AN ALTERNATIVE METHOD FOR LINEAR HYDRODYNAMICS OF AIR CUSHION SUPPORTED FLOATING BODIES

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Introduction

The presence of the air cushions under the floating body can significantly change the overall dynamic behaviour of the body [1]. The idealisation of the problem as a rigid body case is not correct and the effects of the air cushions should be taken into account properly. This is the purpose of the present note. We consider first the hydrodynamic part of the problem which will be solved under the usual assumptions of the linear potential theory. The construction of the boundary value problem for the potential Φ is very similar to the classical problem of the rigid body, except for the condition on the part of the water surface under the air cushion. So we first concentrate on the detailed derivation of this boundary condition.

Boundary condition on the free surface under the air-cushion

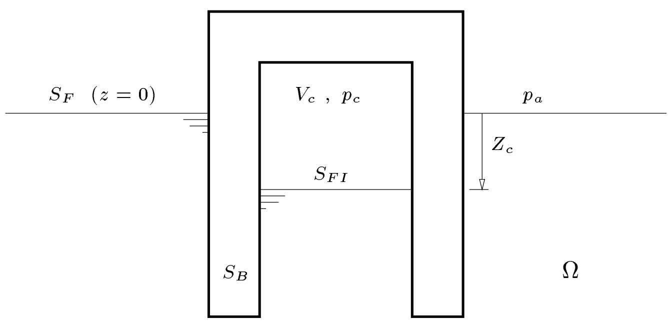

Figure 1: Basic configuration

This boundary condition is derived in the similar way as for the "real" free surface. It is a combination of the following dynamic and kinematic conditions:

$$p_c = -\varrho g(Z_c + \Xi) - \frac{\partial \Phi}{\partial t} \qquad , \qquad \frac{\partial \Xi}{\partial t} = \frac{\partial \Phi}{\partial z} \tag{1}$$

where p_c is the pressure in the air cushion, Z_c is the mean position of the free surface under the air cushion and Ξ is its elevation relative to Z_c.
By taking the time derivative of the first expression in (1) we combine two conditions in one:

$$\frac{\partial^2 \Phi}{\partial t^2} + g\frac{\partial \Phi}{\partial z} + \frac{1}{\varrho}\frac{\partial p_c}{\partial t} = 0 \tag{2}$$

By assuming the adiabatic variation of the pressure inside the air cushion [1] we can write:

$$\frac{\Delta p}{p_{cs}} = -\kappa \frac{\Delta V}{V_{cs}} \tag{3}$$

where p_{cs} is the static pressure in the air cushion $p_{cs} = -\varrho g Z_c + p_a$ (p_a being the atmospheric pressure), Δp is the variation of the pressure inside the air cushion $\Delta p = p_c - p_{cs}$, V_{cs} is the mean volume of the air cushion, ΔV is the volume change and κ is the adiabatic constant ($\kappa = 1.4$ for the air).

Now we can rewrite the equation (2) in the following form:

$$\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial z} - \frac{\kappa p_{cs}}{\varrho V_{cs}}\frac{\partial\Delta V}{\partial t} = 0 \tag{4}$$

Furthermore it is possible to relate the volume change ΔV to the body motions and the surface elevation:

$$\Delta V = \iint_{S_{FI}} w dS \tag{5}$$

where w is the relative free surface elevation:

$$w = \zeta_v - \Xi \tag{6}$$

with ζ_v denoting the vertical displacement of the points fixed to the body and positioned on S_{FI} for $t = 0$.
The interior free surface condition becomes finally:

$$\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial z} - \frac{\kappa p_{cs}}{\varrho V_{cs}}\iint_{S_{FI}} \frac{\partial w}{\partial t} dS = 0 \tag{7}$$

The frequency domain equivalent of this equation is:

$$-\nu\varphi + \frac{\partial\varphi}{\partial z} - \alpha\iint_{S_{FI}} w dS = 0 \tag{8}$$

where $\nu = \omega^2/g$ and:

$$\alpha = -\frac{i\omega\kappa p_{cs}}{\varrho g V_{cs}} \tag{9}$$

Potential decomposition and solution methodology

As in the rigid body dynamics we assume now the following decomposition for the unknown potential φ:

$$\varphi = \varphi_I + \varphi_D - i\omega\sum_{j=1}^{6}\xi_j\varphi_{Rj} \tag{10}$$

It is easy to deduce the boundary conditions on the interior free surface and on the body for each of these potentials:
Diffraction

$$\left\{-\nu\varphi_D + \frac{\partial\varphi_D}{\partial z} = \alpha\iint_{S_{FI}} w_D dS + \nu\varphi_I - \frac{\partial\varphi_I}{\partial z}\right\}_{S_{FI}} \tag{11}$$

$$\left\{\frac{\partial\varphi_D}{\partial n} = -\frac{\partial\varphi_I}{\partial n}\right\}_{S_B} \tag{12}$$

Radiation

$$\left\{-\nu\varphi_{Rj} + \frac{\partial\varphi_{Rj}}{\partial z} = \alpha\iint_{S_{FI}} w_{Rj} dS\right\}_{S_{FI}} \tag{13}$$

$$\left\{\frac{\partial\varphi_{Rj}}{\partial n} = n_j\right\}_{S_B} \tag{14}$$

The boundary value problems are completed by the usual free surface condition on the exterior free surface and the Sommerfeld radiation condition at infinity.
As we can see all BVP-s looks similar. The main difficulty lies in the fact that the relative wave elevation w is not known in advance but depends on the solution of the BVP itself. For the sake of clarity let us

now consider the generic boundary value problem of the form:

$$
\left.\begin{array}{ll}
\Delta\psi = 0 & in \;\; \Omega \\
-\nu\psi + \dfrac{\partial\psi}{\partial z} = 0 & on \;\; S_F \\
-\nu\psi + \dfrac{\partial\psi}{\partial z} = \alpha \displaystyle\iint_{S_{FI}} w dS + C & on \;\; S_{FI} \\
\dfrac{\partial\psi}{\partial n} = v & on \;\; S_B \\
\lim[\sqrt{\nu R}(\dfrac{\partial\psi}{\partial R} - i\nu\psi)] = 0 & R \to \infty
\end{array}\right\} \tag{15}
$$

It is eventually possible (source method plus some "tricks") to solve this BVP directly but the most simple way is to develop w in a form of the series of known basis functions with unknown coefficients. We write:

$$
w = \sum_{i=1}^{N_I} w_i f_i(x,y) \tag{16}
$$

where f_i are the known basis functions and w_i are the unknown coefficients.
The total potential ψ is now decomposed as follows:

$$
\psi = \psi_0 + \alpha \sum_{i=1}^{N_I} w_i \psi_i \tag{17}
$$

The boundary value problems for each of the potentials ψ_i , $i = 0, N_I$ become:

$$
\left.\begin{array}{lll}
\Delta\psi_0 = 0 & \Delta\psi_i = 0 & in \;\; \Omega \\
-\nu\psi_0 + \dfrac{\partial\psi_0}{\partial z} = 0 & -\nu\psi_i + \dfrac{\partial\psi_i}{\partial z} = 0 & on \;\; S_F \\
-\nu\psi_0 + \dfrac{\partial\psi_0}{\partial z} = C & -\nu\psi_i + \dfrac{\partial\psi_i}{\partial z} = \displaystyle\iint_{S_{FI}} f_i dS & on \;\; S_{FI} \\
\dfrac{\partial\psi_0}{\partial n} = v & \dfrac{\partial\psi_i}{\partial n} = 0 & on \;\; S_B \\
\lim[\sqrt{\nu R}(\dfrac{\partial\psi_0}{\partial R} - i\nu\psi_0)] = 0 & \lim[\sqrt{\nu R}(\dfrac{\partial\psi_i}{\partial R} - i\nu\psi_i)] = 0 & R \to \infty
\end{array}\right\} \tag{18}
$$

As we can see the above BVP-s belong to the familly of the classical linear diffraction-radiation problems. The only difference is the condition on the interior free surface but this doesn't influence the methodology of numerical solution so that the well known boundary integral equation method can be used in its most simple form as in [2].
The last remark concerns the choice of the basis functions $f_i(x,y)$. In principle any complete set of basis functions can be used, but the most simple choice are the stepwise functions equal to 1 on one panel and 0 on the others, as used in [1]. The numerical solution becomes then quite straightforward, and the unknown coefficients w_i are found from:

$$
w_i - \alpha \sum_{j=1}^{N_I} (\nu\psi_j^i + S_j) w_j = \nu\psi_0^i + C^i \quad , \quad i = 1, N_I \tag{19}
$$

where S_j is the surface of the jth panel, and superscript "^{i}" means that the value at centroid of the ith panel should be taken.

Comments on the Pinkster's method

The present paper was mainly initiated by the work of Pinkster [1] who treated the same problem using the different methodology. This author use the following decomposition for the potential:

$$
\varphi = -i\omega\{\varphi_I + \varphi_D + \sum_{j=1}^{6} \xi_j \varphi_{Rj} + \sum_{c=1}^{C} \xi_c \varphi_c\} \tag{20}
$$

where ξ_c and φ_c are roughly equivalent to w_i and ψ_i in the present notations (both for the diffraction and radiation). From [1] it can be shown that on the surface S_{FI} the potential φ satisfies the following condition:

$$\frac{\partial \varphi}{\partial z} = \alpha \iint_{S_{FI}} w dS \tag{21}$$

which represents only the kinematic condition and is different from (8).
In fact it seems that the method used in [1] treats the problem as a multibody interaction problem where the panels on S_{FI} are allowed to perform only the rigid body vertical motions. In this way, after satisfying the kinematic condition (21) on the interface, the dynamic condition is satisfied by solving the "motion equations" for the multibody system composed of the rigid body and the massless panels on the interface (Eqn. (10) or (19) in [1]). Thus, at the end, the total potential satisfy the same condition on the interface, so that the two methods are equivalent.

Numerical results

We show now some preliminary results for two limiting cases $\alpha = 0$ and $\alpha = \infty$. The body is the rectangular box with the length of $150m$, width of $20m$, draft of $10m$ and the "wall thickness" of $4m$. On the figure 2 the results for the heave added mass A_{33} and damping B_{33} coefficients are presented. We can appreciate the important differences between two classes of results, which means that, in the general case ($0 < \alpha < \infty$), the influence of the air cushion must be evaluated correctly in order to have an representative model of the body dynamics.

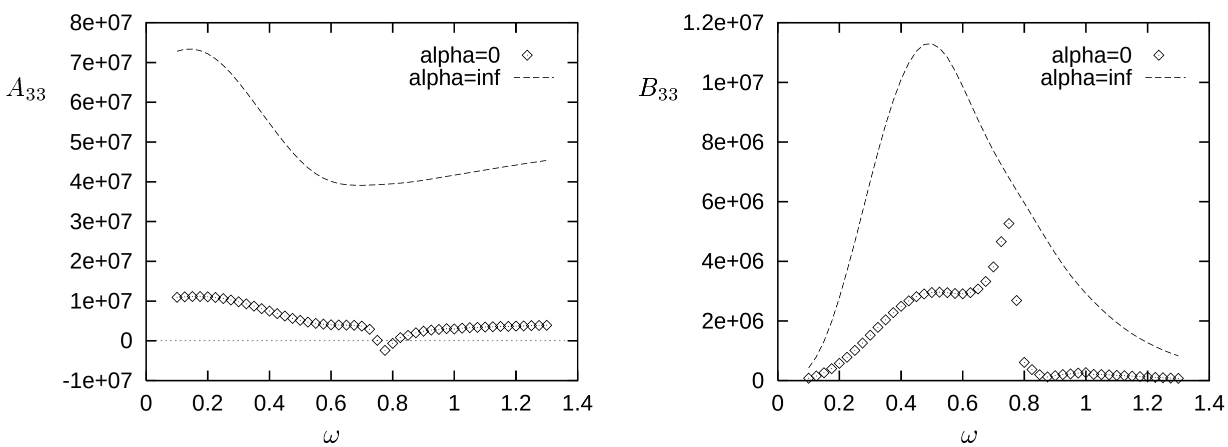

Figure 2: Heave added mass and damping coefficients for rectangular barge.

Conclusion

We presented here an alternative method for solving the linear hydrodynamic problem for air-cushion supported stationnary vessels. In contrast to the original method presented in [1], this method seems to be more direct because the boundary value problems for the potentials include both kinematic and dynamic boundary conditions on the interior free surface. In this way the potential decomposition (10) becomes quite natural (as in the rigid body case) and the methodology for determining the body motions straightforward.

References

[1] PINKSTER J.A., 1997. : "The effect of air cushions under floating offshore structures", BOSS'97, Delft, Netherlands.

[2] CHEN X.B., MALENICA Š. & PETITJEAN F. 1995. : "Offshore hydrodynamics.", Bulletin Technique de Bureau Veritas. Vol.1, pp.47-66.