and integrating in y gives

$$A_m = \frac{1}{\pi\,b\,(1+\delta_{m0})} \sum_n \nu_n\ B_n\ I_{mn} \tag{13}$$

with

$$I_{mn} = \int_{-\infty}^{\infty} du \int_0^b dy \int_0^b dy'\ e^{i\,k_0\,u} \frac{\cos\mu_m y\ \cos\mu_n y'}{\sqrt{u^2 + (y-y')^2}} \tag{14}$$

and, together with the free surface condition, an eigen value problem can be set to obtain the frequencies ω_n and associated free surface shapes. Most likely, like in the longitudinally restricted case, a good approximation of the frequency ω_0 of the inline mode can be obtained by retaining only the $n = 0$ terms, yielding

$$\omega_0^2 = g\,k_0\,\frac{1 + J_0\ \tanh k_0 h}{J_0 + \tanh k_0 h} \tag{15}$$

where

$$J_0(k_0 b) = \frac{k_0 b}{2\,\pi} \int_{-\infty}^{\infty} du \int_0^1 dy \int_0^1 dy'\ \frac{e^{i\,k_0 b\,u}}{\sqrt{u^2 + (y-y')^2}}\ . \tag{16}$$

$J_0(k_0 b)$ can be evaluated as

$$J_0(k_0 b) = 1 - \frac{2}{\pi\,k_0 b} \left(1 - \int_0^1 e^{-k_0 b\,(1-u^2)^{-1/2}}\ du\right). \tag{17}$$

It is also shown on figure 2. It is the limiting case of $J_n(\lambda_n b)$ when n goes to infinity, and the same type of dispersion equation applies:

$$\omega_0^2 = g\,k_0\ \coth(k_0\,h + \beta_0) \tag{18}$$

where $J_0(k_0 b) = \tanh(\beta_0)$. The differences between J_1, J_2, J_n and J_0 are due to end effects, which are most pronounced for the long modes.

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Reference

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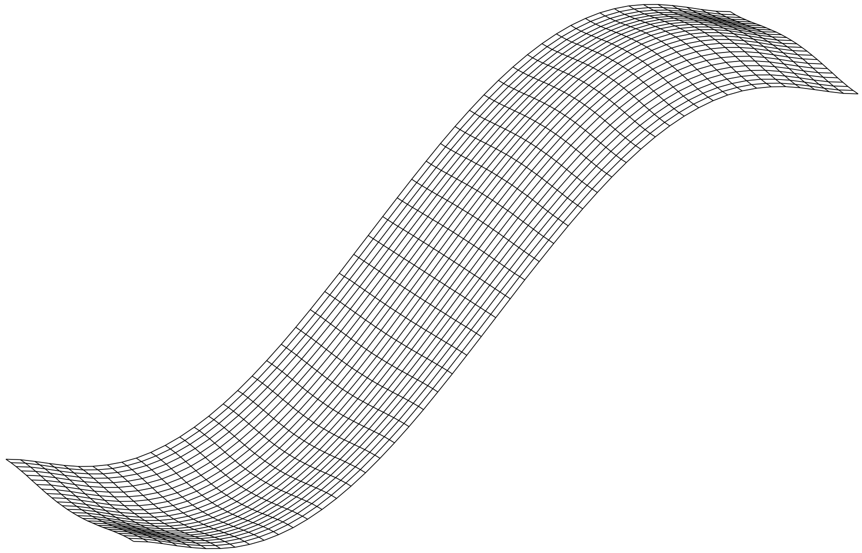

Figure 1: Free surface shape for the first sloshing mode. Moonpool dimensions: 80x20x5.

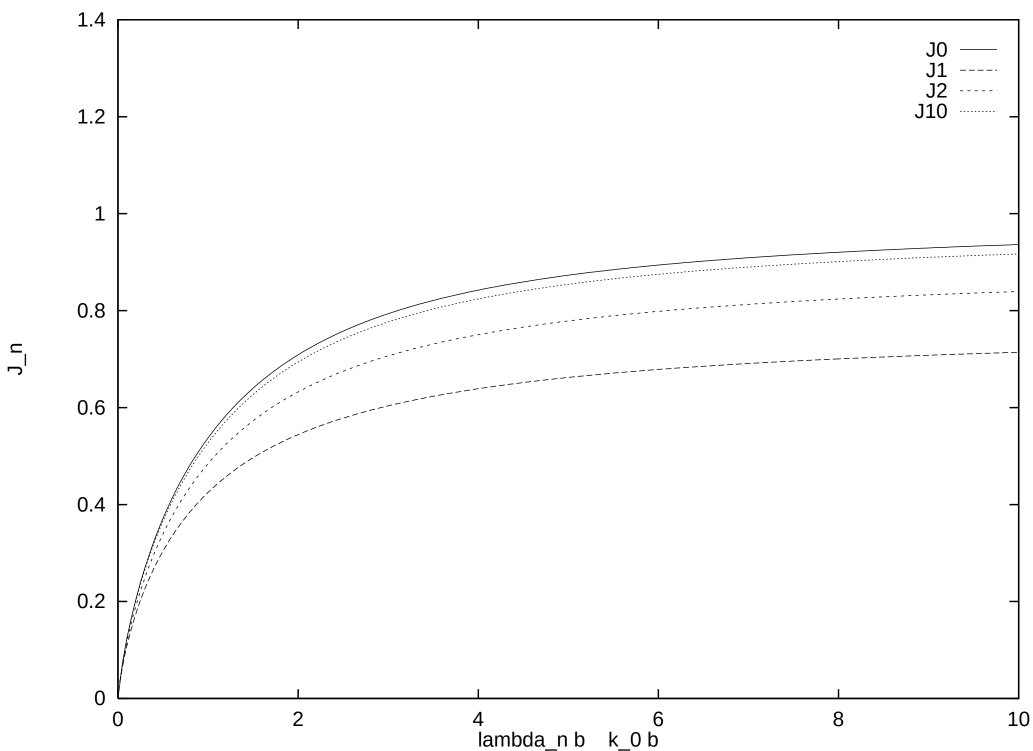

Figure 2: Functions J_1, J_2, J_{10} versus $\lambda_n\, b$ and J_0 versus $k_0\, b$.