LIQUID FLOW CLOSE TO INTERSECTION POINT

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1 Introduction

The plane unsteady problem of water impact is considered with focusing the attention on the flow near the intersection point between a liquid free surface and a rigid contour. The correct description of the flow field in this region is highly expensive in spite of its small influence on the total hydrodynamic load. On the other hand, in numerical study of the slamming problem accurate treatment of jet flow can improve numerical algorithms in use and increase accuracy of the numerical solutions.

We stay within the non-linear potential theory of ideal incompressible liquid flow generated by a floating wedge impact. The liquid initially occupies a lower half-plane ($y < 0$) and is at rest. Initial draft of the wedge is h_0 and the deadrise angle of the wedge is γ. The parts of the liquid boundary $y = 0$, $x < -x_c$ and $x > x_c$, where $x_c = h_0 \cot\gamma$, correspond to the initial position of the liquid free surface. At some instant of time which is taken as the initial one, the wedge begins to move down at a constant velocity V. We shall determine the liquid flow, position of the free surface and the pressure distribution at each instant of time $t > 0$.

The numerical method proposed by Longuet-Higgins and Cokelet for free-surface flows with non-linear boundary conditions is employed. The free surface position is updated at each time step with the help of numerical solution of the corresponding mixed boundary-value problem. This solution may be singular at the intersection points, where the type of the boundary condition changes, and which are usually corner points of the flow domain. The singularities an the intersection points influence the numerical solution at all subsequent time steps, and thus can have disastrous cumulative effects.

In order to avoid the difficulties with the time-stepping numerical method, it is suggested to distinguish small vicinities of the intersection points at each time step and to build there approximate analytical solutions matching them with the numerical solution in the main flow region (section 2). The numerical solution is obtained by the boundary-element method. Initial conditions for the numerical solution are obtained by the method of matched asymptotic expansions in section 3.

Several models have been suggested in the past, which are based on the idea to cut off the jet and to replace it with a suitable boundary condition to be applied on the jet cut. Zhao & Faltinsen [1] suggested to cut the jet there, where the angle between the tangential to the free surface and that to the body contour drops below a small given value. The flow in the jet region is not considered. The cut is orthogonal to the body contour and a linear variation of the velocity potential along the cut is assumed. The normal derivative of the velocity potential obtained by solving the discretized boundary integral equation is used to move the cut. A slightly different approach has been suggested by Fontaine & Cointe [2]. Within this approach it is suggested to cut off the jet there, where the jet thickness becomes smaller than a given limiting value. The normal velocity on the jet cut is assumed to be equal to the tangential velocity on the body contour. Both these models have been found to work well and in good agreement with similarity solutions [3]. It is expected that both models are approximately equivalent to each other for small deadrise angle, which follows from the asymptotic analysis of the wedge-entry problem. On the other hand, both models are not easy to justify for moderate and large deadrise angles, where the "cut-off" technique is also attractive to use. It should be noted that the reliabilities of these models are highly dependent on the limiting values for the jet angle or the jet thickness. The values cannot be too small and, generally speaking, cannot be arbitrary. The best choice of these values is up to the experience of a researcher.

The aim of this study is to develop a more physical way to decompose the flow region without forcing neither the jet angle or the jet thickness. In this paper the velocity potential in the jet region is expressed as a suitable expansion around the intersection point. The coefficients of the expansion are evaluated from the numerical solution of the boundary-value problem, enforcing a matching with the outer solution.

2 Numerical simulation of the water entry

The flow about a symmetric wedge plunging the water surface is numerically studied in the frame of the potential flow assumption. A boundary integral formulation is employed to solve the Laplace's equation in terms of the velocity potential. On the body contour the normal derivative of the velocity potential is assigned while on the free surface the velocity potential is updated according to the unsteady Bernoulli's equation.

The main issue concerns the treatment of the intersection between the free surface and the body contour. In order to avoid an excessive computational effort, the shape of the flow region close to the intersection point is approximated by a wedge and the velocity potential is expressed there as follows:

$$\phi = V \cos\gamma \, r \left[\sin\theta - \tan\beta \cos\theta\right] + \sum_{k=0,k|\cos k\beta \neq 0}^{N} a_k r^k \frac{\cos k\theta}{\cos k\beta} + \sum_{k=0}^{M} c_k r^{\sigma_k} \cos(\sigma_k \theta) \quad , \tag{1}$$

where

$$\sigma_k = \frac{\pi}{2\beta} + k\frac{\pi}{\beta} \quad .$$

Here β is the angle between the body contour and the free surface at the intersection point and r, θ are polar coordinates with origin at this point (Fig. 1).

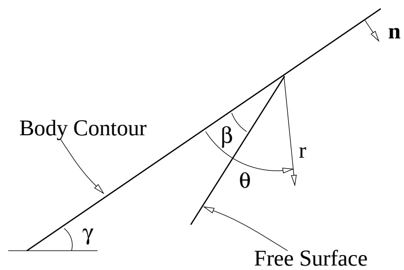

Fig.1 *Scketch of the jet region*

The first term in equation (1) is to satisfy the boundary condition on the body contour, the second term to satisfy the boundary condition on the free surface side and the last term represents the eigen solutions of the Laplace's equation in the liquid-wedge region. This term is of major importance for $\beta > \pi/2$ leading in this case to a singular velocity field at the intersection. For small values of β the eigen-solution part can be neglected in a low-order approximation. The coefficients in expansion (1) are computed directly by solving the boundary-value problem and enforcing the matching between the 'inner' expansion (1) and the 'outer' numerical solution along the matching line. The coefficients c_k are determined by introducing $M + 1$ panels on the matching curve while the coefficients a_k can be recovered either by extrapolation from the computational domain, where the dynamic boundary condition is satisfied, or by introducing additional $N + 1$ panels on the matching curve.

In the case of constant entry velocity, the behaviour of the angle β as function of the wedge deadrise angle was recovered. Besides to the case of constant entry velocity, also the free fall impact is analysed. In this case the unsteady contribution to the pressure field on the body contour is computed by solving another boundary-value problem in terms of the time derivative of the velocity potential. The hydrodynamic load is then used as a forcing term for the dynamic equation of the body motion that is integrated in time to provide the actual entry velocity.

3 Small time analysis

Besides the treatment of the jet region, one problem that causes troubles is the initial transient. Numerical approaches usually start from the undisturbed free surface configuration with the body partially submerged. In this condition a singular velocity field occurs at the intersection and the numerical treatment of this singularity is not straightforward. Due to the velocity singularity at the intersection, the solution that can be obtained up to the time at which the jet develops is not reliable. This unphysical behaviour of the initial transient may affect the solution in the case of free fall impact when a correct estimate of the hydrodynamic load is needed to accurately compute the dynamics of the impacting body [4]. For this reason a small-time analysis of the flow when the body impulsively starts to move down is done. It was shown that, at the leading order as $t \to 0$, the flow close to the contact point is self-similar. The solution of this problem is helpful to derive the initial conditions for the following numerical simulation.

The 'outer' solution of the floating wedge impact problem, which is valid outside the intersection point vicinities, was obtained by Sedov [5] in the form

$$\phi + i\psi = iV(z - \frac{l}{a}\sqrt{\tau^2 - 1}),$$

where $\psi(x, y)$ is the stream function, $z = x + iy$, $l = h_0/\sin\gamma$, τ is the complex variable connected to z by the formula

$$z = \frac{l}{a} e^{i\gamma} \int_0^\tau \left(\frac{\tau_0^2}{1 - \tau_0^2} \right)^{\frac{\gamma}{\pi}} d\tau_0 - ih_0, \qquad a = \frac{1}{\sqrt{\pi}} \Gamma(\frac{1}{2} + \frac{\gamma}{\pi}) \Gamma(1 - \frac{\gamma}{\pi}).$$

Asymptotic behaviour of the complex potential near the right-hand side intersection point $z = x_c(0) = h_0 \cot\gamma$ is

$$\phi + i\psi \approx -iVf(\gamma) h_0^{\gamma - \frac{\gamma}{\pi}} (z - x_c)^{\frac{\pi}{2(\pi - \gamma)}} + iVx_c,$$

$$f(\gamma) = \left(\frac{2\beta}{\pi} \right)^{\sigma_0} (a \sin\gamma)^{\sigma_0 - 1} \quad .$$

As stated above, a singularity of the velocity field at the contact point occurs as it can be recognized from the leading term of the Sedov's solution

$$\phi \simeq A r^{\sigma_0} \cos\sigma_0\theta \ \text{ as } \ r \to 0 \ , \tag{2}$$

where $\sigma_0 = \pi/(2\beta)$ with $\beta = \pi - \gamma$. The constant A can be readily evaluated from the 'outer' Sedov's solution. Equation (2) can also be recovered from expansion (1).

To perform the small-time analysis, local stretched coordinates λ, μ and the 'inner' velocity potential $\varphi(\lambda, \mu, t)$ are introduced as follows

$$x = x_c + a(t)\lambda \quad , \quad y = a(t)\mu \quad , \qquad \phi = a^\nu(t)\varphi(\lambda, \mu, t)$$

with $a(t) \to 0$ as $t \to 0$. Using the methods of asymptotic analysis and the matching conditions, we obtain

$$a(t) = [(2 - \sigma_0)t]^{\frac{1}{2 - \sigma_0}}, \qquad \nu = \sigma_0.$$

Matching the behaviour of the velocity potential in the 'outer' region expressed by equation (2) with that in the 'inner' region, gives the condition at infinity for the 'inner' velocity potential

$$\varphi(\lambda, \mu, t) \simeq A\varrho^{\sigma_0} \cos\sigma_0\theta \ \text{ as } \ \rho \to \infty \ , \tag{3}$$

with $\rho = \sqrt{\lambda^2 + \mu^2}$.

In terms of the new stretched variables the dynamic and kinematic boundary conditions on the free surface take the forms

$$\sigma_0\varphi + \frac{1}{2}|\nabla\varphi|^2 = (\lambda\varphi_\lambda + \mu\varphi_\mu) - a^{2-\sigma_0}\varphi_t \ , \tag{4}$$

$$\nabla\eta \cdot \nabla\varphi = (\lambda\eta_\lambda + \mu\eta_\mu) - a^{2-\sigma_0}\eta_t \ , \tag{5}$$

where the equation $\eta(\lambda, \mu, t) = 0$ describes the free surface shape. Since $2 - \sigma_0 > 0$, the shape of the free surface and the velocity potential do not depend on time in the leading order as $t \to 0$, which means that the flow close to the contact point is approximately self-similar.

In terms of the stretched velocity potential the boundary condition on the body surface is

$$\varphi_\mu = \varphi_\lambda \tan\gamma - Va^{1-\sigma_0}$$

and, as $t \to 0$, the last term can be neglected leading to the following boundary condition on the body:

$$\frac{\partial\varphi}{\partial n} = 0 \ . \tag{6}$$

Equations (4) and (5) indicate that it is convenient to introduce new unknown function $S(\lambda, \mu)$ instead of the velocity potential

$$S(\lambda, \mu) = \varphi(\lambda, \mu, t) - \frac{1}{2}(\lambda^2 + \mu^2),$$

with the help of which the 'inner' problem is reduced to the boundary-value problem for the Poisson's equation

$$\Delta S = -2 \qquad \text{in the flow region,}$$

$$\frac{\partial S}{\partial n} = 0 \qquad \text{on the boundary of the flow region,}$$

$$\left(\frac{\partial S}{\partial \tau} \right)^2 + 2\sigma_0 S = (1 - \sigma_0)\rho^2 \qquad \text{on the free surface,}$$

$$S \simeq -\frac{1}{2}\rho^2 + A\varrho^{\sigma_0}\cos\sigma_0\theta \quad \text{as} \quad \rho \to \infty \quad ,$$

where $\partial S/\partial \tau$ is the tangential derivative of the function $S(\lambda, \mu)$ along the free surface (Fig. 2). It is important to note that the dynamic condition in this formulation can be exactly integrated leading to the direct relation between the value of the function S on the free surface and the free surface shape. The problem for the Poisson's equation is solved by iterations dealing with the corresponding mixed boundary-value problems at each step. The technique outlined in section 2 can be employed to improve the convergence.

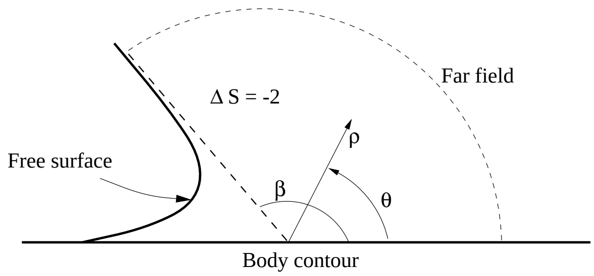

Fig.2 *Scketch of the inner region for the small time analysis*

Acknowledgments

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