ON THE BEHAVIOUR OF STEEP SHORT-CRESTED WAVES IN DEEP WATER AND THEIR EFFECTS ON STRUCTURES

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Most of the studies on the kinematics and dynamics of surface waves were focused for years on two-dimensional wave fields and not many dealt with three-dimensional wave fields. The so-called short-crested waves due to the nonlinear interaction of two similar uniform wave trains coming from two different directions are one of the simplest genuinely three-dimensional waves of permanent form doubly-periodic in two directions of the horizontal plane. These short-crested waves may occur in a number of important maritime situations. Swell being fully reflected off a vertical sea-wall or jetty results in a short-crested wave field being found adjacent to the reflecting wall. Waves propagating down a vertical-walled channel can assume a short-crested wave form when there is a cross-channel variation of the flow pattern. These waves may also occur when a wave train is diffracted behind an obstacle of finite width. These waves are known to have very steep pyramidal shape that may cause serious damage to vessels or off-shore structures.

The study deals with short-crested waves due to two plane waves intersecting at an angle μ, or by a reflection of a progressive plane wave on a vertical sea-wall at an angle of incidence $\theta = (\pi - \mu)/2$. The derived short-crested waves are progressive in one direction and standing in the perpendicular direction. The values $\theta = \pi/2$ and $\theta = 0$ are, respectively, the Stokes wave and the standing wave (the two-dimensional limits). One considers surface gravity waves on an inviscid incompressible fluid of infinite depth. The flow is assumed irrotational. To put all the equations into non-dimensional form, one scales all the variables with respect to the reference length $1/k$ and the reference time $(gk)^{-1/2}$, where k is the wavenumber of the incident wave and g is the acceleration due to gravity. The governing equations are

$$\nabla^2\phi = 0 \quad \text{for} \quad -\infty \le z \le \eta(x,y,t) \tag{1}$$

$$\nabla\phi \to 0, \quad \text{as} \quad z \to \infty \tag{2}$$

$$\eta + \phi_t + \frac{1}{2} \mid \nabla\phi^2 \mid \; = C \quad \text{on} \quad z = \eta(x,y,t) \tag{3}$$

$$\eta_t + \phi_x\eta_x + \phi_y\eta_y - \phi_z = 0 \quad \text{on} \quad z = \eta(x,y,t) \tag{4}$$

$$\phi_y = 0 \quad \text{on} \quad y = 0 \tag{5}$$

where $\phi(x,y,z,t)$ is the velocity potential, $z = \eta(x,y,t)$ is the equation of the free surface, (x,y) are the horizontal coordinates, z is the vertical coordinate and C is a constant. The wave is assumed to propagate in the x-direction without change of shape.

The above problem admits steady short-crested waves. To prove rigorously the existence of short-crested waves is a difficult task, and so far only capillary- gravity waves with high enough surface tension have been shown to exist. However, there is numerical evidence that short-crested waves exist (see, for exemple, the numerical result of Roberts, (1983)). Fuchs (1952), Chappelear (1961), Hsu et al (1979) and Menasce (1994) used perturbation expansions to compute from formal point of view the small-amplitude three-dimensional waves. Ioualalen (1993) and Kimmoun et al.

(1999) extended the computations to higher order. In the frame of reference (X, Y, Z, t) moving with the wave, the surface elevation and the velocity potential are of the form

$$\left\{\begin{array}{lcl}\eta & = & \sum_{i=1}^{\infty} h^i \sum_{m,n} a_{imn} cos(mX) cos(nY) \\ \phi & = & \sum_{i=1}^{\infty} h^i \sum_{m,n} b_{imn} sin(mX) cos(nY) e^{(\alpha_{mn} Z)}\end{array}\right. \tag{6}$$

$$\text{where } \alpha_{mn}^2 = m^2 (sin\theta)^2 + n^2 (cos\theta)^2 \tag{7}$$

The wave steepness of the wave is defined by $h = (\eta(0,0) - \eta(\pi, 0))/2$. The coefficients a_{imn} and b_{imn} are computed up to 27th order.

To study the stability of the short-crested waves, let the perturbative motion be

$$\left\{\begin{array}{lcl}\eta' & = & e^{-\sigma t} e^{i(pX+qY)} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} a_{jk} e^{i(j sin\theta X + k cos\theta Y)} \\ \phi' & = & e^{-\sigma t} e^{i(pX+qY)} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} b_{jk} e^{i(j sin\theta X + k cos\theta Y)} e^{\kappa_{jk} Z}\end{array}\right. \tag{8}$$

$$\text{where } \kappa_{jk}^2 = (p + j sin\theta)^2 + (q + k cos\theta)^2 \tag{9}$$

The infinitesimal disturbances are harmonic perturbations modulated by wave number p and q in the two horizontal directions. Up to $h = 0.30$, Ioualalen & Kharif (1994) have shown that the short-crested waves admit characteristic stability regimes for (i) near standing waves (θ around 0), (ii) fully three-dimensional waves (θ around $\pi/4$) and (iii) near Stokes waves (θ around $\pi/2$). For values of h greater than 0.30 it is found (i) for near standing waves that the dominant instability is a sideband-type instability in the direction of propagation (class Ia), (ii) for fully three-dimensional waves that the dominant instability is a sideband-type instability in oblique direction (class Ib), and (iii) for near Stokes waves that the dominant instability is a *horseshoe-patterned* instability (class IIa). For fully three-dimensional waves, it is interesting to note that instabilities of class IIb become dominant above $h = 0.63$. So it is concluded that the dominant resonant interactions of short-crested waves are four-wave instabilities (class I) exept for steep near Stokes waves, and very steep fully three-dimensional waves for which five-wave processes are predominant (class II).

The force exerted by waves on vertical walls is an important design criterion. Fenton (1985) examined the uniform short-crested wave problem to third order and obtained expressions for the depth integrated wave force on the wall and the depth integrated wave moment about the basis of the wall. Later Marchant & Roberts (1987) via a computer-generated perturbation expansion investigated the uniform short-crested wave to 35th order in wave steepness and extended Fenton's work to oblique incident wave in shallow water. Here the problem is to consider wave forces exerted by more realistic wave patterns such as modulated short-crested. The effect of the modulation for different angle θ and waveheight h is analyzed and results are compared with Fenton and Marchant & Roberts work's.

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