NUMERICAL COMPUTATION OF THREE-DIMENSIONAL OVERTURNING WAVES

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Abstract

We present accurate three-dimensional (3D) numerical computations for strongly nonlinear waves. The program solves fully nonlinear potential flow equations with a free surface, using a higher-order 3D Boundary Element Method (BEM) and a Mixed-Eulerian-Lagrangian time updating. An application to overturning waves over a non-uniform bottom is shown.

1 Introduction

So far, fully nonlinear potential flow (FNPF) equations of water waves have been successfully solved by two-dimensional (2D) Boundary Element methods (BEM). Among many applications, wave shoaling over arbitrary bottom topography up to overturning of a wave crest has been extensively studied (e.g., Grilli *et al.*, 1996, 1997). By comparing results with laboratory experiments (e.g., Bonmarin, 1989), FNPF formulation has been shown to model the physics of wave overturning in deep and intermediate water (e.g., Dommermuth *et al.*, 1988) as well as wave shoaling and breaking over slopes (e.g., Grilli *et al.*, 1997) with a high degree of accuracy. Most recent results on 2D waves deal with a numerical wave tank (NWT) where incident waves are generated at one extremity and reflected, absorbed or radiated at the opposite extremity (e.g., Grilli and Horillo, 1997). Although some 3D NWT have been developed for non-overturning waves over constant depth (e.g., Ferrant, 1998; Celebi *et al.*, 1998), only a few attempts have been reported, which solve FNPF problems for overturning waves in deep water (Xü and Yue, 1992) or over arbitrary bottom (e.g., Broeze, 1993). In general, 3D computations involve more difficult geometric and far-field representation problems than in 2D. In this study, we propose an efficient 3D NWT for calculating nonlinear surface waves according to FNPF theory (Fig. 1). In particular, it is applicable to the simulation of overturning waves over arbitrary bottom topography Γ_b.

2 Numerical approach

The model is based on a higher-order 3D BEM and a Mixed-Eulerian-Lagrangian time updating of the free surface Γ_f. Namely, we consider an incompressible, inviscid and irrotational 3D flow with a free surface. Green's second identity then transforms the Laplace's equation for the velocity potential into the boundary integral equation

$$\alpha(\boldsymbol{x}_l)\,\phi(\boldsymbol{x}_l) = \int_{\Gamma(\boldsymbol{x})} \left\{ \frac{\partial \phi}{\partial n}(\boldsymbol{x})\, G(\boldsymbol{x},\boldsymbol{x}_l) - \phi(\boldsymbol{x})\,\frac{\partial G}{\partial n}(\boldsymbol{x},\boldsymbol{x}_l) \right\} \mathrm{d}\Gamma(\boldsymbol{x})\,, \qquad (2.1)$$

where $\phi(\boldsymbol{x}_l)$ is the velocity potential, $G(\boldsymbol{x}, \boldsymbol{x}_l) = 1/4\pi r$ the 3D free space Green's function, $\boldsymbol{n}$ the outward normal unit vector to the boundary $\Gamma(\boldsymbol{x})$ and $\alpha(\boldsymbol{x}_l) = \theta_l/(4\pi)$ with θ_l the exterior solid angle at point $\boldsymbol{x}_l$. The formulation is completed with the nonlinear kinematic and dynamic boundary conditions on the free surface expressed in Mixed-Eulerian-Lagrangian form. Both spatial and temporal discretizations are direct extensions of Grilli and Subramanya's 2D model (1996). Geometry and field variables in (2.1) are represented on the boundary by 16-node cubic "sliding" 2D elements with continuous slopes. Accurate and efficient numerical integrations are developed for these elements. Discretized boundary conditions at intersections (corner/edges) between the free surface and the lateral walls are well-posed in all cases of mixed boundary conditions. The dense and nonsymmetric linear system resulting from (2.1) is solved iteratively using a preconditioned GMRES (Generalized Minimal Residual) algorithm. For the time integration, we adopt an second-order explicit scheme with adaptive time steps, based on Taylor expansions of the fluid particle position and the velocity potential

$$\boldsymbol{r}(t + \Delta t) = \boldsymbol{r}(t) + \Delta t \frac{D\,\boldsymbol{r}}{D\,t}(t) + \frac{(\Delta t)^2}{2} \frac{D^2 \boldsymbol{r}}{Dt^2}(t) + \mathcal{O}[(\Delta t)^3]\,,$$

$$\phi(\boldsymbol{r}(t + \Delta t)) = \phi(t) + \Delta t \frac{D\,\phi}{D\,t}(t) + \frac{(\Delta t)^2}{2} \frac{D^2 \phi}{Dt^2}(t) + \mathcal{O}[(\Delta t)^3]\,.$$

Higher-order tangential derivatives required for the time stepping are calculated in a local curvilinear coordinate system $(\boldsymbol{s}, \boldsymbol{m}, \boldsymbol{n})$, using 25-node fourth-order "sliding" 2D elements. Arbitrary waves can be generated in the tank by wavemakers on Γ_{r1}, or directly on the free surface. If needed, absorbing boundary conditions are specified on lateral walls Γ_{r2} of the computational domain. Node regridding to a finer resolution can also be performed at any time step in selected areas of the free surface. Details can be found in Grilli, Guyenne and Dias.

3 Results

As an example, Fig. 2 depicts a 3D overturning wave calculated over a sloping 1:15 bottom having a ridge/bump in the y direction, thus focusing wave energy in the middle part of the NWT ($y = 0$). In addition, Fig. 3 shows the vertical cross-section $y = 0$ of the same overturning wave as in Fig. 2 at time t_2 and at an earlier time t_1 before a horizontal tangent occurs beneath the plunging jet.

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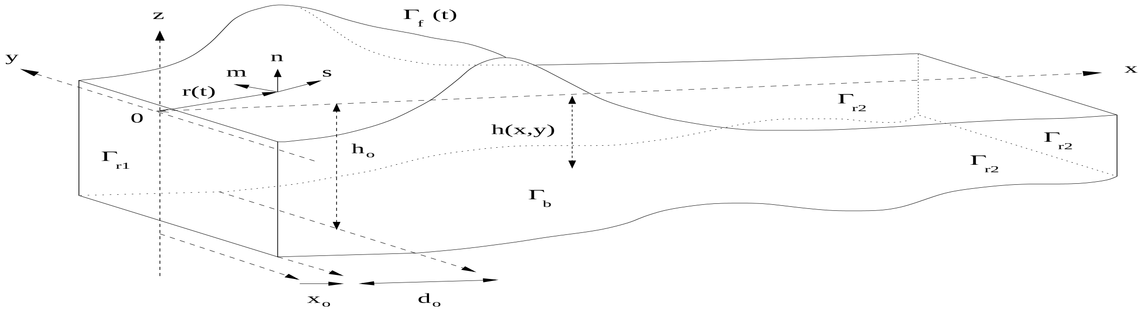

Figure 1: Sketch of the computational domain for 3D BEM solution of FNPF equations.

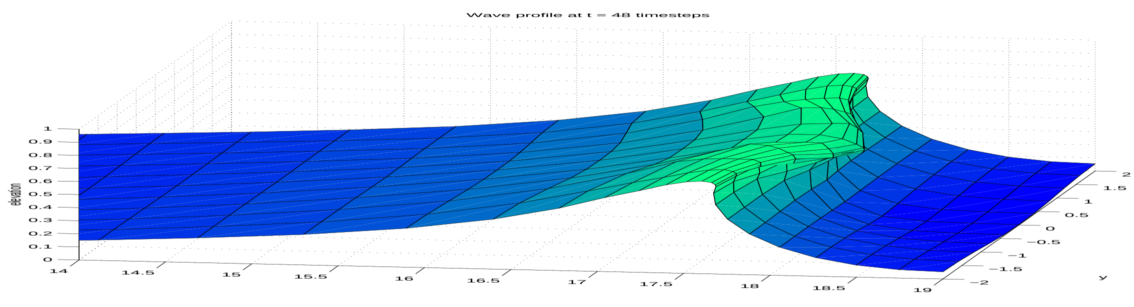

Figure 2: Overturning wave over a ridge modeled as a sloping 1:15 bottom with a $sech^2$ modulation in the y direction.

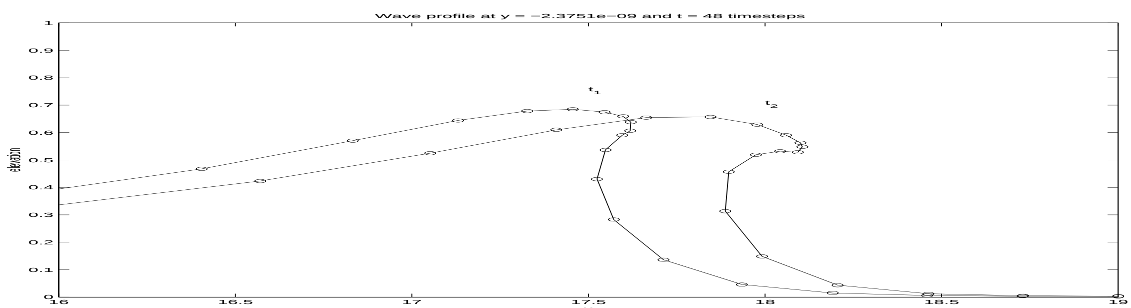

Figure 3: Vertical cross-section at $y = 0$ of the overturning wave of Fig. 2 at time t_2 and an earlier time t_1.