Discussions
of the

FOURTEENTH INTERNATIONAL WORKSHOP

ON WATER WAVES AND FLOATING BODIES

Thomas Edison Inn
Port Huron, Michigan
USA

11 April - 14 April, 1999

Edited by
Robert F. Beck
and
William W. Schultz

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DISCUSSIONS

AND

ERRATA

Hydroelastic Interaction of a Large Floating Platform with Head Seas

M. Ohkusu
Research for Applied Mechanics, Kyushu University, Japan

Discusser: A.J. Hermans

Questions/Comments:

1. Is it possible to take the mass of the plate into account?
2. Can you apply the approach to the case of finite or deep water?

Author's Reply:

1. In our formulation I ignored the effect of the mass of the plate because it is generally of higher order. But if the bending rigidity of the plate is much smaller, the effect of the mass must be taken into account. At such very small bending rigidity the difference of the wave number of the plate deflection and that of the water waves is determined only by the effect of the mass and the hydroelastic deflection of the plate is characterized by this difference in the wave number. It is possible to consider the effect of the mass in the formulation of the plate deflection.

2. I applied a similar approach to the interaction of water waves and the edge of an elastic plate in deep water. Analysis of the plate deflection away from the edge must be a little more complicated but it will not be impossible.

Discusser: M. Tulin

Questions/Comments:

A question for both Prof. Takagi and Prof. Ohkusu. How can the thickness of the structure be entirely neglected? It could seem that the back scatter of wave energy by the plate must depend on the frontal area of the structure. Not only the underwater projection would seem important, but also the freeboard.

Author's Reply:

I assumed the draft of the structure is very small compared with the wavelength. Therefore the effect of the draft (and the effect of the mass which is proportional to the draft) is of higher order in my formulation. The flow "velocity" near the front part of the plate at the limit of the draft approaching zero has a "weak" singularity which represents the back scatter from the frontal area.

Hydroelastic Behavior of a Very Large Floating Structure in Waves

Ken Takagi
Department of Naval Architecture and Ocean Engineering
Osaka University, Japan

Discusser: T. Miloh

Questions/Comments:

In analyzing your corner problem, I believe that you have to look for a local-type solution written in terms of inner variables. The finite thickness of the plate has to be considered and the water depth can not be considered anymore as shallow. A uniformly valid solution can be sought by matching the local (near-field) solution to that which holds in the far-field.

Author's Reply:

In the case of two-dimensional infinite depth problem, I have obtained the inner solution and I have also showed that the relative wave height i.e. the free surface elevation at the edge obtained from the outer solution is valid in the near-field (Tagaki, 1997). In the case of the three-dimensional finite depth problem, I believe that this result is applicable with minor modification except near the corner. In the numerical computation of this work, I have never experienced singular behavior of the solution. This implies that the singularity of the outer solution at the edge is weak.

Reference:

Takagi, K., "On the Slamming of a VLFS," 44th *Sea-Keeping Committee of the Soc. of Naval Architects Japan*, 1997. (English translation of the above mentioned proof is available, however the original article is written in Japanese.)

Discusser: M. Tulin

Questions/Comments (repeated):

A question for both Prof. Takagi and Prof. Ohkusu. How can the thickness of the structure be entirely neglected? It could seem that the back scatter of wave energy by the plate must depend on the frontal area of the structure. Not only the underwater projection would seem important, but also the freeboard.

Author's Reply:

The assumption employed in this study is that the draft of the structure is of the same order as the amplitude of the incident wave. Therefore, the back scatter of wave energy by the frontal area of the structure is higher order in the linearized theory. Geometrically this assumption seems being satisfied since the draft of the structure planed in our project is 1-2m and the wavelength is 50-100 m. This assumption also has been justified by several experimental works, for example Wu et al. and Ohmatsu.

References:

[1] Wu, C., Watanabe, E. and Utsunomiya, T., 1995, "An Eigenfunction Expansion-Matching Method for Analyzing the Wave-Induced Responses of an Elastic Floating Plate," Appl. Ocean Res., Vol. 17, No. 5.

[2] Ohmatsu, S., 1997, "Numerical Calculation of Hydroelastic Responses of Pontoon Type VLFS, J.," Soc. Nav. Arch. Japan, 182 (in Japanese).

Discusser: W. Schultz

Questions/Comments:

The operator on the elastic term is 4th order while those of Prof. Ohkusu and Zilman and Miloh are 6th order. Is this a different model or just different nomenclature?

Author's Reply:

The free surface condition in the plate is the same as those of Prof. Ohkusu and Zilman and Miloh. You can find the 6th order operator on the elastic term in equation (18).

ERRATA:

Equations (27) and (31) in this paper should be replaced as follows:

$$\frac{k_y}{k_x} = \alpha_n \tan \mu_{xn} \quad (27) \quad \longrightarrow \quad \frac{\sqrt{\alpha_n^2 - k_x^2}}{k_x} = \tan \mu_{xn} \quad (27)$$

$$-\frac{k_y}{k_x} = k_0 \tan \chi_{x0} \quad (31) \quad \longrightarrow \quad -\frac{\sqrt{k_0^2 - k_x^2}}{k_x} = \tan \chi_{x0} \quad (31)$$

Hydroelasticity of a Buoyant Circular Plate in Shallow Water: A Closed Form Solution

G. Zilman and T. Miloh
Faculty of Engineering, Tel-Aviv University

Discusser: J.N. Newman

Question/Comments:

The mass is very important for cases where your parameter $\varepsilon = D/2\rho g r_0^4 << 1$. For $\varepsilon \to 0$ the plate of zero draft and zero mass deflects like the incident wave, as expected, but a relatively small draft and proportional mass will magnify the plate deflections. We showed examples of this phenomenon in our paper in the proceedings of Hydroelasticity '98 (Kyushu).

Author's Reply:

Although within the framework of shallow water theory the inequality $\omega^2 m << \rho g$ always holds, Eq. (3)

$$\left(\nabla^6 + a\nabla^2 + b\right)\phi_1 = 0 \qquad \left(a \sim \rho g - \omega^2 m\right)$$

of our abstract does not imply that the term $\omega^2 m$ must be neglected compared with the term ρg. We invoke it here only for one reason, to prove that a is strictly positive which is essential for our further analysis. Nevertheless, it is true that for small values of the nondimensional parameter $\varepsilon = D/\rho g r_0^4$ and high modes even small variations of a can result in considerable changes in the Bessel functions (similar to the behavior of the trigonometric modes in Fourier series). We believe, however, that for shallow water the amplitudes of such short-wave flexural modes do not contribute to the final result, and, thus, the problem remains well posed. Generally, in shallow water the added mass plays a more pronounced role compared to deepwater conditions which may explain the difference between our and your results concerning the importance of the mass-inertia term.

Discusser: B. Molin

Questions/Comments:

Your matching condition $\Phi^+ = \Phi^-$ at the edge of the plate is based on energy flux considerations. Do these considerations account for energy being propagated by elastic waves in the plate?

Author's Reply:

The transition condition between the water and plate regions is somewhat artificial and thus can be interpreted in different ways. We envisage it (Stoker, p. 420) as a requirement that the fluid boundary between these two regions is free of sources and dipoles. That immediately leads to the continuity of the potential and its first derivative which is also equivalent, to the continuity of pressure and velocities on the boundary. Within the framework of our formulation these two conditions result in the continuity of both the mass and energy flux of the fluid across the separating contour. Under such considerations the mathematical treatment is relatively simple. Thus, we believe that there is no need to invoke the energetic approach which incorporates the energy of the plate by itself and, apparently, leads to much more complicated analysis.

Radiation-Diffraction Problem with Forward Speed in a Two-Layer Fluid

Thai Nguyen
Naval Surface Warfare Center, Dahlgren Division, Coastal Systems Station

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Discusser: H. Bingham

Questions/Comments:

How do you evaluate the infinite sum in $G^{(1)}$?

Author's Reply:

The infinite series was simply computed by summing up the individual terms until a certain error criterion is met. The computational effort for this term is relatively small compared to other terms in $G^{(1)}$, and no special treatment was applied. In the case of no forward speed (Nguyen, 1998), the infinite series represents a larger fraction of the numerical evaluation of $G^{(1)}$, and the method described in Newman (1992), which involves the modified Hankel function and Chebyshev polynomials, was used.

References:

[1] Nguyen, T.C. (1998), "Green's Functions for a Two-Layer Fluid of Finite Depth," Ph.D. Thesis, University of California, Berkeley.

[2] Newman, J.N. (1992), "The Approximation of Free-Surface Green Functions," *Wave Asymptotics*, Cambridge University Press, Cambridge, England.

Discusser: J. Grue

Questions/Comments:

In oceanic conditions, the density jump is usually small. Then, for interfacial waves, the free surface may be considered as flat. I wonder, what applications do you think of with regard to the incoming wave conditions, eqs. (9)-(10), in this context? I have also a comment on eq. (21), term 1. Efficient evaluation of this sum may be found in Newman (1992), Wave Asymptotics, Cambridge University Press or in Grue & Bilberg (1993), Applied Ocean Research.

Author's Reply:

Eqs. (9) & (10) represent the two possible incident wave conditions. For $n = 1$, the incident wave is of the surface-wave mode where the maximum displacement occurs at the free surface. This surface wave is similar to surface waves in a single-layer fluid, modified by the presence of the interface. The modification depends on the density difference. For a small density jump, the modification is also small. For $n = 2$, the incident wave is of the internal-wave mode, and for small density difference, the free surface, as you have pointed out, is essentially flat. Eqs. (9) & (10) are general equations, applicable for any arbitrary density jump. When the density jump is small, these equations do give a very small displacement at the free surface as expected. The physical condition for which the density jump can be significant is that of fresh water above muddy water. This condition can be represented by eqs. (9) & (10) as well.

With regard to eq. (21), the author would like to thank Professor Grue for the reference to his more recent treatment of the infinite sum.

Discusser: B.-K. Kim

Questions/Comments:

From a numerical point of view, is the Green function for a two-layer fluid easier to evaluate than the Green function for a single-layer fluid?

Author's Reply:

The Green function for a two-layer fluid is more difficult to evaluate because of the more complicated dispersion relation and dependence on the vertical coordinate z. Also, the Green function in a two-layer fluid includes contributions from both the surface-wave mode as well as the internal-wave mode, and its computation is at least twice as long as that for a single-layer fluid.

Discusser: M. Kashiwagi

Questions/Comments:

You assume that the body does not intersect the interface of the two layers. Is there a particular problem when the body intersects the interface or both the interface and free surface?

Author's Reply:

When the body intersects the free-surface and/or the interface, a line integral along the intersection at the free surface or interface must be accounted for. This line integral is analogous to the line integral in the single-layer fluid case and has the same singular behavior. For a single-layer fluid, the numerical evaluation of this line integral has been addressed by Doutreleau and Chen in this workshop. Similar analysis will be performed for the two-layer in the near future.

Reference:

Doutreleau, Y. and Chen, X. (1999), "Line Integrals on the Free Surface in Ship-Motion Problems," Fourteenth International Workshop on Water Waves and Floating Bodies," Port Huron, Michigan.

Discusser: X. Chen

Questions/Comments:

Concerning the free-surface effect terms of the Green function which are expressed by double integrals in (21) and (22), what are the further formulation for numerical evaluations and your numerical approaches to evaluate them? In particular, when both source and field points are close to or at the free surface or at the interface surface of a two-layer fluid, singular and highly oscillatory terms are expected to exist in G as in the case of one-layer fluid (see paper by Y. Doutreleau and X. Chen).

Author's Reply:

You are correct in noting that when both the source and field points are close to either the free surface or the interface, the Green function becomes highly oscillatory and is very difficult to evaluate. For this reason, the numerical results given in this paper are for a submerged spheroid where both the source and field points are away from the free surface and interface, and direct numerical integration of eqs. (21) and (22) is possible although time intensive.

Uniqueness and Trapped Modes for a Symmetric Structure

Maureen McIver
Loughborough University, Loughborough, Leicestershire, UK

Discusser: D. Evans

Questions/Comments:

Do your results include the results of Mazja? How does your method relate to the work of Fitzgerald and Grimshaw? Have you tried different harmonic functions to obtain more sophisticated geometric conditions?

Author's Reply:

(a) There is an overlap between my results and Mazja's. I think my results cover more general geometries but Mazja deals with the uniqueness of the solution of both the symmetric and axisymmetric problems. (b) I think the method of Fitzgerald and Grenshaw could be modified to cover my results although they considered a completely submerged topography. (c) I haven't tried this but I think it could be done.

Rayleigh-Bloch Surface Waves in the Presence of Infinite Periodic Rows of Cylinders and Their Connection with Trapped Modes in Channels

R. Porter and D.V. Evans
School of Mathematics, University of Bristol, Bristol, UK

Discusser: P. Stansby

Questions/Comments:

What is the maximum force magnification for a cylinder due to periodicity or being part of periodic array?

Author's Reply:

This depends on the size of the (finite) periodic array. The larger the array size, the larger the force on those cylinders near the middle of the army at a frequency very close to the trapped mode frequency for a single cylinder in a channel of width equal to the spacing between adjacent cylinders in the period array.

Discusser: M. Kashiwagi

Questions/Comments:

It seems that you are always thinking bottom-mounted circular cylinders. Do you think the same conclusions can be applied to floating truncated cylinders, concerning the characteristics of trapped modes?

Author's Reply:

There is no doubt that the same phenomenon will occur in this case also, and this has been confirmed for the case of a single truncated cylinder on the centerlines of a channel. However, the effect may be difficult to detect since the mode frequency will be close to the cut-off for the channel as the influence of a truncated cylinder will be less than for a bottom-mounted cylinder.

Discusser: W. Schultz

Questions/Comments:

You find trapped modes with period 2d. Can you find trapped modes of period 2md m = 2,3,... that are not 2d periodic? If so, it would be nice to compare these to the multiple cylinder results in a channel that were in your presentation.

Author's Reply:

By choosing $\beta = n\pi/2N\phi$ in the general Rayleigh-Bloch solution we can construct trapped modes around N cylinders in a channel of width 2Na. Only if n is taken to be N does this reduce to trapped modes with period 2d about each cylinder, so that in effect we have N copies of the untrapped modes about a single cylinder 'glued' together. For general values of n, the modes have period 2Nd.

Non-linear Effects in Trapped Modes with the Case of Gravitational Waves in a Channel with an Elastic Plate

D.A. Indeytsev and E.V. Osipova
Institute of Problems of Mechanical Engineering, St.Petersburg, Russia

Discusser: S. Ando

Questions/Comments:

In the nonlinear regime you are dealing with, isn't there any energy transfer among the trapped modes?

Author's Reply:

Of course in non-linear problems all modes are connected between one with another. But in the first approximation, the energy part of the travelling waves is very small when we have the special parameters of channel and die.

Nonlinear Gravity-Capillary Waves Generated by a Moving Disturbance

Jean-Marc Vanden-Broeck
School of Mathematics, The University of East Anglia

Discusser: J. Newman

Questions/Comments:

In the case where the use of Rayleigh viscosity gives the "wrong" unique solution, vis-à-vis the limit of the nonlinear solution, what is the long-time limit of the initial-value problem and how does this compare with the other solutions?

Author's Reply:

I have not calculated time dependent solutions. I expect the solution with a train of Wilton ripples to be the "long-time limit."

Discusser: F. Noblesse

Questions/Comments:

The classical Rayleigh viscosity approach yields a linear solution that is nonunique. This is not incorrect. In fact, the unique solution obtained using a perturbation analysis is a particular case of the family of linear solutions obtained using the Rayleigh approach. In this sense, this classical approach did not fail (although you have shown that it is not sufficient to determine a unique solution).

Author's Reply:

The difference between the solutions with Wilton ripples and the linear solution satisfying the linear radiation condition is a solution of the homogeneous linear problem.

Discusser: T. Miloh

Questions/Comments:

Is there a physical explanation that by letting $\varepsilon \to 0$ the solution will pick a particular branch of the multi solutions?

Author's Reply:

The solution cannot pick the classical linear solution for $k = 1/2$ when $\varepsilon \to 0$ because one of the trains of waves fail to exist in the nonlinear regime. There are two possible solutions (nonlinear) when $k = 1/2$. Stability should decide which solution is physical.

Discusser: I. Dmitry

Questions/Comments:

Is it necessary to see in this problem the model of fluid not only incompressible but compressible too? In the resonance situation, when we have big displacement we must see not only the non-linear terms but we must see and consider other terms of our equation.

Author's Reply:

It is important not to oversimplify the model by neglecting significant effects. However since the phase velocity of the waves considered here is much smaller than the speed of sound, it is safe to consider the fluid as incompressible.

Discusser: W. Schultz

Questions/Comments:

You showed a nonlinear (or was it linear?) solution with Wilton ripple solutions downstream. Can you find similar nonlinear solutions with the Wilton ripples upstream?

Author's Reply:

The solutions with Wilton ripples are nonlinear. Linear solutions without Wilton ripples cannot be the limit of nonlinear solutions as the magnitude of the disturbance approaches zero.

The Oscillating Submerged Sphere Between Parallel Walls:
The Convergence of the Multipole Expansion

F. Ursell
Department of Mathematics, Manchester University, UK

Discusser: M. McIver

Questions/Comments:

Presumably the determinant of your system of equations vanishes at some frequency, and this corresponds to a trapped mode. Were you able to prove that the determinant vanishes?

Author's Reply:

I have not yet examined the problem of the vanishing of the determinant. We might perhaps expect this for small radii at a frequency near a cut-off frequency, and anti-symmetrical about the mid-plane $x = 0$. If this is so, then the corresponding odd multipole potentials would be needed. We are concerned with a doubly-infinite system, so the determinant would need to be defined with care.

Discusser: T. Miloh

Questions/Comments:

Can you please comment about the convergence of your scheme for small blockage, i.e. as $\ell \to a$? Is it possible to get the limit of zero blockage by considering the integral representation for touching circles/spheres instead of using the multipole expansion?

Author's Reply:

Unfortunately, my scheme will converge very slowly when the sphere is close to a sidewall, and a different approach will be needed. It would be interesting to study this problem but I cannot give any help at this time.

Heave Response of a Semi-Submersible Near Resonance

J.N. Newman and C.-H. Lee
Massachusetts Institute of Technology, Cambridge MA

Discusser: M. Tulin

Questions/Comments:

Your calculations showing the extreme sensitivity of the resonant frequency to draft of the semi (as well as to paneling) are very interesting. Since the draft will be varying due to the motions of the semi near resonance, doesn't this suggest a limitation of linear theory to resolve the resonant frequency, in as much as in that theory the configuration is considered fixed?

Author's Reply:

Firstly, we do not consider that the resonant frequency is 'extremely' sensitive to the draft or paneling, but rather that the precise coincidence of the frequencies of resonance and near-zero excitation has this sensitivity. Certainly in the vicinity of resonance, if the linear theory predicts a large response, one would expect that nonlinear effects and/or viscous damping would be more important than at other frequencies.

Discusser: B. Molin

Questions/Comments:

Do you have a physical explanation for the change of T_0 with the heading? Would one get the same variation when calculating the excitation loads with a simplified method based on infinite fluid coefficients, like Hooft's, or does it have to do with wave diffraction effects?

Author's Reply:

In general the exciting force depends on the wave heading angle, and thus the period where the force is nearly equal to zero will also depend on the heading angle. Since this is the long-wavelength regime, the dependence on heading angle is weak. Hooft's approximate analysis of the exciting force includes the dependence on heading angle, insofar as it affects the phase of the incident wave along the structure.

Discusser: R. Beck

Questions/Comments:

If I recall correctly, WAMIT integrates the wave terms of the Green function over a panel using just one point. How does HIPAN integrate the wave term and could this affect your results?

Author's Reply:

In HIPAN the free-surface part of the Green function is integrated using Gauss-Legendre integration in a systematic manner. Usually we determine the order of this integration to be consistent with the order of the B-splines, and we do not expect this to affect the results.

Discusser: J. Grue

Questions/Comments:

It seems to me that the pontoons of the geometry in consideration have sharp edges. How accurately is the potential obtained close to these edges? How do the edges eventually affect convergence of the exciting force, obtained by pressure integration, and obtained by the Haskind relation? How would this convergence be if the edges were rounded off?

Author's Reply:

The convergences of the linear solutions for a sphere, a cylinder and a cube are presented in Maniar (1996, "A Three-Dimensional Higher-Order Panel Method Based on B-Splines," Ph.D. Thesis, MIT). It shows: i) the convergence is slower for the bodies with corners and ii) the convergence improves significantly with the fine discretization toward the corner. Hsin et al. (1993, "A Higher-Order Panel Method Based on B-Splines," 6th Numerical Ship Hydrodynamics, Iowa City), showed the convergence of the velocity potential on the body surface (including the corner) of a translating 2-dimensional square in an unbounded fluid. The result shows the maximum error of $O(10^{-3})$ using 10-20 unknowns on one quadrant.

We do not think there is a fundamental difference in the accuracy of the forces obtained by the two methods, since both results are obtained from the linear velocity potential. On the other hand, the second-order forces by the momentum conservation converges faster than the pressure integration, since the velocity needs to be evaluated in the latter and it is less accurate than the potential. In the pressure integration care should be taken for the bodies with corners. The velocity is infinite at the corners and it was shown in Lee et al. (1998, "A Geometry Independent Higher-Order Panel Method and its Application to Wave-Body Interactions," Engineering Mathematics and Applications Conference, Adelaide), that one needs to employ appropriate geometric mapping (cosine-spacing discretization in low-order panel method) near the corner to represent the singularity accurately.

We expect that the convergence will improve with the rounded edges as the local radius of curvature near the edges increases. But this practice would, of course, alter the geometry.

Discusser: R. Rainey

Questions/Comments:

I believe it is generally held in the drilling business that the larger the semisubmersible the more the heave tends to occur at the heave natural period. So these calculations, on giant semisubmersibles, ought to be very pertinent. Can you extend them to second-order? In our experience of practical cases at sea, it is often difficult to tell whether the resonant heave is driven by swell, or by second-order vertical forces.

Author's Reply:

It is straightforward to extend HIPAN to compute the second-order solution and we are currently working on it. Whether the resonance is driven by swell, second-order forces, or both should depend on the resonance frequency of the particular semi-sub and the incident wave spectrum.

Waves Generated by a Vertical Cylinder Moving in Still Water

J.R. Chaplin[1], R.C.T. Rainey[2] and C.H. Retzler[1]
[1]City University, London EC1; [2]W.S. Atkins Oil and Gas, London WC1

Discusser: J. Grue

Questions/Comments:

Observations of similar waves as in your experiments have been made earlier, and I have in mind references from the Kyushu University:

M. Tatsuno, S. Inoue and J. Okabe: Transfiguration of surface waves, *Rep. Res. Inst. Appl. Mech.* Kyushu Univ. Vol.17, No.59, 1969, pp.195-216.

S. Taneda: Visual observations of the flow around a half-submerged oscillating sphere, *J. Fluid Mechanics*, Vol.227, 1991, pp.193-209.

S. Taneda: Visual observations of the flow around a half-submerged oscillating circular cylinder, *Fluid Dynamics Research*, 13, 1994, pp.119-151.

That work comprises experiments on a heavily half-immersed sphere. Measurements of a certain pattern of the crests of the ring waves are described.

Author's Reply:

Thank you very much for that reference. I understand from Marshall Tulin that the work on cross-waves on flumes by G.I. Taylor and (later) J.W. Miles may be relevant.

Discusser: M. Tulin

Questions/Comments:

1) The second harmonic radial waves appearing around the cylinder look like non-linear waves appearing due to parametric resonance, which have been much studied for wavemakers, including a sphere oscillating at the water surface.

2) What is the connection between the failure of the plates on the tanker and the wave patterns around the oscillating cylinder? My own impression is that the plate failed due to impact by a deformed breaking wave, with high velocities on the front face of the wave.

3) The linear theory suggests a $\cos\theta$ variation of the wave elevation around the moving cylinder. Did you confirm this variation experimentally, when the radius of the cylinder was of the same order as the wave amplitude?

Author's Reply:

AUTHOR DID NOT RESPOND

Analytical Expressions of Unsteady Ship Wave Patterns

X.B. Chen and L. Diebold
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Discusser: F. Ursell

Questions/Comments:

The expressions (10a) and (10b) given by you are relevant when two points of stationary phase are nearly coincident, as also happens in the steady Kelvin pattern. In your unsteady problem, are there sets of parameters for which there are three (or more) points of stationary phase? If so, the expression would involve Pearcey functions.

Author's Reply:

Thank you for your comments. In the unsteady problem of wave diffraction-radiation with forward speed in deep water, there exists the case that at most two points of stationary phase coalesce. The second terms in (10a) and (10b) involving the Bessel function of fractional order 1/3 can be converted into a form using the Airy function similar to classical results for steady flow. Accordingly, expressions for three coalescing points of stationary phase should involve the Pearcey function.

Errata:

Please replace the paper with the following revised version:

Analytical expressions of unsteady ship wave patterns

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The time-harmonic ship wave patterns are studied by considering free-surface potential flows generated by a point source pulsating and advancing at a uniform forward speed. New expressions in analytically closed form are obtained by an asymptotic analysis on the wave component which is represented by a single Fourier integral along the dispersion curves defined in the Fourier plane by the dispersion relation. These new expressions together with the results given in [1] including simple relations between the dispersion curves and important aspects (wavelengths, directions of wave propagation, phase and group velocities, and cusp angles) of the corresponding far-field waves, build up a complete and analytical picture of unsteady ship wave patterns.

1 Generic representation of wave components

Within an analysis in frequency domain, the time-harmonic velocity potential $\mathcal{G}(\vec{\xi})$ at a field point $\vec{\xi}=(\xi,\eta,\zeta)$ can be expressed in the form $\mathcal{G}^S+\mathcal{G}^F$ where $\mathcal{G}^S$ is defined in terms of distribution of Rankine singularities, and $\mathcal{G}^F$ accounting for free-surface effects. Furthermore, $\mathcal{G}^F$ is decomposed, in [2] and [3], into a nonoscillatory local component $\mathcal{G}^N$ and a wave component $\mathcal{G}^W$ which is defined by the single Fourier integral

$$4\pi i\,\mathcal{G}^W=\int_{D=0}[(\Sigma_1+\Sigma_2)Se^{zk}/\|\nabla D\|]\,e^{-ih\varphi}\,ds \quad \text{with} \quad \varphi=\bar{x}\alpha+\bar{y}\beta\,,\ \ h=\sqrt{x^2+y^2},\ \ (\bar{x},\bar{y})=(x,y)/h \tag{1}$$

along every dispersion curve defined in the Fourier plane (α,β) by the dispersion relation $D=0$. Here, ds is the arc length along a dispersion curve and $\|\nabla D\|^2=D_\alpha^2+D_\beta^2$. The function $\Sigma_1=\text{sign}(D_f)$ associated with the fact to satisfy the radiation condition is obtained by a formal analysis performed in [2], while Σ_2 is expressed in different ways in [3] according to the shape of the dispersion curve. The spectrum function $S(\alpha,\beta)$ is defined in terms of distribution of elementary waves over the mean wetted hull and the mean waterline of the ship. For the sake of simplicity, we consider here the special case of $S=1$ which corresponds to a source at the point (a,b,c), and $(x,y,z)=(\xi-a,\eta-b,\zeta+c)$, the expression (1) represents exactly the wave component of usual free-surface Green functions. Furthermore, the identity $\Sigma_2=\text{sign}(\bar{x}D_\alpha+\bar{y}D_\beta)$ given originally in [2] is used without loss of generality, since the extension to any form of Σ_2 and the spectral function S is straightforward.

The dispersion relation $D=0$ defines usually several dispersion curves. Each dispersion curve is related to a wave system. Wave systems can further be regrouped into different classes according to the type of associated dispersion curves. Analytical expressions of different classes of unsteady ship waves can be obtained by an asymptotical analysis of (1) and summarized hereafter.

2 Unsteady ship wave patterns

For the free-surface time-harmonic ship flows in deep water, the dispersion function $D(\alpha,\beta)$ is given by

$$D=(F\alpha-f)^2-k \quad \text{with} \quad k=\sqrt{\alpha^2+\beta^2} \tag{2}$$

where $f=\omega\sqrt{L/g}$ and $F=U/\sqrt{gL}$ are respectively called nondimensional frequency and Froude number, as ω and U stand respectively for wave encounter frequency and ship's speed, and L and g for ship's length and the acceleration of gravity. The dispersion function (2) shows that the dispersion curves $D=0$ are symmetric with respect to $\beta=0$ and there exist three or two dispersion curves if $\tau=fF=\omega U/g$ is smaller or larger than 1/4, respectively. For $\tau<1/4$, the three dispersion curves intersect the axis $\beta=0$ at four values of α, which are denoted $\alpha_i^\pm$ and $\alpha_o^\pm$ and given by

$$F^2\alpha_i^\pm=\tau\pm(1/2-\sqrt{1/4\pm\tau}) \quad \text{and} \quad F^2\alpha_o^\pm=\tau\pm(1/2+\sqrt{1/4\pm\tau}) \tag{3}$$

such that two open dispersion curves are located in the regions $-\infty<\alpha\le\alpha_o^-$ and $\alpha_o^+\le\alpha<\infty$, and a closed dispersion curve in the region $\alpha_i^-\le\alpha\le\alpha_i^+$. For $\tau>1/4$, we have only two open dispersion curves located in the regions $-\infty<\alpha\le\alpha_i^+$ and $\alpha_o^+\le\alpha<\infty$. In summary, there are two types of dispersion curves : a closed dispersion curve for $\tau<1/4$ and two open dispersion curves for $F>0$. Analytical expressions of ship wave patterns associated with these two types of distinct dispersion curve are given now.

Ring waves - Closed dispersion curve

The ring waves are associated with the closed dispersion curve comprised between α_i^- and α_i^+ for $\tau<1/4$. The dispersion curve is described by a parametric equation

$$\alpha=k(\theta)\cos\theta\,,\quad \beta=k(\theta)\sin\theta \quad \text{with} \quad k=1/(1/2+\sqrt{1/4+\tau\cos\theta})^2 \tag{4}$$

in which the variables (α,β,k) are understood to be multiplied by the frequency-scale factor $1/f^2$. The stationary point $(\alpha_r,\beta_r)=k_r(\cos\theta_r,\sin\theta_r)$ satisfying $\varphi'=0$ is determined by

$$\bar{x}\beta_r-\bar{y}(\alpha_r+2\tau k_r^{3/2})=0 \quad\text{and}\quad \bar{y}\beta_r+\bar{x}(\alpha_r+2\tau k_r^{3/2})<0 \tag{5a}$$

At the stationary point $\theta=\theta_r$, we define

$$\varphi_0^r=\bar{x}\alpha_r+\bar{y}\beta_r,\quad \varphi_2^r=\varphi_0^r[k_r''/k_r-2(k_r'/k_r)^2-1]/2,\quad \varphi_3^r=\varphi_0^r[(k_r'''-3k_r'k_r''/k_r-2k_r')/(3k_r)] \tag{5b}$$

where $k'=dk/d\theta$, $k''=d^2k/d\theta^2$ and $k'''=d^3k/d\theta^3$ are used. The analytical expression obtained from asymptotic analysis for the ring waves is written as

$$\mathcal{G}^R=\exp(-ih\varphi_0^r)A_0^r/\sqrt{(\varphi_3^r/\varphi_2^r)^4+ih\varphi_2^r}\quad\text{with}\quad (\sqrt{4\pi}/f^2)\,A_0^r=-ie^{zk_r}k_r^{3/2}/(2-\sqrt{k_r}) \tag{6}$$

which is of order $O(1/\sqrt{h})$ for $h\to\infty$ consistent with the classical result obtained from the stationary phase method. The fact that the analytical expression (6) has finite values at $h\to 0$ is expected so that it is well suited for numerical evaluations in the near field.

Transverse and divergent waves - Open dispersion curves

Three wave systems associated with open dispersion curves as defined in [3] are the inner-V waves corresponding to the right one located in $\alpha_o^+\le\alpha<\infty$ for $\tau\ge 0$, the outer-V waves related to left open dispersion curve located in $-\infty<\alpha\le\alpha_o^-$ for $\tau<1/4$ and the ring-fan waves associated with left open dispersion curve located in $-\infty<\alpha\le\alpha_i^+$ for $\tau>1/4$. All open dispersion curves can be described by a unique parameter equation as

$$\alpha(u)=\tau-\Sigma_1\sqrt{k},\quad \beta(u)=\sqrt{k^2-\alpha^2}\quad\text{with}\quad k=k_0(1+u^2) \tag{7}$$

for $0\le u<\infty$, in which the Fourier variables (α,β,k) are understood to be multiplied by the Froude-scale factor F^2. Furthermore, $\Sigma_1=-1$ and $k_0=F^2\alpha_o^+$ for the inner-V waves, $\Sigma_1=1$ for both outer-V and ring-fan waves while $k_0=-F^2\alpha_o^-$ for the outer-V waves and $k_0=F^2\alpha_i^+$ for the ring-fan waves.

The open dispersion curve described by (7) has an inflection point at $u=u_c$ determined by

$$k_c^2-(3/2)k_c+\Sigma_1 4\tau\sqrt{k_c}-3\tau^2=0 \tag{8a}$$

with $k_c=k(u_c)$ which gives the cusp angle with respect to the track of the source point

$$\gamma_c=\arctan(1/\sqrt{6k_c-1}) \tag{8b}$$

for both inner-V and outer-V waves, and for the ring-fan waves at $\tau>\sqrt{2/27}$ and

$$\gamma_c=\pi-\arctan(1/\sqrt{6k_c-1}) \tag{8c}$$

for the ring-fan waves in $1/4<\tau\le\sqrt{2/27}$. In fact, $\gamma_c=\pi/2$ at $\tau=\sqrt{2/27}$, i.e. strictly no waves propagate upstream for $\tau\ge\sqrt{2/27}$, an interesting exact result found in [5].

Following the analysis given in [1], there exist two points of stationary phase for $\gamma=\arctan[\bar{y}/(-\bar{x})]<\gamma_c$ at $u=u_t$ located in $[0,u_c]$ and $u=u_d$ in $[u_c,\infty)$ which are determined by

$$\bar{x}\beta_{t,d}-\bar{y}(\alpha_{t,d}+\Sigma_1 2k_{t,d}^{3/2})=0\quad\text{and}\quad \text{sign}[\bar{y}\beta_{t,d}+\bar{x}(\alpha_{t,d}+\Sigma_1 2k_{t,d}^{3/2})]=-\Sigma_1 \tag{9a}$$

with $(\alpha_{t,d},\beta_{t,d},k_{t,d})=[\alpha(u_{t,d}),\beta(u_{t,d}),k(u_{t,d})]$. At the stationary points $u=u_{t,d}$, we define

$$\varphi_0^{t,d}=\bar{x}\alpha_{t,d}+\bar{y}\beta_{t,d},\quad \varphi_2^{t,d}=(\bar{x}\alpha_{t,d}''+\bar{y}\beta_{t,d}'')/2,\quad \varphi_3^{t,d}=(\bar{x}\alpha_{t,d}'''+\bar{y}\beta_{t,d}''')/3 \tag{9b}$$

where $(\alpha'',\beta'')=(d^2\alpha/du^2,d^2\beta/du^2)$ and $(\alpha''',\beta''')=(d^3\alpha/du^3,d^3\beta/du^3)$ are used. Corresponding to the stationary points (α_t,β_t,k_t) and (α_d,β_d,k_d), we may define respectively the transverse waves $\mathcal{G}^T$ and divergent waves $\mathcal{G}^D$. The analytical expressions for both transverse and divergent waves are written as

$$\mathcal{G}^T=\exp(-ih\varphi_0^t)\left(\frac{A_0^t\sqrt{\pi}}{\sqrt{(\varphi_3^t/\varphi_2^t)^4+ih\varphi_2^t}}+\frac{2}{3}\frac{A_0^t|\varphi_2^t/\varphi_3^t|}{(\sigma-ih\varphi_3^t/2)^{\frac{1}{6}}}\,\mathrm{K}_{\frac{1}{3}}\!\left[2(\sigma-ih\varphi_3^t/2)^{\frac{1}{2}}|\varphi_2^t/\varphi_3^t|^3\right]\right) \tag{10a}$$

$$\mathcal{G}^D=\exp(-ih\varphi_0^d)\left(\frac{A_0^d\sqrt{\pi}}{\sqrt{(\varphi_3^d/\varphi_2^d)^4+ih\varphi_2^d}}+\frac{2}{3}\frac{A_0^d|\varphi_2^d/\varphi_3^d|}{(\sigma+ih\varphi_3^d/2)^{\frac{1}{6}}}\,\mathrm{K}_{\frac{1}{3}}\!\left[2(\sigma+ih\varphi_3^d/2)^{\frac{1}{2}}|\varphi_2^d/\varphi_3^d|^3\right]\right) \tag{10b}$$

where $\mathrm{K}_{1/3}(w)$ is the modified Bessel function defined in [4] and σ a positive real constant. For $\gamma>\gamma_c$, we may use the values of (α_c,β_c,k_c) at the inflection point $u=u_c$ to define

$$\varphi_0^c=\bar{x}\alpha_c+\bar{y}\beta_c+\frac{(\bar{x}\alpha_c''+\bar{y}\beta_c'')^3}{3(\bar{x}\alpha_c'''+\bar{y}\beta_c''')^2}-\frac{(\bar{x}\alpha_c'+\bar{y}\beta_c'))(\bar{x}\alpha_c''+\bar{y}\beta_c'')}{\bar{x}\alpha_c'''+\bar{y}\beta_c'''} \tag{11a}$$

$$\varphi_1^c=\bar{x}\alpha_c'+\bar{y}\beta_c'-\frac{1}{2}(\bar{x}\alpha_c''+\bar{y}\beta_c'')^2/(\bar{x}\alpha_c'''+\bar{y}\beta_c''')\quad\text{and}\quad \varphi_3^c=(\bar{x}\alpha_c'''+\bar{y}\beta_c''')/3 \tag{11b}$$

and the transverse and divergent waves by

$$\mathcal{G}^T = \exp(-ih\varphi_0^c)\left(\frac{A_0^c e^{-(h\varphi_1^c)^2(\varphi_1^c/\varphi_3^c)^4}}{2(\varphi_3^c/\varphi_1^c)^2} + \frac{2}{3}\frac{A_0^c|\varphi_1^c/\varphi_3^c|}{(\sigma - ih\varphi_3^c/2)^{\frac{1}{6}}}\,\mathrm{K}_{\frac{1}{3}}\Big[2(\sigma - ih\varphi_3^c/2)^{\frac{1}{2}}|\varphi_1^c/\varphi_3^c|^3\Big]\right) \tag{12a}$$

$$\mathcal{G}^D = \exp(-ih\varphi_0^c)\left(\frac{A_0^c e^{-(h\varphi_1^c)^2(\varphi_1^c/\varphi_3^c)^4}}{2(\varphi_3^c/\varphi_1^c)^2} + \frac{2}{3}\frac{A_0^c|\varphi_1^c/\varphi_3^c|}{(\sigma + ih\varphi_3^c/2)^{\frac{1}{6}}}\,\mathrm{K}_{\frac{1}{3}}\Big[2(\sigma + ih\varphi_3^c/2)^{\frac{1}{2}}|\varphi_1^c/\varphi_3^c|^3\Big]\right) \tag{12b}$$

The amplitude function $A_0^{t,d,c}$ involved in (10a), (10b), (12a) and (12b) are determined by

$$2\pi F^2 A_0^{t,d,c} = -\Sigma_1 i e^{zk_{t,d,c}}(\sqrt{k_0 k_{t,d,c}}/\beta_{t,d,c})\sqrt{k_{t,d,c} - k_0} \tag{13}$$

It can be checked that the second term in both (10a) for $\mathcal{G}^T$ and (10b) for $\mathcal{G}^D$, involving $\mathrm{K}_{1/3}$ decreases exponentially for $h \to \infty$ as far as $\varphi_2^{t,d} \neq 0$ and that both transverse and divergent waves decrease at a rate of order $O(h^{-1/2})$ within the wedge $\gamma < \gamma_c$. Along the wedge $\gamma = \gamma_c$, it can be shown that (10a) and (10b) are of order $O(h^{-1/3})$ as $\varphi_c^{t,d} \to 0$, consistent with the classical results. The amplitude of $\mathcal{G}^T$ represented by (12a) and $\mathcal{G}^D$ by (12b) decreases exponentially for $\varphi_1^c \neq 0$, i.e. $\gamma > \gamma_c$, and at a rate of order $O(h^{-1/3})$ when $\varphi_1^c \to 0$, i.e. $\gamma \to \gamma_c$, and equal to the results by (10a) and (10b).

3 Discussions and conclusions

We have summarized in this study analytical expressions of four ship-wave systems each of which is associated with a distinct dispersion curve by regrouping them into three classes. The first associated with a closed dispersion curve is called ring waves. The second associated with the portion of open dispersion curves limited between two inflection points located symmetrically in the upper and lower half Fourier plane, is called transverse waves. The third class of unsteady ship waves is associated with the portions of open dispersion curves from the inflection points to infinity, and called divergent waves. The ring waves propagate out in all directions for limited values of the Brard number $\tau < 1/4$ and their amplitude decreases at a rate of order $O(h^{-\frac{1}{2}})$ at $h \to \infty$. The transverse and divergent waves are limited by a cusp line whose angle defined by (8b) and (8c) is parallel to the direction of the normal vector at the inflection point of corresponding dispersion curve. Within the wedge limited by the cusp line, the amplitude of transverse and divergent waves decreases at the same rate like $O(h^{-\frac{1}{2}})$ while along the wedge the decreasing rate is of $O(h^{-\frac{1}{3}})$. Outside the wedge, the non-oscillatory local component is dominant since the decreasing rate is of order $O(h^{-1})$ while the wave amplitude falls off exponentially. These important features of transverse and divergent waves are well described by (10a) and (10b) within the wedge, (12a) and (12b) outside the wedge. Furthermore, the expressions (10a) and (12a) for transverse waves as well as (10b) and (12b) for divergent waves provide the same and correct asymptotic values along the wedge so that they are continuous across the wedge, as shown by Fig.1 and Fig.2 which depict $\mathcal{G}^T + \mathcal{G}^D$ of the inner-V waves at $\tau = 1/4$. In both figures, the real and imaginary parts of $\mathcal{G}^T + \mathcal{G}^D$ are presented by the solid and dashed lines, respectively. The value $z = 0$ is used on Fig.1 and $z/F^2 = -0.1$ on Fig.2.

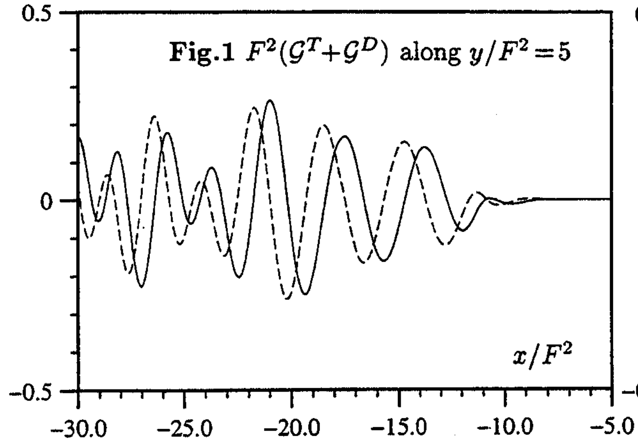

Fig.1 $F^2(\mathcal{G}^T + \mathcal{G}^D)$ along $y/F^2 = 5$

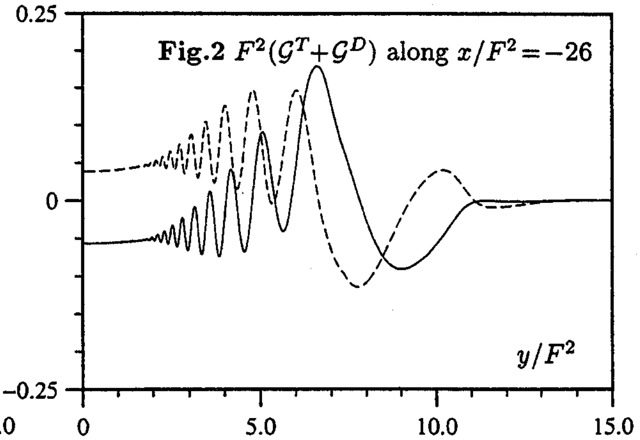

Fig.2 $F^2(\mathcal{G}^T + \mathcal{G}^D)$ along $x/F^2 = -26$

The classical treatments to the phase function which exhibits two coalescing stationary points were presented in [6] to develop uniform asymptotic expansions of an integral. Very fine results can be obtained as presented in [7] in applying to the Neumann-Kelvin steady waves. However, we prefer here forgoing expressions obtained in a different way (which will be presented elsewhere for the sake of space) to describe more complicated *unsteady* ship waves for several reasons.

Firstly, the expressions (10a) for transverse waves $\mathcal{G}^T$ and (10b) for divergent waves $\mathcal{G}^D$ are *explicit* in that the wavenumbers $k^{t,d}$, more exactly wavenumber vectors $(\alpha_{t,d}, \beta_{t,d})$, are defined to correspond to the stationary points $k_0 \le k_t < k_c$ and $k_c < k_d < \infty$ partitioned by k_c given at the inflection point of the corresponding dispersion curve. The decreasing rate is of order $(h^{-1/2})$ since the terms involving $K_{1/3}$ are exponentially small at $h \to \infty$ for a given $\gamma < \gamma_c$. At the wedge $\gamma = \gamma_c$, we have $k_t = k_c = k_d$ and the terms involving $K_{1/3}$ are reduced to those of order $O(h^{-1/3})$ while the first terms in (10a) and (10b) tend to zero as $\varphi_2^{t,d} \to 0$. The transverse waves $\mathcal{G}^T$ and the divergent waves $\mathcal{G}^D$ involved in the inner-V waves, are depicted separately on Fig.3 at $\tau = 1/4$, $z/F^2 = -0.01$ and $\gamma = \gamma_c/2$ with $\tan(\gamma_c) = \sqrt{2/25}$. Fig.4 shows the sum $\mathcal{G}^T + \mathcal{G}^D$ and compared with the line integral (1). It is shown that the difference between analytical expressions (10a) and (10b) and the line integral (1), represented by the thick solid and dashed lines, is negligible for $h/F^2 > 10$. In both Fig.3 and Fig.4, the solid and dashed lines represent the real and imaginary parts, respectively.

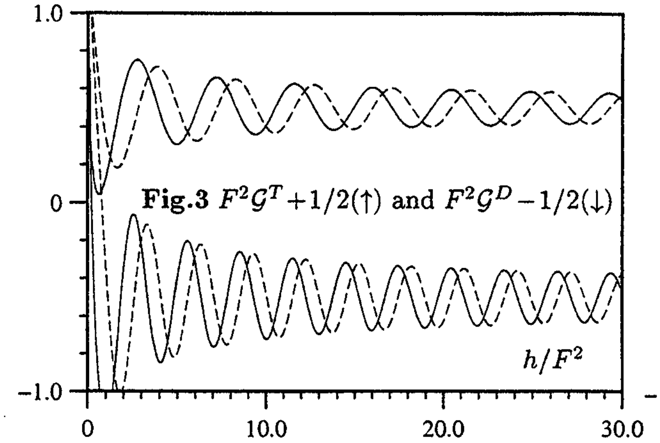

Fig.3 $F^2\mathcal{G}^T + 1/2(\uparrow)$ and $F^2\mathcal{G}^D - 1/2(\downarrow)$

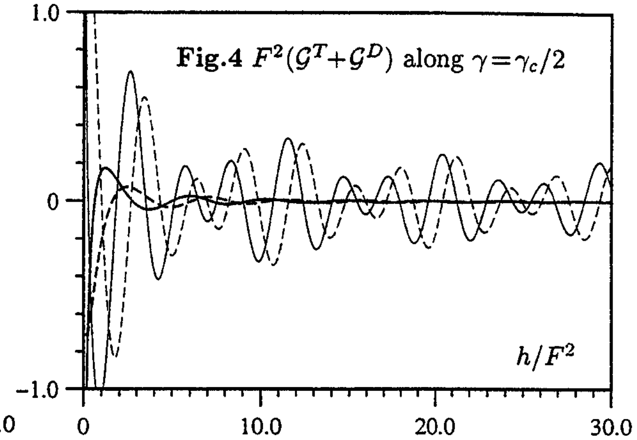

Fig.4 $F^2(\mathcal{G}^T + \mathcal{G}^D)$ along $\gamma = \gamma_c/2$

Concerning divergent waves defined by (10b), as already noted, the decreasing rate is of $O(h^{-\frac{1}{2}})$ in a given direction within the wedge and $O(h^{-\frac{1}{3}})$ along the wedge as well as the transverse waves. However, for a field point in the downstream $\bar{x} < 0$ with $\bar{y} \to 0$, the values of the stationary point (α_d, β_d, k_d) defined by (9a) are given approximately

$$\alpha_d \sim \Sigma_1 \bar{x}/(2\bar{y}), \quad \beta_d \sim \bar{x}^2/(4\bar{y}^2) \quad \text{and} \quad k_d \sim \bar{x}^2/(4\bar{y}^2) \tag{14}$$

as the leading term. The amplitude function A_0^d defined by (13) decreases exponentially for a field point approaching to the track of an *immerged* source point $(z<0)$. Furthermore, if $z=0$ and $\bar{y} \to 0$, i.e. a field point approaches to the track of a source located *at* the free surface, the divergent waves are highly oscillatory with infinitely increasing amplitude and infinitely decreasing wavelength, since $\varphi_2^d \sim k_0\bar{y}$ and $\varphi_3^d/\varphi_2^d \sim 0$ involved in (10b). This singular and highly-oscillatory properties of ship waves are analyzed in [8] in great detail.

Secondly, the expressions (6) for ring waves, (10a) and (12a) for transverse waves, and (10b) and (12b) for divergent waves are *regular* in the near field, unlike the results from classical asymptotic analysis which are singular for $h \to 0$. Far-field waves represented by these analytical expressions are then extended to the near field and complementary to the local component. Finally, the last but not the least concerns calculating the line integral (1) in a complete way. Indeed, the asymptotic analysis performed to obtain the analytical expressions, provides formulations well suited for numerical evaluations of the remaining term - another line integral with the integrand of (1) after subtracting the terms related to the analytical expressions.

In summary, we have given the new expressions of unsteady ship wave patterns in an analytical form. These expressions are critically important in calculating the ship-motion Green function in the far field. They may be very useful as well in a number of analyzes such as estimating wave-damping and wave-resistance components.

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Application of the Fourier-Kochin Theory to the Farfield Extension of Nonlinear Nearfield Steady Ship Waves

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Discusser: S. Ando

Questions/Comments:

Can you apply the F-K method to a ship with a bulbous bow? Does the bulb cause any complication at all?

Author's Reply:

The F-K theory can be applied to a bulbous-bow ship without any particular complication.

Discusser: J. Grue

Questions/Comments:

Can the effect of viscosity be included in the near-field part of your method? And, to what extent will it be a coupling between the far-field flow and the (viscous) near-field flow?

Author's Reply:

Yes, the Fourier-Kochin flow representation can be coupled to a viscous near-field flow solver. As a matter of fact, calculations based on a strong (two-way) coupling between RAMS near-field flow solver and the Fourier-Kochin representation are currently pursued at the University of Nantes in France and at M.I.T.

A Nonlinear Method for Predicting Wave Resistance of Ships

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Odd M. Faltinsen
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Discusser: M. Kashiwagi

Question/Comments:

In the $2\frac{1}{2}$D theory, the condition $\zeta = 0$ and $\phi = 0$ may be used as the starting condition at the bow. But I understand you use the 3-D results for the starting-point condition. Could you say how much difference will be induced in the final resistance result, if the condition of $\zeta = \phi = 0$ is used?

Author's Reply:

Roughly speaking it can matter up to 20% of the nonlinear contribution from the $2\frac{1}{2}$D theories. The nonlinear contribution may be up to 30% of the total wave resistance.

Discusser: R. Beck

Questions/Comments:

How do you match the $2\frac{1}{2}$D nonlinear domain to the 3-D domain in the stern?

Author's Reply:

The $2\frac{1}{2}$D solution is never used after midships. The $2\frac{1}{2}$D solution has not been formally matched to the 2-D solution in all parts of the ship. What is done is to combine formulas for wave resistance based on $2\frac{1}{2}$D solutions and 3-D solutions. An important point is that the results of wave resistance by $2\frac{1}{2}$D theory are not sensitive to where one stops to use the theory. Further, the 3-D theory gives similar results as the $2\frac{1}{2}$D theory in the bow part of the ship where it contributes most to the wave resistance. It has been shown by Faltinsen (1983, JSR) that the linear $2\frac{1}{2}$-D solution matches with the 3-D solution in the bow region. So formally what is lacking is to match the $2\frac{1}{2}$-D solution away from the bow region.

A Non-linear Wave Prediction Methodology for Surface Vessels

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Discusser: M. Tulin

Question/Discussion:

With regard to the density of panels, have you done any convergence testing to find the density needed to get accurate wave profiles on the hull at the bow? I notice that your Series 60 comparison showed results for the peak of the wave not far below the measurement.

Author's Reply:

The wave profile comparison shown in the abstract is for the Model 5415 at a Froude number of 0.28. I believe that this is the figure you're referring to. The Series 60 data/prediction comparison was for a longitudinal cut adjacent but off the side of the model.

A convergence study of panel density has only been made for the Series 60. Results indicate that for this hull form at a Froude number of 0.316 a panel density of 50 to 60 panels per transverse wavelength would be required to obtain complete convergence. Computational resource limitations prevented investigation of panel densities over 50; however, at this density the predicted wave profiles were not changing significantly and were consistent with the data.

No formal convergence study has been made for Model 5415. Panel density was mandated by the available computational resources. The wave profile at the Froude number of 0.28 was generated at 35 panels per transverse wavelength and yielded reasonable results. A similar comparison at a Froude number of 0.41 showed that a panel density of over 50 panels per wavelength was insufficient. We hypothesize that panel density will be a function of Froude number and the shape of the bow entry. For bows and speeds where there is a very divergent plunging bow wave observed, such as Model 5415 at Fn = 0.41, very high panel densities will be required. For bows and speeds where a more transverse spilling bow wave is observed or no breaking is observed, such as the comparisons shown here, a reasonable density of 50 panels per transverse wavelength should be sufficient to match the wave profile.

On the Generation of Upstream Large Amplitude Internal Waves at a Bottom Topography in the Ocean

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Discusser: D. Indeytsev

Question:

1. Have we first of all the standing wave and then the travelling waves? If it's so, this is a trapped mode.

2. Did you see the influence of the vertical velocity of the sill?

Author's Reply:

1. Above the sill, a pronounced depression occurs, which is kind of trapped. On this depression, waves with a length corresponding to that of steady lee waves appear. Ahead of the sill a longer elevation of the interface occurs. At the end of the half of the semidiurnal period, the depressions with the "lee"-waves, move upstream of the sill. If the volume of the depression is sufficiently large, one or more solitary waves may be generated upstream. If the depression is small, no solitary wave appears. The "lee"-waves behind the sill are still present.

2. I have not investigated vertical oscillations of the geometry. This may be interesting.

3. I thank the discusser for bringing up the aspect of trapped modes. I will have to think about this aspect.

Discusser: T. Miloh

Questions/Comments:

When you say that you find a solitary wave what do you really mean? Is it related by any means to the various small-amplitude solitary waves that are known to exist in a two-layer fluid arrangement for which there exist some analytic expressions? Did you determine the dependence of the velocity of such waves on their amplitude?

Author's Reply:

The solitary wave solution is obtained from the fully nonlinear equations of the two-layer model. They may be found in references 1 and 2 (JFM 1999, Vol. 380). These solutions are experimentally verified in reference 2. When the amplitude is small we recover KdV-solitons ($a/h_2 \lesssim 0.4$) and BO-solitons ($a/h_2 << 1$, $h_1/h_2 = \infty$). The range of validity of the BO-solitons is very small ($a/h_2 \sim 0.05$). See reference 1, figure 3. See also the work by Koop and Butler (1981) JFM.

Discusser: H. Bingham

Questions/Comments:

Your condition on the ocean surface is a wall condition?

Author's Reply:

Relevant to oceanic conditions where $\Delta\rho/\rho$ is small, the ocean surface may be regarded as flat, i.e. we may apply the rigid wall condition there. This approximation is also confirmed by experiments, see reference 2, JFM 1999, Vol. 380.

Slam Forces and Pressures on a Flat Plate Due to Impact on a Wave Crest

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Discusser: Y. Kim

Questions/Comments:

Can you give more details on the type of pressure transducers used? Some pressure sensors, i.e. Piezo-type, are sensitive to temperature and may give different results when the surface is wetted before impact. Did you consider this problem?

Author's Reply:

The question was prompted by results showing that peak pressures were reduced by as much as 30% if the surface of the plate was wet when impacting upon a wave crest. The sensors used were Endevco 8530B piezoresistive pressure transducers capable of measuring up to 1000 psi. The transducers have a resonance frequency of over 1 MHz and are, therefore, well suited for the capturing the transient pressure response of the impacts. The transducers feature an active four-arm strain gauge bridge diffused into a sculptured silicon diaphragm. The authors are aware of pressure transducers, which exhibit spurious outputs upon initial contact with water due to differences in response of individual bridge elements to the step change in temperature and thermal conductivity. Tests by Souter and Krachman (1978) have shown that the 85-- series of Endevco pressure transducers give accurate results when immersed in water due to the sensor gauges exhibiting identical thermal characteristics meaning that the bridge balance is not upset by thermal changes. The thermal sensitivity shift, from -18°C to 93°C, is below 1.5% for all transducers used. The authors believe that the pressure reduction is due to the wet plate acting as a 'rough' surface when hitting the wave. The surface of the wave is broken leading to a more enhanced air cushion effect and hence a reduced peak pressure and correspondingly longer rise time (Shih and Anastasiou 1992).

Discusser: W. Schultz

Questions/Comments:

Your results show 30% pressure reductions for wet plates. This might indicate contact line effects. Are you aware of any model that might include this?

Author's Reply:

We are unaware at present of any model that is available to predict the features seen in the experiments.

Discusser: M. McIver

Questions/Comments:

Do you notice any deformation of the water surface before the plate impact?

Author's Reply:

The question of how the water surface responds before impact occurs is a very interesting one. Several previous studies, including Chuang (1996) and Miyamoto and Tanizawa (1985), have used high-speed cameras to film drop tests of flat plates hitting initially calm water. Prior to impact, a depression of the water surface below the plate was observed to form. The depression was found to be greater at the centreline of the plate than at the edges, so that when impact occurs a layer of air is trapped between the plate and the water, which cushions the impact and reduces pressures. The jet of escaping air can first be observed at the edges of the plate as it mixes with water blown off the top of the rising water surface there. A detailed account of this process is presented in Lewison (1970). A series of drop tests with a flat water surface were filmed using high-speed video at Manchester and the depression of the water surface before impact was clearly visible. With impact onto a wave crest, however, the evidence was not so clear. From careful scrutiny of the video, no evidence of surface movement before impact was found although in some cases an air-water jet was observed to form before impact suggesting that some disturbance of the surface was occurring. In addition, recent tests with a pressure transducer mounted below the impact zone on the bed of the flume, show that pressure at the bed begins to increase approximately 5 ms before impact occurs.

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Souter, K. and Krachman, H.E. (1978), "Measurement of Local Pressures Resulting from Hydrodynamic Impact," *ISA International Instrumentation Symposium*, Albuquerque, New Mexico (available as Endevco technical paper TP269).

Pressure Near Wave Impacts and the Role of Trapped Air

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Discusser: R.C.T. Rainey

Questions/Comments:

You say "the pressure at the rear of the caisson was relatively high" but I understand it was only $3kN/m^2$ or 30cm head of water. Is this high enough to overturn the caisson, bearing in mind the short duration? More generally, what engineers often used is some measure of the energy their structure had to be designed to absorb. Is there any prospect of you giving this rather than the pressure-impulse?

Author's Reply:

During the experiments the largest overall moment seaward about the toe was 178Nm per m run, which was significantly larger than the maximum landward overturning moment about the heel of 112Nm per m run. The structures which have reduced mass as they are designed to reduce landward acting wave impact loads could in particular be at risk. These structures have less mass to resist the seaward forces generated during the presence of the trough at the front, and because they allow overtopping. In these cases, our measurements have shown that these structures should be assessed for seaward stability. The Mustapha breakwater collapsed in a seaward direction (see Minikin, 1950), in addition Oumeraci (1994) describes several breakwaters tilting in a seaward direction. It is possible that the plunging jet is at least part of the mechanism involved in these types of failure.

These calculations were carried out in terms of pressure-impulse as the engineers involved in the MAST 3, PROVERBS (Probabilistic design tools for vertical breakwaters) project were particularly interested in the prediction of pressures and forces on these structures. It is difficult to assess immediately how much energy the caisson must be designed to absorb since much energy can be give to a jet or splash created on impact. See Cooker and Peregrine (1995) for more details on this.

Discusser: W. Schultz

Questions/Comments:

The picture you showed of the varying pressure beneath the caisson, lowest toward the sea, would seem indicative of possible failure with the caisson washing ashore rather than to sea as you mentioned. Any comments?

Author's Reply:

The previous picture (also Van der Meer's, private communication) shows the pressure distribution under the caisson when the wave initially hits the front of the caisson. At this stage the highest pressure is at the front of the caisson and drops to zero at the back of the caisson. By the same reasoning as posed in the question this would indicate seaward failure at this time. It is clearly unlikely that we could have seaward failure at the point where the wave hits the front of the caisson. The high pressure at the back of the caisson coincides not only with the impact of the jet at the back of the caisson, but also with a trough at the front of the caisson. It is therefore extremely unlikely that the caisson experiences landward failure at this point in time.

Seaward forces and overturning moments (about the toe) were measured during our experiments. See the answer to R.C.T. Rainey's question for more details.

<u>References</u>:

Cooker, M.J. and Peregrine, D.H. (1995), "Pressure-Impulse Theory for Liquid Impact Problems," *J. Fluid Mech.*, 297, 193-214.

Minikin, R.R. (1950), "Winds, Waves and Marine Structures," Charles Griffin and Company, London.

Oumeraci, H. (1994), "Review and Analysis of Vertical Breakwater Failures – Lessons Learned," Coastal Engineering, 22, 3-29.

Discusser: J. Grue

Questions/Comments:

Can you be so kind to describe how the pressure in the trapped air was determined, and also how sensitive the problem is to this pressure?

Author's Reply:

The boundary condition (value of pressure impulse) along the boundary between the trapped air and the previously unabsorbed water was set to be 0.8 of the value of the pressure impulse at positions under the jet impact. This value was found by examining a pressure-time graph for a transducer just below still water level at the back of the caisson. The peak negative pressure was approximately 0.8 that of the initial rise in pressure.

The problem is reasonably sensitive to the choice of this value. However 0.8 gave excellent results for predictions of the pressure impulse both down the back of the caisson and along the plate/beam for all three cases considered.

The choice of 0.8 is likely to be conservative for most marine events.

Numerical Simulation of the Linear and Second-Order Surface Flows Around Circular Cylinders in Random Waves

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P.D. Sclavounos
Massachusetts Institute of Technology

Discusser: A.H. Clement

Questions/Comments:

Could you give us some precision about the incident wave model you use? Is it a simple superposition of linear 1st order Stoke's waves or a more sophisticated model?

Author's Reply:

The application of 2nd order incident wave with an analytic form is very time-consuming when the number of frequencies is great.

In our computation, we generated the incident wave (2nd order) "numerically". At $t = 0$, all initial values are given to the linear problem. Then, we can get initial values of the corresponding second-order problem.

Discusser: B. Büchmann

Questions/Comments:

In your paper you examine the difference between various orders of the B-spline basis (Fig. 1). However, in your comparisons the errors seem to be dominated by the temporal discretization rather than the spatial discretization. In my opinion it would make more sense to examine the spatial discretization in the limit of small time step size ($\beta \rightarrow \infty$). Can you comment on this?

Author's Reply:

It is correct that the method presented here, has greater error from time discretization than space discretization. Most error comes from time integration and it is important to keep large β, in particular near the disturbance. However, discrete dispersion relation is a function of both Δx and Δt. Some limit cases when $\Delta x, \Delta t \rightarrow 0$ are described in my thesis, and please refer to it.

A B-Spline Panel Method Using NURBS Surfaces

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Discusser: S.B. Chen

Questions/Comments:

You mentioned using the PART method in the near field. What is the principle of this Projection and Angular & Radial Transformation? There exists another interesting method presented by A. Guillerin et al. ("Efficient Evaluation of the Ranking Singularities in Higher-Order Methods," OMAE '98, Lisbon) which provides a uniform accurate method to evaluate the simple singularity terms in both near and far fields. Do you think it is interesting to compare these methods in your approach?

Author's Reply:

The PART method is very efficient for evaluating the influence coefficients for the near field, especially when the field point is very close to the panel. This method uses relatively less numerical quadrature points than usual polar coordinate transformation method for the desired accuracy.

Discusser: C.H. Lee

Questions/Comments:

ϕ_n is approximated by B-spline and you will lose the exact representation of NURBS. I wonder why you need to introduce B-spline, while you can calculate the normal value directly from NURBS.

Author's Reply:

Depending on the type of given boundary conditions (e.g. Neumann or Dirichlet B.C.), the control net a_{ij} for ϕ or b_{ij} for ϕ_n are the unknowns to be determined. I need to approximate ϕ_n by equation (4) if the given boundary condition is Dirichlet type.

Uniform Solution for Water Wave Diffraction-Radiation at Small Forward Speed in Water of Finite Depth

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Discusser: W. Schultz

Questions/Comments:

Can you describe a physical reason for the need to have a long scale in the x direction to suppress secularities?

Author's Reply:

The reason for introduction of the second scale in the formulation is a rather mathematical one and is not due to the physics. In fact, the appearance of the secularity is due to the nonuniformity of the perturbation by τ which produce a kind of resonant interactions between the consecutive orders of perturbation. More details about this kind of problems can be found in:

Bender, C.O. and Orszag, S.A., 1978, "Advanced Mathematical Methods for Scientists and Engineers," *McGraw-Hill.*

FFT Acceleration of the Rectangular Dock Problem

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Discusser: P. Sclavounos

Questions/Comments:

The simplicity of the circulant Toeplitz matrix invites the question as to whether there is a closed form solution to the integral elevation for the dock, or an analytical formulation which accepts a fast solution primarily by analytical means.

Reply (by Evans):

The dock problem in 2-dimensions has received a lot of attention in the literature — for example, two papers by R. Holford in *Proc. Camb. Phil. Soc.* (1964) — but no explicit solution has been found. So it seems unlikely that the 3-D dock has such an explicit solution.

Discusser: T. Miloh

Questions/Comments:

With regard to the scheme that you presented for calculating the pressure on large floating platforms, I wonder how do you determine the deflection of the plate? I believe that you have to use a particular dry mode expansion where the plate stiffness enters into the formulation. There is, however, a direct way of formulating an integral equation which accounts for the boundary condition at a free-free edge. How would you compare these two approaches from the numerical point of view?

Author's Reply:

I used the integral equation for the pressure as an example equation for acceleration. The exploitation of the Toeplitz structure of the problem is equally applicable to the direct formulation for the dock. It is the uniform discretization of this rectangular geometry that allows the straightforward application of the FFT for acceleration, not the particular integral form, nor the choice of Green function.

Discusser: M. Kashiwagi

Questions/Comments:

It seems to me that the preconditioning needs some know-how and is very sensitive to the CPU time. Could you give us some helpful suggestions in using the preconditioning?

Author's Reply:

Preconditioning is almost always important for the efficient solution of linear systems arising from integral equation formulations. The most effective methods we have tried are the overlapping-block strategies (e.g. Vavasis, "Preconditioning for Boundary Integral Equations," SIAM J. Matrix Analysis and Applications, 1992, 13, 3, pp. 905-925). These are particularly suited for use with acceleration algorithms as they require a similar spatial sorting of the elements.

Discusser: F. Noblesse

Questions/Comments:

Do you think that the technique you presented would work for free-surface flows with forward speed, for which the Green function has a continuous spectrum of wavelengths (rather than a single wavelength for diffraction-radiation at zero forward speed)?

Author's Reply:

I think I answered too quickly in the affirmative at the Workshop. The method is straightforward to apply to translation invariant kernels such as 1/r, and the extension to kernels with depth dependence (such as the zero-speed frequency domain Green function I use here) is not difficult. However the forward-speed Green functions have a directional dependence that adds a further complication I do not presently know how to handle.

Stability of Time-Domain Boundary Element Models; Theory and Applications

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Jesper Skourup
Danish Hydraulic Institute

Discusser: P. Sclavounos

Questions/Comments:

We and others have extensively used our codes with the stability condition enforced and have not seen instabilities. The sawtooth oscillation is of different nature than a fundamentally unstable solution. It arises from the zero group velocity of the panel-panel waves and can be treated rationally by periodic filtering.

Author's Reply:

First, let me state that the intent of the present paper is to initiate a general discussion on the stability behaviour of boundary element models and especially a caveat against blindly following obtained stability criteria. Thus, the paper should not be viewed as a criticism of any particular existing model. In fact, our guess is that you have, in SWAN-II, a reliable and well-tested model. Furthermore, we agree with you that sawtooth instabilities may be treated rationally by applying (low-pass) spatial filtering. However, we disagree with you on the nature of the instabilities. In the present paper we have given evidence that the instabilities are caused by eigenvalues with positive real part in the spatially discretized system. Other instabilities are basically of the same nature. The only fundamental difference between this instability and a "normal" fundamentally unstable solution is that the stability analysis only predicts the latter. Recalling that a number of assumptions were made in the stability analysis, and that these assumptions are generally not fulfilled in an actual application, it should be clear that the stability criterion obtained by the analysis can be viewed as a necessary, but not sufficient (!), condition for stability. Still, in a particular model, the sawtooth instability may rarely – if ever – be seen, and as stated earlier, filtering appears to be a rational and most effective way to eliminate the problems.

Discusser: Y. Kim

Questions/Comments:

Authors described an eigenvalue problem with respect to $\tilde{\phi}^{fs}$, which are the weights of the basis function, in eq. (2) and (3). I think it is the eigenvalue problem in very local sense. Since the eigenvalue varies, (i.e. + and -) depending on the position of collocation point and considered panel. Therefore, this description may not be proper to describe the instability issue. As you showed in Table 1, the eigenvalues of a global matrix is more meaningful to explain the instability issue.

Author's Reply:

I am glad to hear that you agree with our points on the origin of the instabilities. We cannot say if solving eigenvalue problems which are local in space would be applicable in practical terms. It might be difficult to figure out how small such an area could be without losing important information. Still, solving the full eigenvalue problem for every conceivable geometry and discretization is clearly not a feasible solution. Instead a few simple cases could be investigated for each particular program, to see how the model is expected to behave in practice. However, if an efficient local solution can be obtained, then some light may be shed on the efficiency of applying also the filtering only locally. In an earlier model, such local filtering has been used with good results (see Buchmann et al., 1998). Basically, a filtering technique was applied only very close to the body intersecting the free surface, where the non-uniformity of the mesh (the basis) was large.

Comparison Between Three Steady Flow Approximations in a Linear Time-Domain Model

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Discusser: P. Sclavounos

Questions/Comments:

We have carried out similar computations a while ago for America's Cup yachts and found indeed a large discrepancy between linear theory and using the nonlinear wave elevation at the bow and stern. However, we found the added resistance to decrease with nonlinear effects. Does the type of hull and flare matter?

Author's Reply:

The size, the shape and the speed of a ship strongly influence the computated added resistance. It is therefore not realistic to compare results for a fast sailing cup yacht with results for a bulbous tanker. All I can say is that simpler linearizations lead to an underprediction of the amplitude of the bow-wave and the amplitude of the stern wave. Because the computation of the added resistance involves an evaluation of the relative wave height at the waterline, this strongly influences the added resistance.

Discusser: M. Ohkusu

Questions/Comments:

Since high added resistance at short waves has not been accurately predicted, agreement presented in this paper of the computed and measured added resistance is encouraging. Computation of wave height at the bow with RAPID flow as a basis steady flow seems to explain high wave height at the bow, observed in my experiment (Okushu & Weu, 21st ONR; Okushu, 22nd ONR), which was not predicted by other linear theories. My question is: why did you not try a Neumann-Kelvin solution as a basis steady flow before trying RAPID nonlinear flow? Does not Neumann-Kelvin flow as a basis flow give almost the same results for added resistance and wave height at the bow as RAPID does?

Author's Reply:

We could have tried Neumann-Kelvin flow of course, but it is hard to implement this flow in our code because it completely changes the linearization and a totally different free-surface condition shows up. If we would have tried it, the results would probably have been somewhere between the results with RAPID flow and the results with double-body flow.

Discusser: M. Tulin

Questions/Comments:

It was very good to make these comparative studies, especially of added resistance. I wonder how the comparison with experiment depends on the hull shape, particularly for ships with high flares? I imagine that for high flare (in view of your discovery about the importance of wave height elevation at the bow) it will be necessary to go beyond ordinary 3-D panel methods, i.e. perhaps to a method 2D+t techniques as mentioned earlier by Faltinsen. Here I point out work of Maruo on ships in headseas using the non-linear flow theory (2D+t), reported at the 1994 ONR Symposium.

Author's Reply:

As long as RAPID can still give a reasonable prediction of the bow wave, as is the case for this ship, our linearization method is applicable. Of course if RAPID breaks down or gives inaccurate results, something else must be tried. Our method has to be set up again in another way then because we make use of the fact that the steady flow satisfies the non-linear steady free-surface condition. If 2D+t techniques are used, the free surface condition that we use will become very different and a whole new simulation program will have to be set up. I'm afraid there is no time left for me to do that.

Discusser: R. Beck

Questions/Comments:

Can you give any reason for the poor agreement at the long wavelength? Normally, you would expect trouble at the short wavelengths.

Author's Reply:

In all our calculations we used the same free surface grid, which stretched from $x=-L$ to $x=L$ and from $y=-L$ to $y=L$. A long wave, with wavelength larger than one, therefore does not fit properly on the grid and some inaccuracies may occur. Furthermore, at the low frequencies the restoring forces become very important to predict the ship's motions. These restoring forces are estimated from the steady fluid velocities on the hull and involve numerical differentiation of these velocities over the hull. There might be inaccuracies in the calculations of these derivatives, especially near the stagnation point at the bow.

Downward Lifting Force of Submerged Body with Minimum Drag

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Shigeki Nagaya
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Discusser: R. Beck

Questions/Comments:

In the viewgraph you showed of the drag of a "winged body," where were the wings placed and were there effects due to longitudinal location?

Author's Reply:

It is true that the location and shapes of foils affect the reduction of resistance. Some preliminary experiments support this. They may encourage us to study about the optimum configuration of body and foil under free-surface.

Discusser: M. Tulin

Questions/Comments:

Comment: I have shown in many 70's reports on wave-free bodies that for three-dimensional slender bodies the zero wave resistance is realized when the downward force equals the buoyancy; this corresponds to a proportionality between the horizontal doublet strength and the vertical lift. These bodies seem useless, since they have no payload. I have also described a successful singularity system producing no resistance in 2-D and low resistance in 3-D, see 9th ONR Symposium Proceedings "Ships of Small Wave Resistance."

Author's Reply:

Thank you for the valuable comments. The present work supports theoretical works by the discusser. We are now extending our studies to 3-D cases.

On the Piston Mode in Moonpools

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Discuser: T. Miloh

Questions/Comments:

The moonpool problem which you have presented is related to the bottomless harbor problem previously discussed in the literature by Garret and Miles, some 25 years ago. In 1983, I presented a paper on the circular bottomless pool in a meeting in UC Berkeley honoring the 70th birthday of John Wehausen. In the proceeding paper, I provided an analytical solution for the piston $(m = 0)$ and slashing $(m = 1)$ modes which I believe are similar to the results that you showed. There is also a closed form solution for the resonance frequency as a function of the geometry and the incident wavelength. Also given are added-mass and damping coefficients as well as force measurements on a model.

Author's Reply:

The geometries I consider are different from your solar pond. I do not solve the problem of water motion in the moonpool under external wave action. I just formulate an eigenvalue problem, and solve it, to obtain the natural frequencies of oscillation and the associated shapes of the free surface.

Discusser: R. Rainey

Questions/Comments:

Is this "piston mode" excited by a current, as you showed at the Oxford workshop?

Author's Reply:

At the Oxford workshop I showed a video, illustrating resonance in the moonpool induced by forward speed of the ship. This was due to alternate vortex shedding at the moonpool edges, which were rounded — with sharp cornered no current induced resonance was observed.

Discusser: J. Newman

Questions/Comments:

Why didn't you use a panel code to analyze this problem directly? I might add that we have used two different approaches: (1) direct with the moonpool free surface treated as part of the global free surface, and (2) with a "lid" on the moonpool, including one or more generalized modes for this lid (piston, sloshing,...). Two nice features of the later are (a) the ill-conditioning at resonance is transferred from the large linear system for ϕ to the small set of equations of motion for the lid modes, and (b) it is easy to add external damping of the lid mode(s).

Author's Reply:

Our intention is to tackle the problem of joint barge and moonpool motion in waves with a panel code, as you suggest, assuming the moonpool to be solid. Given the large dimensions and shallow draft of the considered moonpools, we wanted beforehand to ascertain the validity of the solid moonpool approximations, and check how flat the free surface is at resonance.

ERRATA:

Equations (12) and (13) are off by a factor of 2.

Replace with:

$$C = \frac{1}{\pi}\frac{1}{b\,l\,h}\left(b^2\, l \operatorname{argsinh}\frac{l}{b} + b\, l^2 \operatorname{argsinh}\frac{b}{l} + \frac{1}{3}\left(b^3 + l^3\right) - \frac{1}{3}\left(b^2 + l^2\right)^{3/2}\right) \qquad (12)$$

$$\omega_{00} = \sqrt{\frac{g}{h + 0.47\, b}} \qquad (13)$$

An Experimental Verification of the Wave Drift Damping Formula

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Discusser: R. Rainey

Questions/Comments:

You show only stable yaw equilibria in Fig. 1. Is there not a problem of unstable yaw equilibria — generally known in the oil industry as "fishtailing"?

Author's Reply:

In the numerical simulation, with current action only, we sometimes observed a "limit cycle" that corresponds to the "fishtailing instability" known in Naval Engineering. These simulations can be found in works by Simos, Tannuri & Pesce, presented at OMAE'98 Conference and, in general, the observed instabilities have a small amplitude. The dynamic behavior obviously depends on the assumed hydrodynamic model. If a standard bi-linear damping law is taken, a small self-excited oscillation (amplitude around 1 or 2 degrees) is observed, but only when the equilibrium yaw angle is the trivial one (namely 0 degrees); however, if a linear yaw damping is also included, such oscillations are unlikely to occur around the trivial equilibrium, at least for typical mooring pre-tensioning levels. Despite not yet a closed result, an extended work, addressing a comparison among different hydrodynamic models, will be published by Garza-Rios, Matsuura, Bernitsas & Nishimoto at OMAE '99. I do not recall if the same pattern was numerically observed when a following sea is also present. In the experiments the "fishtailing instability" was not observed, eventually because it was too small to be detected.

Discusser: J. Grue

Questions/Comments:

The authors claim the acceptance of formula (1b). I have two comments to that: Formula (1b) was first proposed by Clark, Malenica and Molin AOR (1993). They found that the formula reproduced the results due to the method of Emmerhoff and Sclavounos JFM (1992), for an array of fixed vertical cylinders. I think that, if formula (1b) should have a name, it should be called the Aranha, Clark, Malenica and Molin – formula.

My second point is that the formula is not general. This means that it will fail in many cases. Turning to the steps in Aranha's work JFM 1996, he first proposes a formula for the far-field amplitude of the ring waves. I have not seen a single case where his far-field amplitude agrees with a complete formulation. Documentation is found in Finne and Grue JFM 1998. In his second step, Aranha plugs his far-field amplitude into the "Oslo-formulation" (Nossen, Grue and Palm, JFM 1991), obtaining formula (1b). This formula may be useful for engineering applications, but it is not complete. Documentation of this statement may be found in Malenica and Molin AOR 1995, Finne and Grue JFM 1998, work by Hermans and co-workers, ONR – Meeting in Trondheim 1996.

Author's Reply:

1. I didn't call the formula "Aranha's formula." Others did, including J. Grue in the paper he has written with Finne. I could not deny that this denomination is important to my work but one should agree that, except in some few circumstances, in the majority of the cases "Aranha's formula" has appeared in the literature in a critical context. I have been just fighting these last nine years to give to this formula a more even appraisal, that I certainly believe it deserves, and I interpret Grue's claim about the denomination as an indication that the formula is gaining some credibility even among their more fervors opponents;

2. With respect to the "priority" of the work one should just recall some facts: the first version of the paper was submitted to JFM in February of 1990 and it was accepted for publication four years later, in February of 1994. Meanwhile, I presented the formula in the IWWWFB that took place at Woods Hole (1991) and, to my understanding, Clark & Malenica & Molin short note published in the Applied Ocean Research, in 1993, was based on my presentation at Woods Hole, although the umbilical link between their work and mine was not very clearly stated in their paper. Apparently, after observing the perfect agreement between the formula and numerical results when waves and current are collinear they just tried, in an "ad hoc" manner and monitored by their numerical results, to find an extension of the formula when waves and current were not collinear. As the authors say in their paper, they had not, at that time, a single plausible explanation why the formula works;

3. "I have not seen a single case where the far field amplitude (analytically derived, he means) agrees with a complete formulation:" Grue's assertion is numeric in essence and it has a weak theoretical basis. For how could one obtain "exact results" (even he agrees with this in some cases) placing a completely "wrong amplitude" in the far field expressions? This is a bizarre result that he should explain, not me. This same question has already been raised in Marseille Workshop, where I gave the same answer, and this is another aspect of this already long controversy: we are making very boring this dispute since no new ideas or facts are incorporated into the discussion, and when they are, as the experimental results we have shown in the present Workshop, they are ignored and dismissed;

4. In my point of view, it is a daring position to stick with (some) numerical results and reject the formula without any more persuasive theoretical argument; in spite of the fact that numerical analysis has become very sophisticated and widespread lately, I myself, having to confront between Green's Theorem, the basis of the mathematical demonstration, and some numerical results, I would stick to Green's Theorem to the end. Or, in other words: when a

100% disagreement between the formula and numerical results are found, as claimed by Grue in some cases, I would say that the numerical result is 100% wrong. I only will be convinced of the contrary if a theoretical flaw is pointed out and I ask the community to find it; because, in the other way, how could one give up from a mathematically proven result, obtained from the basic set of equations and that provides, with the simplicity that it has, an impressive agreement with experimental results with practical relevance for the oil industry? How could one dare to give this up?

5. Within the scientific world my work is only a small side remark. But even in this small size it stands I will not refrain to quote here a phrase from a first rank scientist: Planck once said that "a new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents finally die and a new generation grows up that is familiar with it." I hope this will not be our fate because even if we are not a first rank scientists we should, at least, have some commitments with the real world, with problems that cannot wait until our death to be solved.

Discusser: S. Malenica

Questions/Comments:

This paper is interesting because, once again, it demonstrates the importance of the wave drift damping calculations, the neglect of which can lead to completely wrong results. However, I disagree with the authors conclusions that the obtained results present the final proof for the validity of the simple formula [1,3] for wave drift damping, which is claimed by the authors to be an exact formula. In fact I think that it just happens that the formula works quite well for this case but in any case this is not the final proof that it should work for arbitrary case.

This is an old question, well known to the participants of this Workshop, and I would like to present some results which can hopefully help in an improvement of the formula. On figure 1 the analytical results for 4 fixed bottom-mounted cylinders are presented. This figure is taken from [1], where the 3D formula was discovered for the first time, and the agreement between the formula and the direct analytical calculations is perfect. The same is true for the single fixed cylinder (or arbitrary number of cylinders) for any incidence. But if we allow the cylinder to move under the action of the waves (fig. 2 from [2]) there is a large difference between the formula and the direct semi-analytical calculations, which in turn agree very well with the numerical calculations of Grue. One interesting point is that the difference between the direct calculations and the formula is proportional to cos 2β, with β denoting the incident angle. We note however that this is the case of the finite water depth, which is not treated by Aranha, but the equivalent simple formula was proposed in [2].

In the case of a fixed truncated cylinder (fig. 3), where again a semi-analytical solution can be found, and the formula and direct calculations disagree but this difference is less pronounced.

The same is true for the case of the sphere as shown on the same figure. However in this case the difference between the formula and direct calculations is no more proportional to cos 2β.

Finally we take the case of a simplified FPSO geometry (similar to the case treated in this paper) and on figure 4 we show the results for the surge wave drift damping coefficient for the fixed and freely floating case. We can observe that the formula works reasonably well. However, knowing the uncertainties in evaluating the viscous part of the forces, it seems to me that the agreement between the experiments and the calculations cannot be qualified as a final proof of the formula, especially because it is claimed that the formula is exact under the assumptions of the potential theory.

It seems that the formula works perfectly only in the case where there are no local waves in the zero forward speed solution for the potential i.e. only the case of the fixed complete cylinders. When local waves are present in the zero forward speed solution (freely floating complete cylinder, fixed truncated cylinder, sphere...) the formula disagree with the direct calculations.

My feeling is that there is something missing in the formula, an additional term either simple and explicit or including some numerics, in order to account for local waves.

Anyway, I don't think that the discussions will stop here, and we will certainly discuss again about the formula on the next Workshops and this is rather good (exciting). I guess Jose's answer but...

References

1. Clark, P.J., Malenica S. & Molin B. 1993. An heuristic approach to wave drift damping. *Appl. Ocean Res.* 15, 53-55.

2. Malenica S., Clark P.J. & Molin B. 1994. Wave and current forces on a vertical cylinder free to surge and sway. *Appl. Ocean Res.* 17, 79-90.

3. Aranha, J.A.P. 1996. Second order horizontal steady forces and moment on a floating body with small forward speed. *J. Fluid Mech.* 313, 39-54.

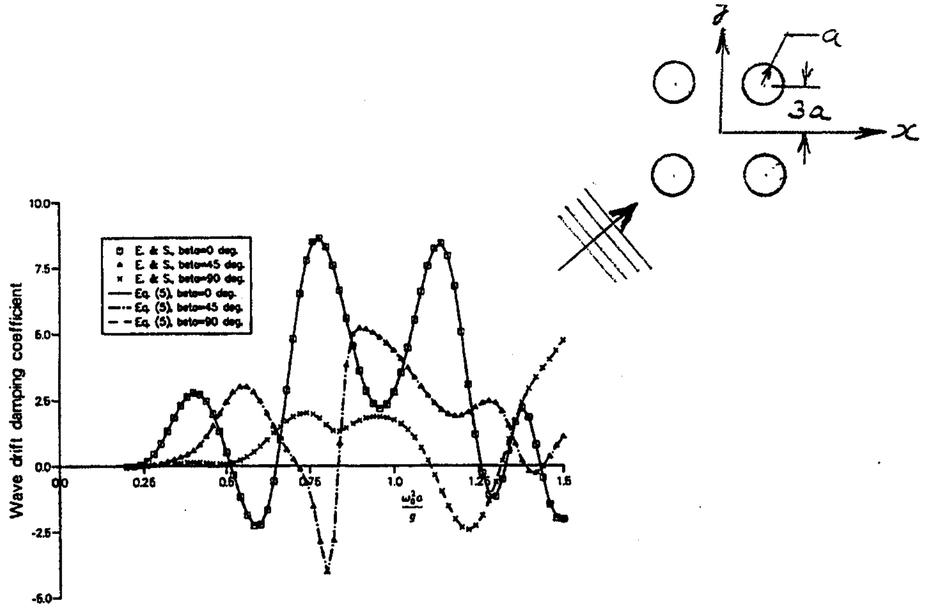


Figure 1: *Wave drift damping coefficient B_{11} for four fixed, bottom mounted circular cylinders [Eq. (5) means simple formula and E&S means the direct analytical calculations].*

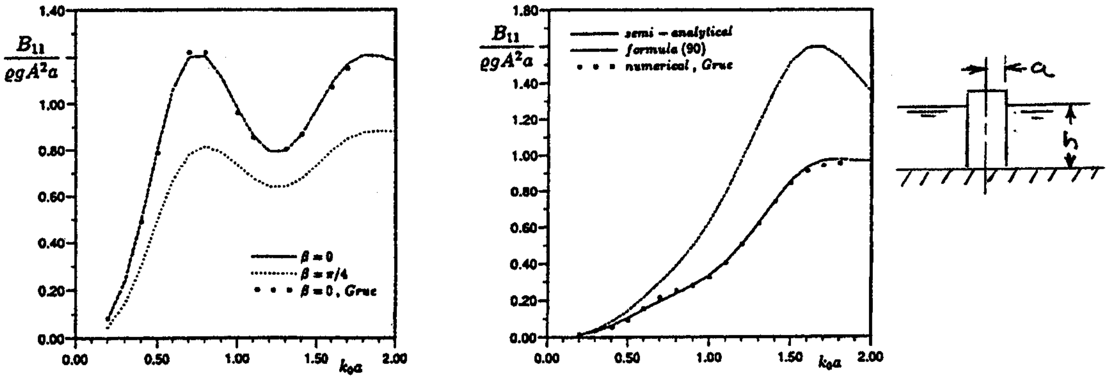


Figure 2: *Wave drift damping coefficient B_{11} for single cylinder. Left - fixed cylinder, right - freely surging cylinder.*

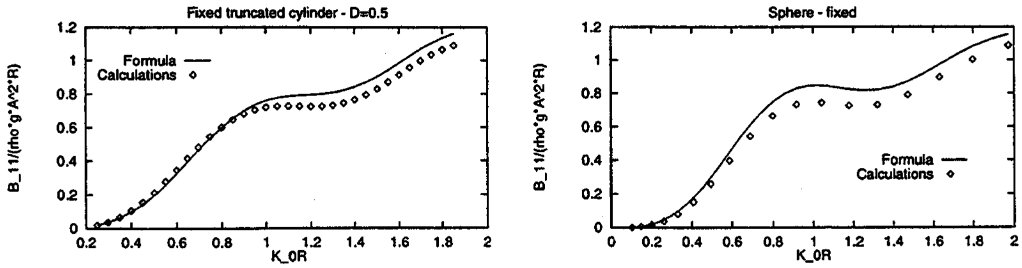
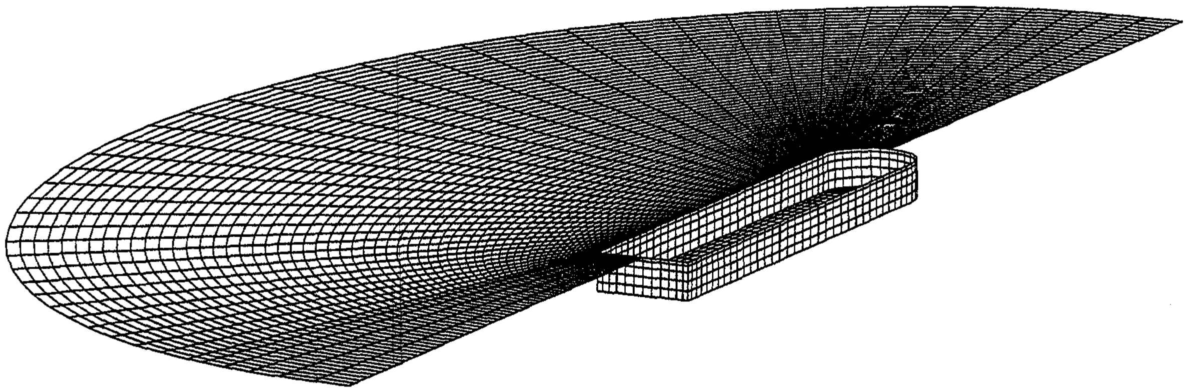


Figure 3: *Wave drift damping coefficient B_{11} for fixed truncated cylinder for draft radius ratio equal to 0.5 (left) and for fixed sphere (right).*



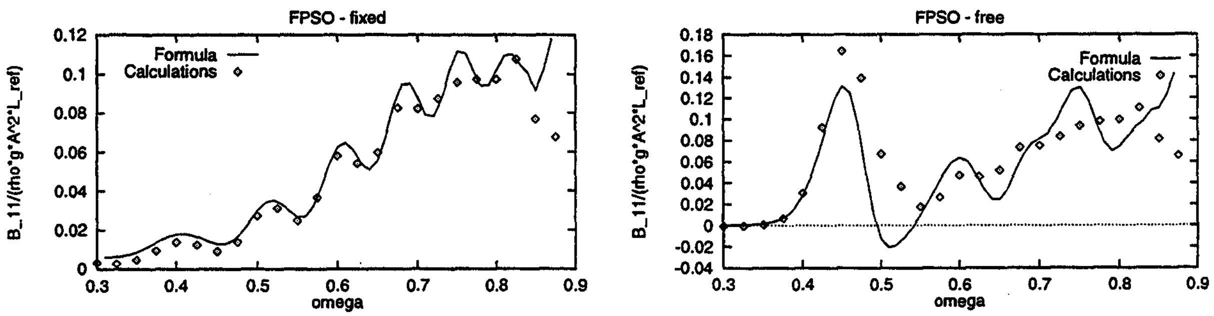


Figure 4: *Wave drift damping coefficient* B_{11} *for simplified FPSO geometry,* $(L = 257.5m$, $B = 51m$, $T = 20m)$.

Author's Reply:

1. I never claimed that the experiments would be a "final proof" for the validity of the formula; as it is known, they never validate completely any theoretical results, they can only disproof them (see K. Popper, "The Logic of Scientific Discovery"). In a positive sense, experiments are used to give more credibility to a theoretical result, the confidence increasing with the exposure of the theory to more and more different experimental set ups. In the other hand, it does not seem sound to ignore the fact that the only known experiments about WDD (decay test and the present one) give support to the formula. In particular, one specific point should be noted in this context: Finne & Grue claim that for a ship like structure the disagreement between the numerical results and the formula is large for frequencies in the range $kL > 6$, where L is the ship length; the numerical results presented by Malenica for a FPSO free to move (fig. 4 in his question) also show, in a similar frequency range ($kL > 5$ in the case) and for the same B_{11} coefficient, a large disagreement between the numerical results and the formula. Recently, however, Trassoudaine & Naciri (AOR, 1999) analyzed the experimental results obtained at MARIN for a FPSO and observed a very good agreement between the formula and the experimental results in the frequency range $0 < kL < 16$ ($0 < \omega < 0.8$ rad/s in Malenica's fig.4). This comparison has been shown in the Workshop and, in my understanding, there are here three options left: either their claim that the disagreement is systematic is not correct, and then the numerical results and the formula are coincident at least (and by chance, eventually) for the Marin's FPSO, or else the experiments are not correct, or even, as a last option, their numerical results are wrong. In any case the burden is on them to show what is happening here.

2. The "uncertainties in evaluating the viscous forces" have in general been used, sometimes in an improper way one must say, to explain differences between theory and experiment. This is the first time, however, I have seen one insinuate that viscosity should be blamed for the observed agreement. It is worthwhile in this context to review briefly the three distinct levels we have been through while discussing the formula. First, we have a *physical model*, that in essence says the following: when terms of order U^2 are ignored, the wave amplitude in the far field can be obtained looking only to the interaction between the far field "free waves" and the current. This model played an essential role in foreseeing results and it should not be forgotten that, in science, models, assumptions, hypothesis are essential to open new routes, to help us to see the problem in a new light; as Novalis once said "hypothesis are like nets, only throwing them one can get something out." But, in spite of its well-established affiliation, most workers in this topic have never seriously considered this model. Second, the final result has been *mathematically proven*, first in 2D and after in 3D; although the 3D proof presented in a 1996 paper has some algebraic inaccuracies, already corrected, the 2D proof was accepted without restriction and it should be already enough to convince people about the correctness of the physical model; but it didn't. Third, *experimental results*, in two different set ups, obtained by different laboratories (Marin and IPT) and analyzed by different groups, give support for the formula but they are now ignored or disqualified. In this way it is almost impossible, within the traditional science, to go a step further in this subject.

3. What has been left by them as the only criteria to accept or not the formula are the numerical results, with their peculiarities; but even here, even restricting our sight to the smoky guerilla warfare of numbers, let's see what Malenica has as novelties in this scenario. In fig. (4) he presents results comparing the formula and numerical calculation for a FPSO; comments about these results have already been made in item (1) above, suggesting that Malenica's result is in doubt, at least. In fig. (3) he compares a circular cylinder and a sphere; these results are old and show, in my understanding, very good agreement, not a disagreement, unless he believes that numerical results are always "exact." For instance, Kinoshita & Bao (1996), JMST, 1:155-173, used also a semianalytical method to obtain perfect agreement between the formula and numerical results for a fixed truncated cylinders; these authors also observed that the rate of convergence of the semianalytical method is much slower when the cylinder is free to oscillate, an observation that has been apparently ignored by both Grue and Malenica. In fig. (2) he uses again the same numerical result he had shown four years ago in Oxford. It deals with a circular cylinder free to surge with radius a, draft $d=a$ and placed in a water with depth $h=a$. In my understanding this is a pathological example that should be looked with caution. On one hand, when a small clearance between the sea bed and the cylinder is allowed one observes an extremely sharp variation of B_{11} with the frequency, a behavior well captured by the formula, what turns difficult to understand why the formula fails only when $d=h=a$ and the body is free to move (see Kinoshita & Bao for the cases $d=1.75a;h=2a$ and $d=2.0a;h=2.5a$). On the other hand, the $\cos 2\beta$ behavior of the difference, observed only in the case $d=h=a$, is too well behaved to be accidental and may be a symptom of a pathological behavior. I do not have a clear answer for this puzzling result although I have some possible hints: first of all, I am not at ease considering a free floating cylinder without clearance, I do not feel this is a clearly stated problem (notice, in particular, that heave radiation damping should go to zero as the clearance does and the response should then become unbounded in this limit); second, I suspect that secular behavior at the far field, inherent in Malenica's and Grue's formulations, may be a problem when the cylinder imposes to the whole fluid depth the surge motion, even more in shallow water. Anyway, I don't believe that this example alone can be taken as a major case against the formula.

4. "The 3D formula was discovered for the first time by Clark & Malenica & Molin:" this phrase would be correct if instead of "discovered" one uses "applied." For, what they have done was just to apply the formula to a circular cylinder and compare with numerical results obtained with a program developed by Emmerhoff & Sclavounos. Once a perfect agreement was observed, they used a "trial and error" method, monitored by Emmerhoff & Sclavounos program, to extend the formula for the case where waves and current were not collinear. This is not a guess or an impression: Malenica himself told me it, in several conversations since the Oxford Workshop.

5. As a matter of fact, the word "discovery" should be applied with restriction even to the formula I have obtained. In my point of view, the only real discovery in this topic was made by Wichers, 20 years ago, who not only observed the phenomenon but also give it the proper explanation; this is really an important work, the formula I obtained having the only merit to make very easy the computation of the wave-current interaction effect (although it has some theoretical interest in itself).

Numerical Investigation of Shallow-Water Wave Equations of Boussinesq Type

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Discusser: H. Bingham

Questions/Comments:

Would you please elaborate on how you include the ship in your solution?

Author's Reply:

We thank Dr. Bingham for his question which gives us the chance to summarize the two methods applied to approximate the ship influence on the ambient flow. Due to the small value of the slenderness parameter (defined by $\delta = \frac{\sqrt{S_o}}{L} = 0.072$ with the main sectional area S_o and the length L on the water line) of the subject inland passenger-ferry, we apply the classical slender-body theory to approximate the near-ship flow. According to the technique of matched asymptotic expansions, first introduced by Tuck (1966), the depth-averaged transverse velocity at the ship location can be written as

$$-v\,|_{y\to 0^\pm} = \pm\frac{1}{2}\frac{V}{h_o} S_x(x)$$

where $S(x)$ denotes the local cross-sectional area, see e.g. Mei and Choi (1987). It defines the boundary condition at the ship location and can be easily implemented in our numerical scheme for the modified Boussinesq equations in the far field.

On the other hand, the underwater form of the subject ship is also characterized by a large beam-draft-ratio $\left(\frac{B}{T} = 7.37\right)$ which suggests a reasonable approach proposed by Hogner (1932), namely replacing the hull boundary condition by a pressure distribution p_s proportional to the local draft $T(x,y)$:

$$p_s = \rho g T(x,y)$$

which can be directly included into the modified Boussinesq equations.

References:

Hogner, E., "An Interpolation Formula for the Wave Resistance of Ships (in German)," Jahrbuch der Schiffbautechnischen Gesellschaft (STG), Vol. 33, 1932, pp. 452-456.

Mei, C.C. and Choi, H.S., "Forces on a Slender Ship Advancing near Critical Speed in a Wide Canal," *Journal of Fluid Mechanics*, Vol. 179, 1987, pp. 59-76.

Tuck, E.O., "Shallow Water Flows Past Slender Bodies," *Journal of Fluid Mechanics*, Vol. 26, 1966, pp. 81-95.

Discusser: J. Wehausen

Questions/Comments:

Both the photograph of an experiment made by the authors and the data look very similar to those occurring in a paper by R.C. Ertekin, W.C. Webster and J.V. Wehausen [Proc. 15th Symp. Naval Hydrodynamics, 1984, pp. 347-364]. In that paper the computations were restricted to two dimensions and the moving disturbance to a pressure distribution. Both the Green-Naghdi (hereafter G-N) equations and T.Y. Wu's version of the Boussinesq equations were used in order to compare the results predicted by the two equations and to compare these with the experimental results. The restriction to two-dimensional computations was removed in a later paper [J. Fluid Mech. 169 (1986), 275-292], but here also the moving disturbance is a pressure distribution and not a solid body. Since the authors of the present paper have replaced the hull boundary condition for a ferry by a pressure distribution proportional to the local draft, following a suggestion of Hogner's, it seems evident that their computation leading to Figure 2 could be repeated using our G-N program. This might be a worthwhile project, allowing comparison of the results obtained from Boussinesq and G-N equations for a 3-dimensional problem. The authors may also be unaware of a more recent paper by Ertekin, Qian, and Wehausen [Proc. 7th Internat. Offsh. & Polar Engrg. Conf., 1997, pp. 238-246], where Boussinesq equations are used and exact body boundary conditions are satisfied, but for a wall-sided ship.

For those who have forgotten the basis of the G-N equations, we recall that all boundary conditions and conservation laws are satisfied exactly, some of the latter integrated over depth, but that the field equations are approximated. For those who are uneasy about this latter approximation, it is suggested that they consult a paper by Shields and Webster [J. Fluid Mech. 197 (1988), 171-199], where computations for steady periodic waves over a flat bottom compare the results for three levels of the G-N equations with those from conventional approximations.

Author's Reply:

We greatly appreciate Prof. Wehausen's references to the important contributions made by his group at Berkeley and by Professor Wu's group at Caltech toward the understanding of ship waves in shallow water through model experiments and comparative computations using both

Green-Naghdi restricted theory and Boussinesq type equations. We quite agree that it would be useful to carry out a similar comparison for a 3-dimensional situation, either pressure patch or ship hull. Our passenger ferry seems to be a suitable candidate for such benchmark studies. We concur with Professor Wehausen on the merits of the Green-Naghdi theory and will try to incorporate it into our own shallow water wave studies in the near future following his suggestion.

A Boussinesq-Panel Method for Predicting the Motion of a Moored Ship in Restricted Water

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Discusser: J. Newman

Questions/Comments:

Have you considered the (hopefully small) errors in the Haskind relations due to the different free-surface conditions associated with ϕ_j and ϕ_I ?

Author's Reply:

My assumption is that the incident wave is of mild enough slope near the ship that it essentially satisfies the linear free-surface conditions, and hence these errors are small. This may be a problem for very long waves (shallow water waves) where the linear condition is only realistic for truly infinitesimal waves.

Discusser: R. Beck

Questions/Comments:

Have you considered conducting regular wave experiments that would avoid the complications associated with measuring random wave spectra?

Author's Reply:

Yes, regular wave experiments, as well as strictly linear test cases are all included in the paper which is referred to in the abstract.

Discusser: S. Malenica

Questions/Comments:

How do you model your wall (L type) in your frequency domain diffraction radiation code?

Author's Reply:

WAMIT models have an infinite wall, (or two semi-infinite walls meeting at a 90° angle) using the method of images.

Discusser: Tao Jiang

Question:

1. Could you show one example of harmonical case (for instance at f=.08 Hz) ? I think it would help to know the experimental validation of your work.

2. How do you simulate the wavemaker?

Author's reply:

1. I refer you to the Coastal Engineering paper cited in the abstract for such results. One of the results presented there is a linear case where the ship is forced by a spectrum of very mild-slope waves. From this we demonstrate that the linear RAO (i.e. the frequency-response divided by the Fourier coefficients of the incident wave) are recovered in all modes.

2. For the results presented here, the wavemaker is simply a flux applied at the boundary of the Boussinesq domain.

The Spinning Dipole: An Efficient Unsymmetrical Numerical Wavemaker

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Discusser: T. Korsmeyer

Questions/Comments:

At what depth do you set the dipoles? And does the depth depend on frequency?

Author's Reply:

We have made some test to vary the immersion depth as a function of wave frequency. Such variation results in some minor changes in the near field kinematics, and in the far field wave amplitude which must be adapted to reach the desired amplitude. But, in all cases, the singularity must be located at a minimum distance below the moving free-surface to avoid the exit of the dipole from the fluid domain when the free surface moves down. At last we found it more practical to keep it at a fixed location (precisely at mid-depth in the reported computations).

Discusser: B. Büchmann

Questions/Comments:

Would it be an advantage to consider a continuous sheet of dipoles spanning the water column?

Author's Reply:

No it wouldn't. You must consider that the final goal is to permit simultaneous generation and absorption, which requires to minimize the interaction of the waves reflected by the body with the D.I.S. generator. These waves must just "pass through" the generator without "feeling" it. This interaction would be important if one uses a sheet of singularities rather than discrete singularities as proposed here.

Discusser: D. Evans

Questions/Comments:

(Comment): This idea reminds me of the work of Ogilvie (JFM 1963) who proved in linear theory that a submerged solid horizontal cylinder which makes small circular motion generates waves in one direction only. In order to construct the physical cylinder, he needed to use an infinite series of multipoles, even for heave motion, and odd for sway motions. By combining these $\frac{\pi}{2}$ out of phase he obtained wave cancellation in the same way as the author. This idea formed the basis for the development of the Bristol Cylinder wave energy device (Applied Ocean Research, 1979, Vol. 1, No. 1, Evans, D.V. et al).

Discusser: P. Stansby

Questions/Comments: Is there potential for using the spinning dipole for wave absorption?

Author's Reply:

Using the spinning dipole for wave absorption implies an instantaneous adaptation of its characteristics to the incident wave field to be absorbed. This would not be a problem in frequency domain. But, because we work, here, in the time-domain, such a feedback law is not available, nor easily conceivable.

Discusser: J. Newman

Questions/Comments:

In linear theory it is clear that a singularity consisting of two superposed symmetric and anti-symmetric singularities with a 90° phase difference will radiate uni-directionally. The spinning dipole is one example. Is it preferable to others, e.g. the sum of a source and vortex?

Author's Reply:

Such an association of a source and vortex would indeed have the same property of uni-directional wave radiation. Nevertheless, we prefer using dipoles in order to preserve the mass-balance at every time step in this time domain simulation code.

A Hierarchical Interaction Theory for Wave Forces on a Great Number of Buoyancy Bodies

Masashi Kashiwagi
Research Institute for Applied Mechanics, Kyushu University, Fukuoka, Japan

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Discusser: M. Murai

Questions/Comments:

In Section 3 in this paper, you named "hierarchial interaction theory" and cited some related references in the presentation. We have already published the almost same mathematical idea in Japanese about one year ago.

Author's Reply:

I am aware of your work published in Journal of the Society of Naval Architects of Japan (Vol. 183, June 1998). My first paper introducing the present idea and numerical examples was published in the Proceedings of 14th Ocean Engineering Symposium in Japan (July 1998). I think these works were done independently, and in fact not completely the same; that is, there are a couple of differences. For instance, 1) Evanescent wave modes are neglected from the outset in your paper, which is not exact; 2) Solution method for the radiation problem in the present paper is based on the mode-expansion method, which is different from your work; and 3) Numerical results for up to 5120 cylinders are first shown in the present paper.

Experimental Investigation on the Wave Decay Characteristics Along a Long Array of Cylindrical Legs

Hiroshi Kagemoto, Motohiki Murai and Masanobu Saito
Department of Environmental and Ocean Engineering, University of Tokyo, Japan

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Discusser: T. Miloh

Questions/Comments:

In your talk you introduced an additional damping force which appears as a term $-N\phi$ in your modified Bernoulli equation (2). For a slightly viscous fluid and laminar flow this term should be $-2\mu\nabla_h^2\phi$ where ∇_h^2 is the horizontal Laplacian and μ is the dynamic viscosity. For a turbulent flow μ is probably two orders of magnitude larger, i.e. around 100 times. How good is this value compared with the empirical value which you found for N?

Author's Reply:

We think our $-N\phi$ should be compared with $-2\nu\nabla_h^2\phi$, where ν is the kinematic viscosity. Supposing that $\nabla_h^2\phi \approx k^2\phi$ where k is the wave number, then, since $k^2 \approx 100m^{-2}$ and $\nu \approx 10^{-6}m^2/sec$, $2\nu\nabla_h^2\phi \approx 2\times10^{-4}sec^{-1}$ while our N is 10^{-1} sec^{-1}. Therefore our N is 500 times larger than the value you suggested. However, as you also suggested, the effective viscosity can be several orders of magnitude larger if the flow is turbulent.

Discusser: B. Molin

Questions/Comments:

We congratulate the authors for having undertaken such an impressive (and much needed) experimental study. We agree with them that viscous effects are responsible for the discrepancies between potential flow numerical results and measurements. However we feel that viscous dissipation takes place in the boundary layers on the cylinders, not everywhere in the fluid. Here we propose a potential flow approach that allows for a controllable energy dissipation at the cylinder walls. It consists in making them slightly porous, and stating that the traversing velocity is related to the ambient pressure. Such boundary conditions are widely used in coastal engineering, to simulate partially reflective boundaries.

The boundary condition that we have used is

$$\partial\phi/\partial R = -i\varepsilon/a\phi \qquad \text{(A1)}$$

where $\Phi = \Re\{\phi \exp(-i\omega t)\}$, a is the cylinder radius, and ε is a small constant.

To determine realistic values for ε we make reference to the well known case of a 2D cylinder in infinite fluid, in sinusoidal flow $U(t) = U_0 \cos\omega t$.

We assume the flow to be laminar and non-separated. In such case the additional force due to viscosity is (Stokes, 1851)

$$F = \Re\left\{(1-i)\rho\omega U_0 D^2 \sqrt{\frac{\pi}{\beta}} e^{-i\omega t}\right\} \qquad \beta = \frac{D^2}{\nu T} \tag{A2}$$

Taking (A1) as the boundary condition, we easily obtain the velocity potential as

$$\phi = \left[U_0 R\cos\theta + U_0 \frac{a^2}{R}(1+2i\varepsilon)\cos\theta\right] e^{-i\omega t} + O\left(\varepsilon^2\right)$$

and the force acting on the cylinder

$$F = \Re\left\{-2i\rho\omega\pi a^2 U_0 (1+2i\varepsilon) e^{-i\omega t}\right\} \tag{A3}$$

Identifying energy dissipating terms in (A2) and (A3) yields the ε value

$$\varepsilon = \frac{1}{\sqrt{\pi\beta}} = \frac{1}{\sqrt{\pi}} \frac{\sqrt{\nu T}}{D}$$

With a diameter of 0.165 m and a period of 0.7 s we obtain $\varepsilon = 0.003$. However we know (e.g. see Sarpkaya, 1986) that at such β numbers as achieved here $(\beta \simeq 40\,000)$ the flow is no longer laminar, but turbulent. This results in the drag coefficient being about 5 times the laminar case value, or, as suggested by Troesch and Kim (1991), the kinematic viscosity ν having to be replaced by an 'effective' viscosity about 50 times larger.

Here we deal with wave flows, and it is not very clear how energy dissipation in the boundary layer is affected. Further the cylinders are truncated and some flow separation results, inducing further dissipation. So we feel that a value of 0.3 for ε is plausible.

It is straightforward to extend the Linton & Evans method to take account of the modified boundary condition (A1). In fact, the only things that change is the linear system of equations for the interaction coefficients A_m^k:

$$A_m^k + \sum_{j\neq k}^{N_c} \sum_{n=-\infty}^{\infty} A_n^j Z_{n0}^{(1)j} e^{i(n-m)a_{jk}} H_{n-m}\left(k_0 R_{jk}\right) = -I_k^{(1)} e^{im(\pi/2-\beta)}$$

which becomes:

$$A_m^k \left[1 + i\frac{\varepsilon}{k_0 a}\frac{H_m}{H_m'}\right] + \sum_{j\neq k}^{N_c} \sum_{n=-\infty}^{\infty} A_n^j Z_{n0}^{(1)j} e^{i(n-m)a_{jk}} H_{n-m}\left(k_0 R_{jk}\right)\left[1 + i\frac{\varepsilon}{k_0 a}\frac{J_m}{J_m'}\right] =$$

$$-I_k^{(1)} e^{im(\pi/2-\beta)}\left[1 + i\frac{\varepsilon}{k_0 a}\frac{J_m}{J_m'}\right]$$

The expression for the potential remains the same i.e.:

$$\varphi_D^{(1)} = f_0^{(1)}(z) \sum_{k=1}^{N_c} \sum_{m=-\infty}^{\infty} A_m^k Z_{m0}^{(1)k} H_m\left(k_0 r_k\right) e^{im\theta_k}$$

The cylinders are assumed to be bottom mounted, but this does not have a large affect on the comparison with the experimental results of Kagemoto et al., at low wave periods. We present on figures 1 and 2 the results that we have obtained with $\varepsilon = 0$ (perfect fluid), $\varepsilon = 0.003$ (laminar boundary layer) and $\varepsilon = 0.015$ or 0.03 (turbulent boundary layer). We have also reproduced the experimental points given in the figures of Kagemoto et al. It can be seen that the two latter values of ε produce results in reasonable agreement with the experimental data.

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Troesch, A.W. and Kim, S.K., 1991, "Hydrodynamic Forces Acting on Cylinders Oscillating at Small Amplitudes," *J. Fluids Structures*, **5**.

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Author's Reply:

As you suggest, much of the viscous dissipation may come from the boundary layers on the cylinders. But it is also true that the dissipation takes place in the fluid (Navier-Stokes). Anyway, the required damping force to subdue the surface elevations is so tiny that it can come from anywhere in the fluid domain. In other words, it should be understood that the dissipation is not a result of a single cause but a result of multiple causes. In this sense, our damping force added to the Euler's equation may be considered to represent the dissipation macroscopically by a single term.

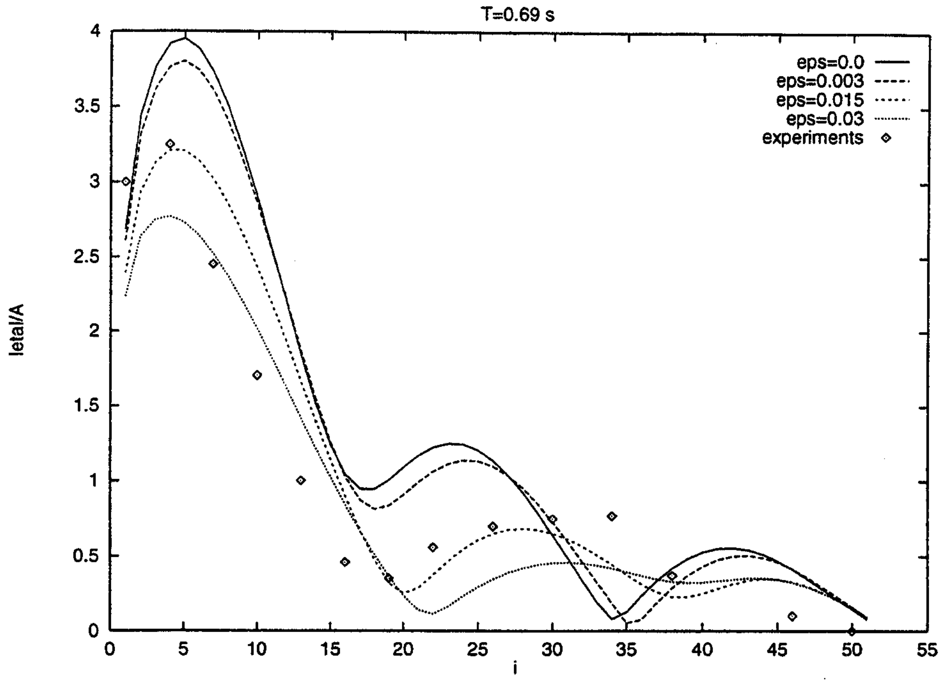

Figure 1: *The distribution of surface elevation amplitudes along the array for* $T = 0.69$

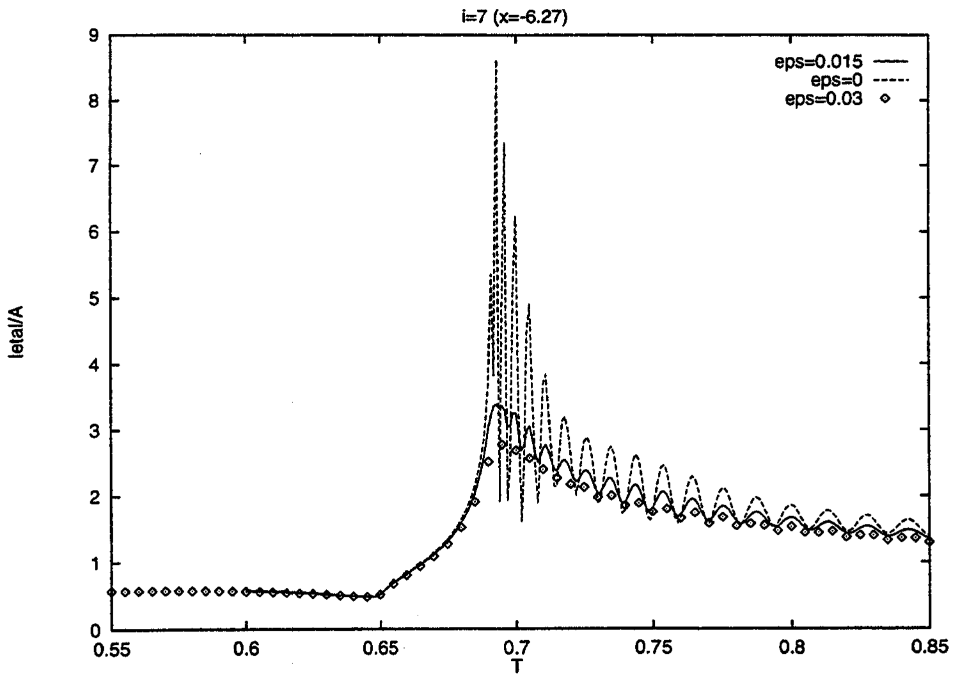

Figure 2: *The surface elevation amplitude vs period for* $i = 7$ *(*$x = -6.27$ *if we assume the configuration to be symmetric about* y *axis)*

Water-Wave Propagation Through an Infinite Array of Cylindrical Structures

P. McIver
Department of Mathematical Sciences, Loughborough University, UK

Discusser: T. Miloh

Questions/Comments:

There are similar schemes (homogenization) used in composite materials and two-phase media to determine the effective properties of periodic or randomly oriented structures under a so-called dilute or non-dilute limits. These are known in the literature as the cell method, self-consistent or generalized self-consistent. You can then find the effective property of a periodic structure under random excitation or a randomly oriented structure under monocromatic excitation. Is it possible to extend your theory to also account for these cases?

Author's Reply:

I have been specifically exploiting the periodicity to make deductions about the complete structure from the properties of one 'cell.' I think the introduction of randomness into the structure would require a different approach to that described here.

Line Integrals on the Free Surface in Ship-Motion Problems

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X.B. Chen
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Discusser: J. Newman

Questions/Comments:

The paper by Peters and Stoker (Communication on Pure and Applied Mathematics, Vol. X, pp. 399-490, 1957) derives and discusses the line integral at infinity.

Author's Reply:

Thank you for informing us the previous work. After reading the mentioned paper, we realize that the analysis on the integrals (surface and line) at infinity is based on the usual assumption of both the Green function and velocity potential decreasing at the rate inversely proportional to the square root of distance from the ship. Their analysis is then not complete as we have a singular and highly-oscillatory term involved in the Green function when a field point approaches to the track of the source point located at the free surface. To our knowledge, our paper is the first to study the line integrals on the free surface involving this peculiar term of ship-motion Green functions.

Discusser: M. Kashiwagi

Questions/Comments:

According to the past work, the line-integral singularity may be cancelled out with the singularity from the body-surface integral as the field point approaches to the intersection point. I hope you will work on the singularity-cancellation and incorporate the result into the panel code.

Author's Reply:

Thank you for informing us the past work on the line integral. To our knowledge, the classical line integral can be, only partially, cancelled out with the body-surface integral as shown in [a]. Although the ship-motion Green function contains a singular and highly-oscillatory term when the field point approaches to the track of the source point located at the free surface, its integration along a contour on the free surface is not singular as shown in the paper.

In the case where the free-surface condition is linearized above a basic flow such as the double-body flow (more realistic) instead of the uniform stream, as presented in [b], the waterline integral may be eliminated at the price to include surface integrals on the free surface in a region enclosing the waterline up to a certain distance where the basic flow is nearly as uniform stream. The efficient evaluation of the free-surface integral is indeed our goal of this study on singularities of ship-motion Green functions when both field and source points are at the free surface, and their integration.

The line integral along a contour at infinity has been considered to tend to zero by assuming that both the Green function and velocity potential decrease at the rate inversely proportional to the square root of distance from the ship such as presented in [c]. By using the similar "radiation condition," the line integral at infinity may be cancelled with the surface integral at infinity. However, these analyses are not complete as we have a singular and highly-oscillatory term involved in the Green function when a field point approaches to the track of the source point located at the free surface.

References:

[a] F. Noblesse, C. Yang, and X.B. Chen (1997), "Boundary-Integral Representation of Linear Surface Potential Flows," *J. Ship Research*, 41, 10-16.

[b] M. Kashiwagi (1994), "A New Green-Function Method for the 3-D Unsteady Problem of a Ship with Forward Speed, *Proc. 9th WWWFB*, 99-103.

[c] A.S. Peters and J.J. Stoker (1957), The Motion of a Ship, as a Floating Rigid Body in a Seaway," *Comm. Pure & Applied Math.* X, 399-490.

On the Use of Free-Surface Distributions of Havelock Singularities

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Discusser: R. Beck

Questions/Comments:

How many panels do you use in a typical computation? What type of linear equation solver do you use?

Author's Reply:

The typical number might be on the order of 1000 panels on the hull and a similar number of panels on the free surface. However, the number of panels used does depend quite strongly upon the forward speed of the vessel. To maintain the same number of panels per wavelength, it becomes necessary to use a greater number of panels for lower speed cases.

Running on Macintosh PowerPC, we have found that it is faster to use an incore equation solver; we are simply using a standard linear equation solver (matrix decomposition) from the IMSL package. This limits the number of panels to a few thousand, depending upon the available memory of a particular machine. If larger numbers of panels are required, we use an out-of-core iterative solver.

Discusser: Tom Korsmeyer

Questions/Comments:

How do you handle transom sterns? How important is it to get the transom flow right if you are interested in the wave pattern?

Author's Reply:

The greater the transom depth, the more impact it will have on the wave pattern. For shallow transom submergence, simply extending the streamlines along hull buttock lines to the position of the undisturbed free-surface seems to work reasonably well. For deeply submerged transoms, we have found that linear free-surface approximations do not adequately handle the situation and we employ our nonlinear code Das Boot discussed earlier by Don Wyatt.

Discusser: X.B. Chen

Questions/Comments:

Your work is very interesting to us as we are implementing a method based on the linearization of free surface condition on double-body flow to solve ship-motion problem. In this method, similar to yours for Neumann-Kelvin problem, we place panels on the free surface in the region near the ship and we need to evaluate influence coefficients between these panels, which is not a trivial work. The question is how you calculate the A_{22} terms numerically? And have you a line integral along the border contour of the part of F.S. where you place the Havelock panels?

Author's Reply:

The calculation of the A_{22} terms is trivial in the Neumann-Kelvin problem. The elements are either 4π or zero depending upon whether the element corresponds to the panel's influence upon itself or on some other free-surface panel. For the Dawson F.S. condition, this calculation is not trivial since Havelock panels do not satisfy Dawson's B.C. The influence coefficients are calculated numerically using a spectral description of the free-waves for the single integral terms (Scragg and Talcott, 1991, Proceedings of the Eighteenth ONR Symposium) and using Newman's polynomial approximation (Newman, JSR, June, 1987) for the double integral terms.

We do not include any line integral along the border of the computational domain. A line integral would result from an application of Stokes theorem to the integral over the free surface. However, we do not follow this approach. Instead we deal with the integration of Havelock singularities over (conceptually) the entire free surface. At some relatively short distance from the hull, the singularity densities tend rapidly toward zero, and we argue that their contribution to the Kelvin wave field can be ignored. Therefore, we approximate the integral over the infinite domain by the integration over the local domain in which the Havelock singularities have significant strengths. This is equivalent to the physical argument that all Kelvin waves must originate from the local region immediately surrounding the hull.

Diffraction Waves of a Blunt Ship with Forward Speed Taking Account of the Steady Nonlinear Wave Field

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Discusser: W. Schultz

Questions/Comments:

I understood you to say that the small τ problem required reverting to the Green function due to difficulties in the radiation condition in your desingularized method. What would you propose to do for large amplitude waves for the small τ case?

Author's Reply:

I propose the combined method of RPM in near field and GFM in far field. I have already presented this method in 1991 although the steady nonlinear wave field had not been taken into account in the calculation at that moment. The extension of this method employing the present RPM, which can capture the influence of the nonlinear steady field, will make it possible to get reasonable solution even for small τ problem.

Impulsive Tsunami Green Functions for Two-Dimensional Basins

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Discusser: A. Clement

Questions/Comments:

If I understand well, you solve a space problem at $t=0$; no time variable appears. So, is the variable η a wave elevation, or a vertical velocity of the free surface, as it seems to be defined in the abstract?

Author's Reply:

In the abstract we assumed the flow to be integrated over a short impulsive period. With this interpretation, the variable η would be the integrated surface displacement just at the end of the impulsive period. Thus the boundary-value problem is without any time variable. However, we now realize that it may be better to consider the instantaneous initial incompressible flow just as the source is turned on. Then ϕ denotes the initial velocity potential, and η (still defined by eq. 3) denotes the initial surface velocity. Then the time variable is reintroduced, appearing only in the velocity of the surface. This interpretation of η as the immediate surface velocity is fruitful in connection with the initial value problem for water waves. In the classical Cauchy-Poisson problem the initial surface velocity and surface elevation was taken as given a priori. By the present theory we may take one step back and investigate the causality behind the initial state, if it is due to a sudden normal deformation at the bottom.

Discusser: M. Tulin

Questions/Comments:

I notice that the surface displacement for a source on the bottom was EVERYWHERE positive; the implication is that a sink (dropping of the bottom) could not produce a soliton. (The reason for this is that the initial displacement must include a positive area, since the soliton is a wave of elevation.) Is it true that dropping of the bottom could not produce a tsunami?

Author's Reply:

Our general problem is that of a boundary source (180 degrees section of a source) at a given boundary contour below the free surface plus an image boundary contour with an image boundary sink. Since all the flow starts in the source and ends in the sink, it is fairly obvious that the free-surface velocity (or displacement if we integrate it in time) is positive everywhere. This may possibly be proven in general by Green's theorem. A sudden dropping of the bottom will thus produce only negative initial surface displacement. Assuming that only solitons are of interest as tsunami solutions in the full nonlinear Cauchy-Poisson problem, there may be no tsunamis generated. It is well-known that no Korteweg-de Vries solitons may be produced by an initial "well" (with no negative initial displacement everywhere). Although it is likely, I am not sure if this is true for all other types of solitons.

Interaction of Regular and Extreme Waves with Submerged Cylinders

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Discusser: W. Schultz

Questions/Comments:

Do you ever introduce vorticity at the free surface? What effects do you think it would have?

Your streamlines do not look right to me. You often show only one stagnation streamline. When I sketch in the one on the other side, the flow speed seems much higher on one side. Any comments?

Author's Reply:

Since inviscid free-surface boundary conditions are used, generation of vorticity on the free surface is not included in the current numerical model. Compared with the strength of the vorticies generated around the body, the effects of the free-surface shear layer are estimated to be relatively weak, that is as far as forces on the body are concerned. The effects of vorticity on the free surface signature however can be quite complex, which was examined by a more complete formulation in Ananthakrishnan and Yeung (Wave Motion, 1994).

The two-stagnation point model, pointed out for comparison purpose is only applicable to inviscid flow with a circulation. The presence of a bluff-body wake changes the "fore-aft" symmetry of the idealized inviscid flow. The "higher velocity on one side" that you noticed can be caused by an unsymmetrical wake as well as that the incident flow is not truly uniform.

Discusser: J.R. Chaplin

Questions/Comments:

In the regular wave case, did the forces ever achieve a steady state? In experiments this seems to happen quite quickly.

Author's Reply:

The discrete vortex model, even with a finite core size does not generate fluctuations in a time scale of the random walk and we do not use a time filter. As a result, the forces are not as steady as experimental records. However, the temporal average of the hydrodynamic coefficients do agree with your experimental measurements (Chaplin, 1984b) very well.

A Second Order Initial Value Solution of Two-Dimensional Sloshing in Rectangular Tanks

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Discusser: T. Korsmeyer

Questions/Comments:

Are 3-D effects important in sloshing problems, and would you like to move to 3-D models?

Author's Reply:

The importance of 3-D effects depend on how natural periods of the fluid motion are relative to the frequency contents of the wave excited ship motions. If the dimensions in length and breadth directions are similar, the highest natural period in length and beam direction are similar. It means that 3-D effects must be considered. If the length and breadth are quite different, which often will be the case, 3-D effects are not important for a rectangular box shaped tank. This means 2-D flow effects are important for a broad class of tank configurations. Since the problem is strongly nonlinear, it is important from a numerical point of view to start out with the 2-D analysis. The procedure that we have followed can be generalized to 3-D flow.

Discusser: B. Molin

Questions/Comments:

Is there any reason why you use a fixed coordinate system – It seems to me that a moving coordinate system would produce homogeneous Neumann conditions at the walls at second-order.

I also mention some work done some years ago by Cointe, Nays and myself on the second-order initial value problem: "Non-linear and Second-Order Transient Waves in a Rectangular Tank," given at BOSS '88 in Trondheim.

Author's Reply:

We now have developed a third order method. We then found it convenient to use a body-fixed coordinate system.

Thank you for informing us of your previous work.

Discusser: P. Sclavounos

Questions/Comments:

Would there be insurmountable difficulties to treat the 2-D or 3-D sloshing problem with a BEM method with proper damping mechanisms?

Author's Reply:

We have tried to use a 2-D completely nonlinear BEM method. A difficulty is instabilities (physical and numerical) that occur close to the tank walls. In addition we have the general difficulty of plunging breakers and what to do after the tip of the plunging breaker enters the free surface. There is also the problem of high resolution in time and space when studying impact due to sloshing. Since the objective is to obtain very long time simulations in order to obtain a statistical estimater, there is also the problem of time efficiency of the numerical code.

We are presently trying to combine a nonlinear method with a BEM formulation. The BEM will be used when heavy impact occurs in the tank. The initial conditions are gotten from the nonlinear method and a Wagner's approach is used at the moment of impact.

Discusser: M. McIver

Questions/Comments:

You are assuming that the forcing can be switched on in a smooth fashion. Do you think that how the forcing is started in practice will be important?

Author's Reply:

We have recently been working with a third order method based on different ordering of terms and a Bateman-Luke variational principle. The pressure is used in the Lagrangian of the Hamilton principle. The result is a system of nonlinear ordinary differential equations in time. The unknowns are generalized coordinates of the free surface elevation.

We then studied the influence of the initial conditions and found that the results were insensitive to these after some oscillation periods. However, the effect of the initial period of forcing is important and does not die out. If the frequency of oscillation is slightly changed during the first few oscillation periods this can give a pronounced effect on the long-term response.