BEHAVIOR OF A SHIP WITH ELASTIC DISTORTIONS IN PERIODIC WAVES .

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1 Introduction

In this presentation we study the rigid body motion and the elastic behaviour of a ship in periodic waves. In an earlier paper on the motion of large elastic platforms it was found that it is possible to derive a formulation to describe the rigid body motion and the elastic behaviour by means of one differential-integral equation. This equation for the elevation can be solved numerically, without splitting the problem in the various rigid motion and eigenmode components, as is standard in the field of linear ship motions. One of the goals of this project is to apply similar ideas to an elastic ship.

In our case the ship is not a rigid structure and we focus on the shears, bending moments and distortion due to the waves excitation. We consider the ship as an elastic beam and we only study the time dependent distortions in the hull. It means that we don't take in account the strains due to gravity and buoyancy forces in still water. In this presentation we finaly restrict ourselves to the influence of head-seas on the heaving and pitching rigid-body motions. Furthermore, to obtain the bending motion we take an elastic beam as an elastic model for the ship.

We first derive a general formulation for a 3D ship, and then give some results for a ship in head waves for heave and pitch motion, and associated elastic distortion of the hull. In principle we can extend the formulation for the effect of waves on a ship travelling with steady forward speed.

2 Mathematical model

In this section we derive a formulation for a ship without forward speed. We assume the flow to be a potential flow and introduce the velocity potential $\mathbf{V} = \nabla\Phi(x,t)$ where $\mathbf{V}$ is the fluid velocity vector. We get for the potential $\Phi(x,t)$ the Laplace equation $\Delta\Phi = 0$ in the fluid domain Ω.

We have at the linearized free surface $z = 0$, the linearized kinematic condition $\Phi_z = \eta_t$, and the dynamic condition $\Phi_t = -g\eta$ where $\eta(\mathbf{x},t)$ denotes the wave elevation. On the hull, we have the following kinematic and dynamic conditions:

$$\frac{\partial\Phi}{\partial n} = v_n = \tilde{w}_t(x,t)\ \vec{e_z}\cdot\vec{n} + \tilde{\theta}_t(x,t)\,y\,\vec{e_z}\cdot\vec{n} - \tilde{\theta}_t(x,t)\ z\,\vec{e_y}\cdot\vec{n} \tag{1}$$

$$\frac{p}{\rho} = -\Phi_t - g\left(\tilde{w}(x,t) + \tilde{\theta}(x,t)\,y\right) \tag{2}$$

where v_n is the normal velocity of a point at the hull of the ship, $\tilde{w}(x,t)$ the deflection and $\tilde{\theta}(x,t)$ the angular rotation due to torsion respectively.

The ship is assumed to behave like a beam with no thickness. We use the linear beam theory to describe its deflection $\tilde{w}(x,t)$ and its angular rotation $\tilde{\theta}(x,t)$. Then we have the equations:

$$\mu(x)\frac{\partial^2\tilde{w}(x,t)}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(D(x)\frac{\partial^2\tilde{w}(x,t)}{\partial x^2}\right) = Z(x,t) \tag{3}$$

$$I_s\frac{\partial^2\tilde{\theta}(x,t)}{\partial^2 t} - \frac{\partial}{\partial x}\left(c\frac{\partial\tilde{\theta}(x,t)}{\partial x}\right) = \Gamma(x,t) \tag{4}$$

where D is the flexural rigidity, μ the mass per length, I_s the rotational inertia, Z and Γ the vertical force and the moment per length acting on the ship. The shear and bending moment vanish at the ends of the beam. Thus, we have for the boundary conditions:

$$D\frac{\partial^2 \tilde{w}}{\partial x^2} = 0 \qquad \frac{\partial}{\partial x}\left(D\frac{\partial^2 \tilde{w}}{\partial x^2}\right) = 0 \tag{5}$$

$$\frac{\partial \tilde{\theta}}{\partial x} = 0 \tag{6}$$

We consider a harmonic wave propagating in the direction β with respect with the main axis of the ship. The harmonic wave potential, deflection and angular rotation can be written as $\Phi(\mathbf{x}, t) = \phi(\mathbf{x})\, e^{-i\omega t}$ $\tilde{w}(x, t) = w(x)e^{-i\omega t}$ and $\tilde{\theta}(x, t) = \theta(x)e^{-i\omega t}$. The incident plane wave potential equals:

$$\phi^{inc} = \frac{g\zeta_0}{\omega_0}\exp\{ik_0(x\cos\beta + y\sin\beta) + k_0 z\} \tag{7}$$

The potential function is split in a incident wave potential and a diffracted wave potential.

$$\phi(\mathbf{x}) = \phi^{inc}(\mathbf{x}) + \phi^{D}(\mathbf{x}) \tag{8}$$

We notice that in most theories the diffracted potential is defined for the fixed ship, while here it also contains the effect of the rigid- and elastic-body motions.
We finally introduce the Green's function that fulfills $\Delta\mathcal{G}(\mathbf{x}, \xi) = 4\pi\delta(\mathbf{x} - \xi)$, the free surface condition and the radiation condition.

For $\mathbf{x} \in \partial\Omega$:

$$2\pi\phi^D = \int\limits_{\partial\Omega}\left(\phi^D\frac{\partial\mathcal{G}}{\partial n} - \mathcal{G}\frac{\partial\phi^D}{\partial n}\right)dS \tag{9}$$

Equations (1), (2) and (7) lead to a system of integral equations:

$$-\mu\omega^2 w + \frac{\partial^2}{\partial x^2}\left(D\frac{\partial^2 w}{\partial x^2}\right) - i\rho\omega\left\{\int\limits_{\mathcal{C}(x)}\phi^D\vec{n}\,dl\right\}\cdot\vec{e_z} + \rho g b(x)\, w = i\rho\omega\left\{\int\limits_{\mathcal{C}(x)}\phi^{inc}\vec{n}\,dl\right\}\cdot\vec{e_z} \tag{10}$$

$$-I_s\omega^2\theta - \frac{\partial}{\partial x}\left(c\frac{\partial\theta}{\partial x}\right) - i\rho\omega\left\{\int\limits_{\mathcal{C}(x)}\overrightarrow{OM}\wedge\phi^D\vec{n}dl\right\}\cdot\vec{e_x} + \mathcal{K}(x)\,\theta = i\rho\omega\left\{\int\limits_{\mathcal{C}(x)}\overrightarrow{OM}\wedge\phi^{inc}\vec{n}\,dl\right\}\cdot\vec{e_x} \tag{11}$$

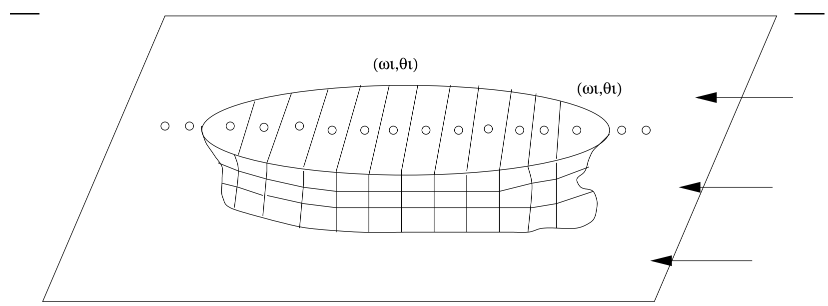
where $b(x)$ is the width of the ship at abscis x, $\mathcal{K}(x)$ the restoring moment for a slice at abscis x and $\mathcal{C}$ the line integral over the wetted hull at abscis x.

$$2\pi\phi^D - \int\limits_{\partial\Omega}\left\{\phi^D\frac{\partial\mathcal{G}}{\partial n} + i\omega\mathcal{G}\left(w\,\vec{e_z}\cdot\vec{n} + \theta\, y\,\vec{e_z}\cdot\vec{n} - \theta\, z\,\vec{e_y}\cdot\vec{n}\right)\right\}dS = \int\limits_{\partial\Omega}\mathcal{G}\frac{\partial\phi^{inc}}{\partial n}dS \tag{12}$$

3 Numerical method

The differential equations are discretised by means of a difference scheme, while the integral equation is discretised by means of a piece-wise constant panel distribution. The final set of equations is a matrix equation for the coupled unknowns. For the purpose of calculation, the hull is divided into N slices. Each slice is supposed to have a constant deplacement (w_i, θ_i) .
As we are using a finite difference approach to solve the dynamic equations for the beam, we introduce 4 supplementary points for the mesh describing the beam in order to represent the fourth order derivative for the beam equation. We denote the diffracted potential on each of the M panels of the mesh describing the ship's hull by ϕ_i^D.
Equations (3), (4), (5), (6) (10) and (11) lead to a linear system for the discretized problem. The solution vector is written in the form

$$\vec{X} = \{\phi_1^D, \phi_2^D, \cdots \phi_M^D, w_{-1}, w_0, w_1, w_2, \cdots w_N, w_{N+1}, w_{N+2}, \theta_0, \theta_1, \theta_2, \cdots, \theta_N, \theta_{N+1}\} \qquad (13)$$



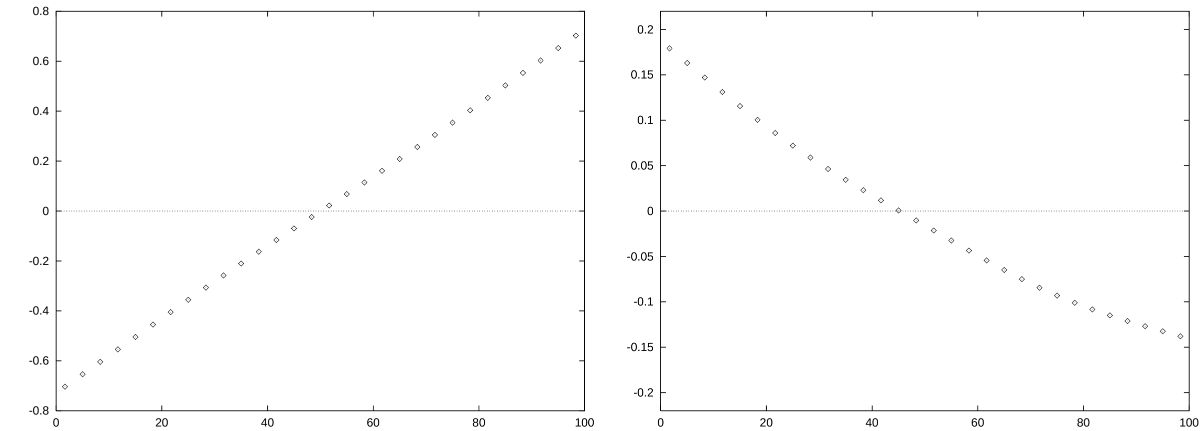
4 Numerical results for a paralepipedic barge

We present some numerical results for a thin barge of dimension lx=100m, ly=10m, lz=5m. The ship is cut in 30 slices and has a constant flexural rigidity of $10^{11} Nm^2$. The computations are carried out for a wave direction parallel to the ship direction, hence, $\theta(x) \equiv 0$. The incoming wave has a wave length of 45m and a unit amplitude. We are so here only interested in the motion associated with the heave and pitch motion and confined in a vertical plane. The first two graphics represent the real part and the imaginary part of the total vertical deflection, ie the deflection at $t = 0$ and $t = T/2$ including the heave, pitch motion and the bending distortion due to the flexibility of the ship.
The graphs of figure (2) represent the bending distortion and moment.
We can clearly see the first two principal bending modes of an elastic vibrating uniform beam.
The graphs of figure (3) represent the amplitude of the bending moment. for a wave length equal to the length of the ship, the amplitude is maximal at mid-point of the ship.

References

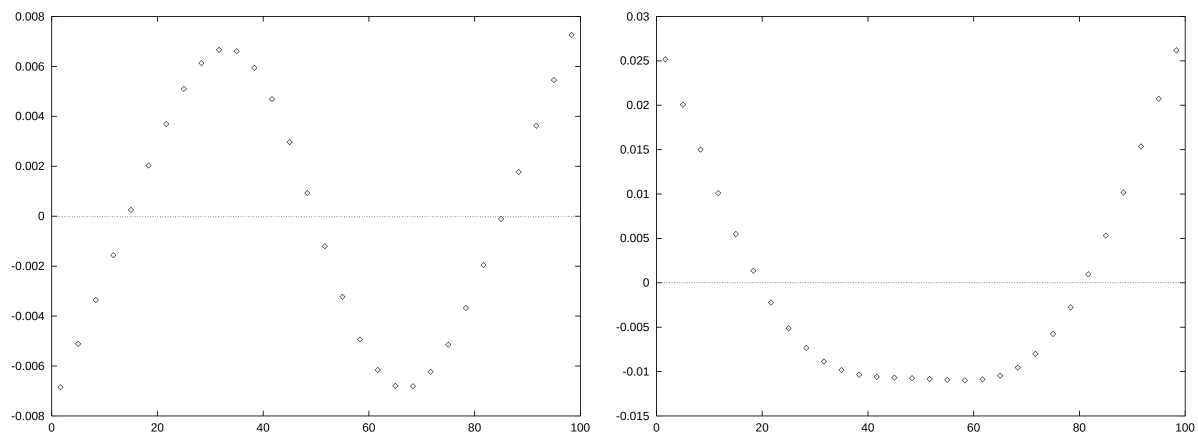
[1] Aad J. Hermans. A boundary element method for the interaction of free-surface waves with a very large floating flexible platform, *J. of fluids and structures*, to appear.

[2] R.E.D Bishop and W.G Price (1978) . *Hydroelasticity of ships*, Cambridge University Press.



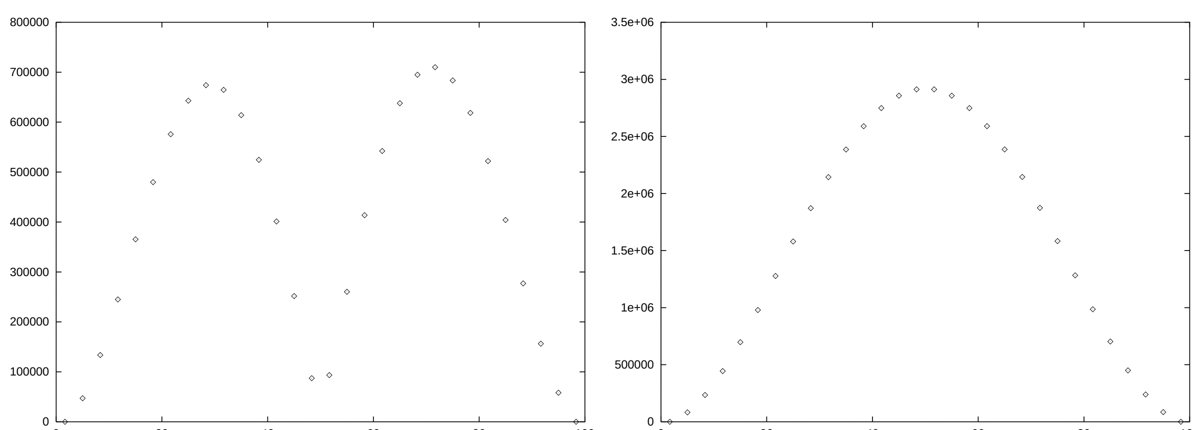
Real part of the total deflection | Imaginary part of the total deflection

Figure 1: total deflection



Real part of the flexural deflection | Imaginary part of the flexural deflection

Figure 2: flexural deflection



$\lambda/L = 0.45$ | $\lambda/L = 1$

Figure 3: Amplitude of the bending moment