NON-LINEAR MOTION OF A SUBMERGED BODY IN WAVES

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Background and Introduction

In this paper the time-domain solution of wave diffraction and the unsteady free motion of a body in waves are addressed. The 'shell-function' method of solution, described in Hamilton & Yeung [3], is extended to handle the diffraction problem and the exact kinematic boundary condition on the body under wave-excited motion. The simplification of a linear free surface condition is retained. The radiation problem with prescribed body motion was solved in [3], and as noted therein, the method has excellent computational efficiency and the shell coefficients constitute a perfect radiation boundary. The essential features of this solution method are briefly reviewed below.

The shell function method is a hybrid integral equation method which combines a Rankine source integral equation formulation with a time-dependent free-surface Green-function formulation. As is well known, a Rankine singularity as the source function yields integrals over all boundaries of the fluid domain. The resulting numerical method requires discretization of the entire area of the disturbed free surface in addition to discretization of the body geometry. The major advantage of this formulation is the low computational requirements of such a source function, making the inclusion of nonlinear effects more accessible. Alternatively, a more sophisticated choice of Green function eliminates the need to discretize any surfaces except that of the body. Typically, this function satisfies the linearized free-surface condition and an appropriate far-field condition. The computational effort required to evaluate this function is high, making the treatment of nonlinear body boundary conditions rather impractical.

The shell function method combines these two integral equation formulations in a way that retains the advantages of both. The fluid domain is split into two regions separated by an imaginary surface surrounding the body, labeled the shell surface. The simple source method is used in the resulting interior region and the time-dependent Green-function formulation is used to represent the exterior problem. Matching the two solutions on the shell surface provides a complete solution for the flow exterior to the body. Additionally, choosing an axisymmetric shell surface geometry and a constant time-step size reduces the number of unique integrals of the unsteady Green function to a number that can be readily pre-computed and stored. An axisymmetric shell imposes no restriction on the body geometry.

If the body is surface piercing, the shell surface separating the interior and exterior regions will be surface piercing and the free-surface boundary condition will need to be advanced by numerical integration. If a non-linear free-surface condition is desired, a mixed Eularian-Lagrangian method can advance the position of the free surface. The non-linear solution of the free-surface solution in the interior region will also have to be matched to the linear solution in the exterior region. This is sensible in three-dimensions as the height of wave disturbances will diminish as they travel away from the body. To date, the three dimensional shell method has only been implemented for bodies and shells that do not pierce the surface but the surface-piercing case is not a difficult hurdle to overcome (see Figure 1).

Formulation and Numerical Procedure

Considered here are the effects of an incident wave on a submerged body which responds freely to the wave excitation. Figure 1 illustrates the problem geometry and indicates the separation of the two fluid domains by the imaginary shell surface. In each domain the velocity potential must satisfy the usual boundary conditions as well as a matching condition on the shell surface. Applying Green's theorem to the interior region with a simple source Green function provides:

$$C(P)\phi(P,t) + \int_{B_o \cup F_o \cup S} \phi(Q,t) \left(\frac{1}{r}\right)_\nu dS_Q = \int_{B_o \cup F_o \cup S} \phi_\nu(Q,t) \left(\frac{1}{r}\right) dS_Q \qquad P \in S \tag{1}$$

In the discrete form of a typical panel method, this implies:

$$A_{nm}\phi_m = B_n \tag{2}$$

where,

$$A_{nm} = C(P_n)\delta_{nm} + \int_{S_m} \left(\frac{1}{r}\right)_\nu dS_Q \qquad B_n = \sum_{m=1}^{M} (\phi_m)_\nu \int_{S_m} \left(\frac{1}{r}\right) dS_Q \tag{3}$$

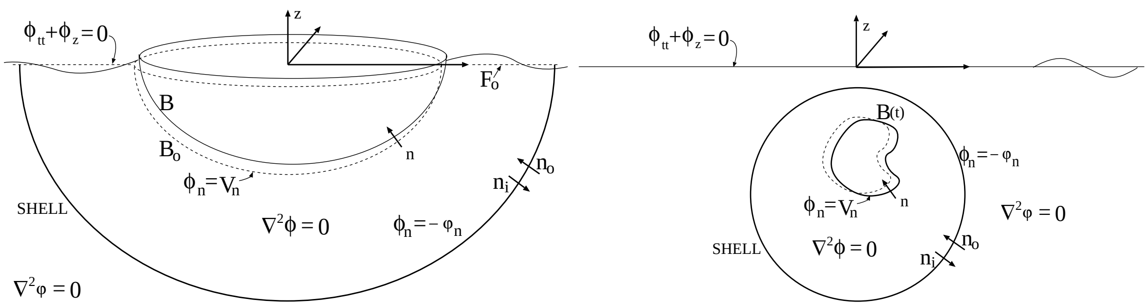

Figure 1: Schematics of the surface piercing shell method and the fully submerged shell method.

with m being the index associated with the surface unknown, and n the point at which the integral equation is satisfied. For this source formulation, ϕ_ν is provided by the boundary conditions of the domain, equal to the normal velocity of the body on the body surface, and provided by the shell coefficients on the shell surface. A perfect shell would represent the appropriate radiation and memory effects associated with wave motion.

Application of Green's theorem in the exterior region to the usual time-dependent Green function G [2] and φ_τ provides:

$$\begin{aligned} -C(P)\varphi(P,t) &= \int_S [G_\nu(P,Q,0)\varphi(Q,t) - G\varphi_\nu]\, dS_Q + \\ & \int_0^t d\tau \int_S [\varphi_\nu(Q,\tau)H_\tau(P,Q,t-\tau) - \varphi H_{\nu\tau}]\, dS_Q \end{aligned} \tag{4}$$

Discretization of this integral equation provides a linear relationship between φ and φ_ν on the shell surface (see [3] for details).

$$C_{nm}[\varphi(t_K)]_m + D_{nm}[\varphi_\nu(t_K)]_m = E_n \tag{5}$$

The matrices C_{nm}, D_{nm}, and the vector E_n are summations of integrals of the free-surface Green function. If an axisymmetric shell is selected, and if the time-step is chosen as constant, the number of distinct integrals (in C_{nm}, D_{nm}, E_n) that need to be calculated is greatly reduced. When these "shell coefficients" are pre-computed and stored, evaluations of the free-surface Green function during *(any subsequent)* time-domain simulations is eliminated. E_n is a convolution of the velocity potential with respect to past time, involving integrals of the unsteady Green function. This term contains the "memory effects" of the fluid problem.

Solving (5) for φ_ν and applying the matching conditions between the two domains provides the necessary terms to complete the formation of the right hand side of (2) for the shell surface. If a superposition of solutions in the exterior region is needed, then the matching conditions are modified to reflect this. For example, in our case of an incident wave potential being present (φ^i), the matching conditions are:

$$\phi(\mathbf{x},t) = \varphi(\mathbf{x},t) + \varphi^i(\mathbf{x},t) \qquad \frac{\partial\phi(\mathbf{x},t)}{\partial n_i} = -\frac{\partial\varphi(\mathbf{x},t)}{\partial n_o} - \frac{\partial\varphi^i(\mathbf{x},t)}{\partial n_o} \tag{6}$$

Thus, the linear equations for ϕ_ν provided by the exterior solution become:

$$[\phi_\nu]_m = -[D_{nm}]^{-1}E_n + [D_{nm}]^{-1}C_{nm}\left([\phi]_m - [\varphi^i]_m\right) + [\varphi^i_\nu]_m \tag{7}$$

This equation provides the necessary terms on the right hand side of (2) in terms of quantities that are either known or are being solved for. Time-stepping of the solution is thus possible. It is well known that the effects of incident waves can be incorporated by directly modifying the body boundary condition, but it is envisioned that such a manner of treating the incident wave potential will provide more effective extensions of the method described in the conclusion section.

For the free body motion problem, the equations of motion for the body must be solved in conjunction with the hydrodynamic problem. On the body surface, ϕ_ν on the right hand side of equation 2 is provided by the normal velocity of the body surface

$$\phi_\nu = V_n = \dot{\mathbf{x}} \cdot \mathbf{n} \tag{8}$$

For the radiation problem this term is known but is unknown for the free motion problem. Thus, allowing translational motion creates three additional unknowns on the left hand side of the linear system (2). Numerical integration of the equations of motion of the body provides the required three additional equations.

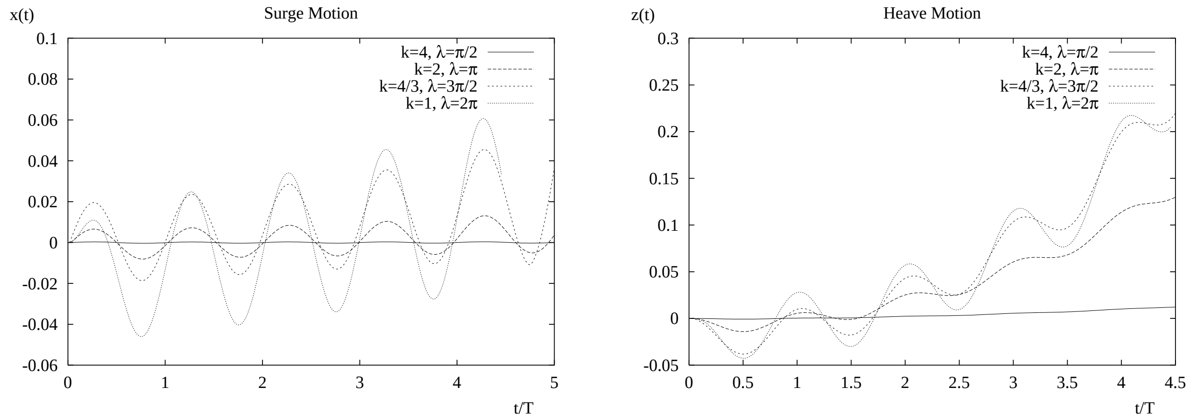

Figure 2: Submerged sphere in incident waves, motion relative to initial position, time is measured relative to incident wave period.

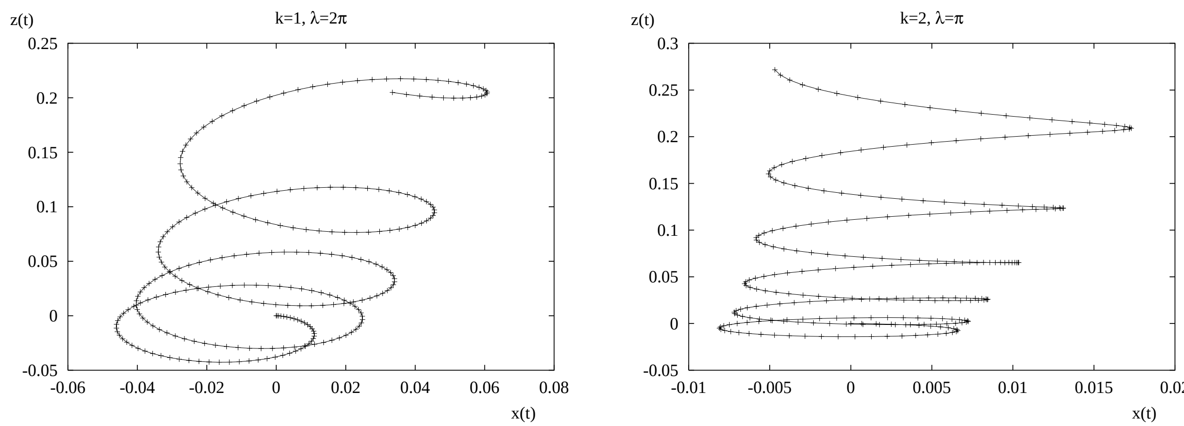

Figure 3: Path of motion for body free to move for k=1 and k=2.

This procedure is similar to that employed in two dimensions by Yeung [6]. An explicit Adams-Bashforth multi-step method is well suited to integrating the equations of motion because the of the shell method's constant time step restriction.

In this work, no rotational motion of the body is considered. The results below are for a sphere and so the pressure on the body is always directed towards the center of buoyancy, resulting in no angular moments.

Demonstrative Results

The method described is used to compute a variety of results for the motion of a sphere submerged in the presence of plane progressive waves. All quantities shown are nondimensionalized by the primary variables density $\bar{\rho}$, body radius $\bar{a}$, and gravity $\bar{g}$. The incident wave potential is comprised of simple plane progressive waves whose amplitude grows smoothly from zero to 0.15 in the first one quarter wave period. Transient effects of this are minimal. The test cases consist of a sphere submerged $1\frac{1}{2}$ radii below the free surface, this provides an initial vertical gap between the sphere and the calm water surface of $\frac{1}{2}$.

Two situations are considered, one in which the sphere is free to move in all directions, and one in which it is restrained to move only in surge. Figures 2 and 3 shows the surge and heave motions at different wave numbers. The amplitudes are proportional to the fluid motion at the depth of the sphere. i.e. The $k = 4$ case corresponds to shorter wavelengths and the body is largely below the $\lambda/2$ depth of wave effects. In each case, the body experiences drift in the direction of the waves but also in the vertical direction, towards the free-surface. As the body rises closer to the free surface, the amplitudes of motion increase. When the sphere has risen 0.3, it hits the submerged shell surface and the simulation has to be halted.

Figure 4 shows results for the same geometry but with the motion restricted to the surge direction. As expected, there is drift in the direction of the waves, however, the amplitude of the motion does not increase because the body is not free to rise to the free surface. Again the longer wavelengths produce larger motions and after a few periods of oscillations, the drift velocity has become constant. The vertical force on the body is also computed. As seen in figure 4, this force has a small positive average, causing the body to rise if not restrained.

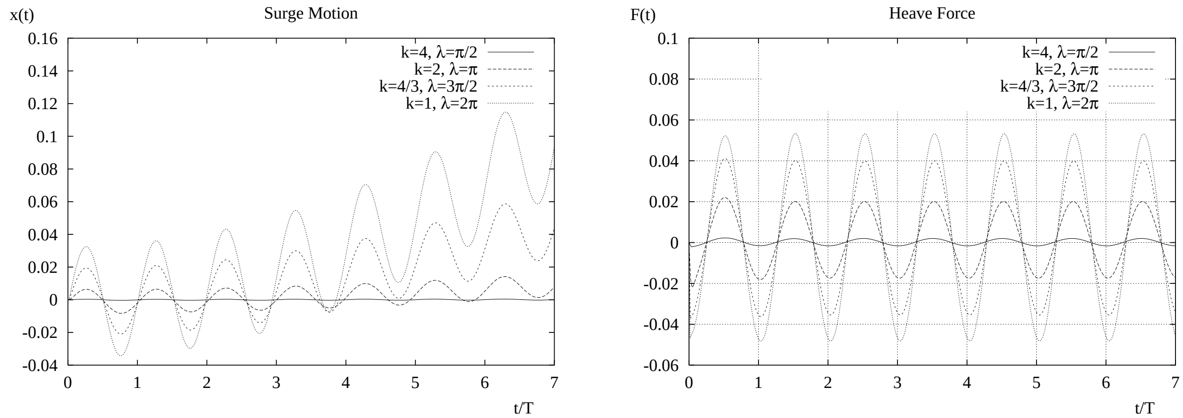

Figure 4: Submerged sphere restricted to surge motion.

Conclusion

Although results presented here are limited to the submerged case, the shell method can be implemented for a surface piercing shell. The shell function method provides a far-field boundary condition which limits the size of the region near the body for which the hydrodynamic problem needs to be solved. To date, a 'simple source' boundary integral method has been used for the interior region. Extension of the method to include a more nonlinear treatment of the interior region, such as that used by Scorpio and Beck [4], is a possibility. The inclusion of effects of viscosity by a Navier-Stokes solver such as [9] or a three-dimensional analog of the FSRVM [8] is also a viable alternative. The superior feature of this formulation is that no real-time computations of the unsteady free-surface Green function and its memory effects are needed during the simulations. All the runs presented here used the same file of shell coefficients and runs of 1000 time-steps can be completed within one day on a medium power workstation with no restrictions on the body geometry.

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