A nonlinear method for predicting wave resistance of ships

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ABSTRACT

A nonlinear theory to predict wave resistance of ships with low block coefficients is developed. The theory is based on a linear and second order 3-D theory and an additional nonlinear correction by a fully nonlinear $2\frac{1}{2}D$ solution in the bow region. The numerical code is verified by analytical solutions and thin ship theory. The theory is validated by comparing with model tests of a series of passenger ships and container ships.

INTRODUCTION

The motivation for developing a new numerical code to predict wave resistance of conventional ships with low block coefficients is that current numerical codes do not predict the wave resistance satisfactorily, especially in the design stage when evaluating different concepts with small differences in the geometries.

Linear and nonlinear numerical methods to predict wave resistance have been studied by many authors. Raven(1996) gave an extensive review about the subject. A short summary about different methods will be given here. The linear methods consist mainly of two solutions, namely Neumann-Kelvin and Dawson(1977) methods. In the Neumann-Kelvin problem a linear classical free surface condition is used and the Dawson method is based on a linearization on the double body potential. The nonlinear solutions are based on the fully nonlinear free-surface conditions and the solutions are obtained by iterations. The nonlinear methods predict better wave elevation around the ship, but not necessarily better wave resistance. Limited results of nonlinear wave resistance calculations have been presented for real ships. Generally speaking the linear methods predict fairly well the wave resistance for ships with low block coefficeints and the nonlinear solutions may give better results, but not always get convergence and correct results.

We are looking for a numerical method for ships with block-coefficient C_B lower than 0.6 to 0.7. That means that the ship is fairly slender. The method should also be robust and easy for an engineer to use. Since the linear solutions predict quite well the wave resistance for relative slender ship, our idea is to introduce nonlinear corrections for predicting better wave resistance. We start with the linear problem(Neumann-Kelvin) with linear free-surface condition satisfied on the mean water surface. When one calculates wave resistance, nonlinear terms will be included in a similar way as for the second order problem of mean drift force. That means that the quadratic term in the Bernoullis' equation and a water line integral are included. For the wave resistance problem it is believed that the strong nonlinear effects are only located in the bow region. Then one can use a linear and second order and nonlinear $2\frac{1}{2}D$ methods to predict additional nonlinear contribution at bow region. The $2\frac{1}{2}D$ method means that one use two-dimensional Laplace equation and three-dimensional free-surface conditions. The $2\frac{1}{2}D$ solution is valid for high Froude number, so the method can only apply in the bow region with a high local Froude number.

THEORY

A potential theory is used to solve the wave resistance problem since the viscous effects are neglected here. The problem is solved as a steady problem. The effects of trim, sinkage and transom stern are not included in the analysis.

A right-handed coordinate systems $\vec{x} = (x, y, z)$ has been chosen. $\vec{x} = (x, y, z)$ is a coordinate system fixed in the vessel. The surface z=0 is the mean water surface when the forward speed of the ship is zero. A velocity potential $\phi_T = \phi + Ux$ is introduced. Here U is the forward speed of the ship and ϕ satisfies the Laplace equation

$$\nabla^2\phi = 0 \tag{1}$$

in the fluid domain.

Following Newman(1976) the nonlinear dynamic free-surface condition on the exact free surface can be written as

$$U\frac{\partial\phi}{\partial x}+\frac{1}{2}[(\frac{\partial\phi}{\partial x})^2+(\frac{\partial\phi}{\partial y})^2+(\frac{\partial\phi}{\partial z})^2]+g\zeta=0$$
$$on\ z=\zeta(x,y) \quad (2)$$

and the kinematic free-surface condition is

$$U\frac{\partial\zeta}{\partial x}-\frac{\partial\phi}{\partial z}+\frac{\partial\phi}{\partial y}\frac{\partial\zeta}{\partial y}+\frac{\partial\phi}{\partial x}\frac{\partial\zeta}{\partial x}=0$$
$$on\ z=\zeta(x,y) \quad (3)$$

Here $\zeta(x,y)$ is the free surface elevation.
The body boundary condition on the exact wetted body surface S_B can be written as

$$\frac{\partial\phi}{\partial n}=-Un_1 \quad on \quad S_B \quad (4)$$

where $\vec{n}=(n_1,n_2,n_3)$ is the normal vector on the body surface. Positive direction of $\vec{n}$ is into the fluid domain.
The problem will be solved in two levels. The first approximation is based on the linear classical free-surface condition on the mean water surface. Then a local nonlinear correction near the bow region is carried out by using $2\frac{1}{2}D$ methods.

Linear and second order 3-D solution

The first approximation is based on the linear classical free-surface conditions on the mean water surface. Here one assumed that the ship is fairly slender. Neglecting the nonlinear terms in the equation (2) and (3) one obtains the linear classical free-surface condition

$$U^2\frac{\partial^2\phi}{\partial x^2}+g\frac{\partial\phi}{\partial z}=0 \quad on \quad z=0 \quad (5)$$

The velocity potential ϕ for the flow is solved by using Green's second identity with 3-D Rankine sources and dipoles.. The numerical procedure and radiation condition are similar as Dawson(1977).
The pressure P on the wetted body surface can be obtained by the following equation

$$P=-\rho U\frac{\partial\phi}{\partial x}-\rho\frac{1}{2}[(\frac{\partial\phi}{\partial x})^2+(\frac{\partial\phi}{\partial y})^2+(\frac{\partial\phi}{\partial z})^2]-\rho gz \quad (6)$$

The wave resistance R_w can be written as

$$R_w=-\int_{S_B}Pn_1dS \quad (7)$$

Here S_B is the wetted body surface. The first term in equation (6) is the leading order term for the pressure. The reason to take into account the nonlinear terms in equation (6) will be explained in the following text. It is assumed that $-\rho U\frac{\partial\phi}{\partial x}$ is the leading order term for the pressure. This pressure term oscillates along the ship like the free surface elevation along the ship. This means positive and negative contributions to the wave resistance from different parts along the ship. The term $-0.5\rho[(\frac{\partial\phi}{\partial x})^2+(\frac{\partial\phi}{\partial y})^2+(\frac{\partial\phi}{\partial z})^2]$ gives lower contribution to the pressure compared to the first term in eq.(6), but it has negative sign along the ship. This means the integrated contribution to the wave resistance is important relative to the first term in eq.(6). These two pressure terms are first integrated to the calm water surface. Then the nonlinear contribution on the real wetted surface should be corrected. The terms $-\rho U\frac{\partial\phi}{\partial x}-\rho gz$ give contribution from the calm water surface to the real free surface.

Additional nonlinear correction

The additional nonlinear corretion is based on $2\frac{1}{2}D$ methods, which are based on two-dimensional Laplace equation and three dimensional nonlinear free-surface conditions. Here one introduces a slenderness parameter ε, the ratio between the beam(or draught) and the ship length. One assumes that $\frac{\partial f}{\partial x}=O(f\varepsilon^{-\frac{1}{2}})$, $\frac{\partial f}{\partial y}=O(f\varepsilon^{-1})$ and $\frac{\partial f}{\partial z}=O(f\varepsilon^{-1})$, where f is any flow variable caused by the body in the region closed to the body. Further one assumes that $n_1=O(\varepsilon^{\frac{1}{2}})$ where n_1 is the x-component of a unit normal vector to the wetted part of the body surface. The assumptions follow the similar approach of Faltinsen and Zhao(1991) in solving the problem of ship motion for high speed vessels. Based on the assumptions the three-dimensional Laplace equation(eq.(1)) will be two-dimensional Laplace equation for each cross section. The nonlinear dynamic and kinematical free-surface conditions can then be written as

$$\frac{\partial\phi}{\partial x}=-\frac{1}{2U}[(\frac{\partial\phi}{\partial y})^2+(\frac{\partial\phi}{\partial z})^2]-\frac{g}{U}\zeta$$
$$on\ z=\zeta(x,y) \quad (8)$$

$$\frac{\partial\zeta}{\partial x}=\frac{1}{U}\frac{\partial\phi}{\partial z}-\frac{1}{U}\frac{\partial\phi}{\partial y}\frac{\partial\zeta}{\partial y} \quad on \quad z=\zeta(x,y) \quad (9)$$

where $\zeta(x,y)$ is the free-surface elevation.
The Green's second identity based on 2-D Rankine sources and dipoles can be applied for each cross section. A solution can be found by starting at the bow, use equation (8) and (9) to step the solution of the free surface elevation ζ and the velocity potential ϕ. For each cross section a two dimensional problem is solved. The start conditons are $\zeta=0$ and $\phi=0$ in Faltinsen and Zhao(1991). Later Fontaine and Faltinsen(1997) pointed out that the ζ is different from zero even for a very thin ship bow. Since the linear three-dimensional problem is solved first, one may use it as start

condition. In our numerical solution the start conditions of ζ and ϕ are based on the linear three-dimensional solution. It is found out that the start conditions are important for the wave resistance of passenger and container ships, but not significant for slender bodies as the Wigley hull and catamarans. The $2\frac{1}{2}D$ solution is only applied in the local region near the bow. A similar linear velocity potential can be obtained by neglecting nonlinear terms in the free-surface conditions (8) and (9). Then the free surface conditions will be the same as in solving linear three-dimensional problem and it will be satisfied on the calm water surface. It was shown by matched asymptotic expansion by Faltinsen(1983) that the linear $2\frac{1}{2}D$ solution is an approximate solution for the linear 3D solution in the bow region of conventional slender ships. The additional nonlinear effect can be obtained by the difference between the fully nonlinear $2\frac{1}{2}D$ and linear and second order $2\frac{1}{2}D$ solutions. The $2\frac{1}{2}D$ methods is valid for Froude number is order of O(1). The theory can be used for the whole ship for high speed vessels. For practical applications one assumes that the additional nonlinear effect is only important in the bow region and the $2\frac{1}{2}D$ methods are used from the bow to the section with local Froude number large than Fn_{loc}^{min}, but not for the sections after midship. Here one defines a local Froude number based on the length from the bow to the actual section. We choose $Fn_{loc}^{min} = 0.6$, because $2\frac{1}{2}D$ methods predict well wave resistance of catamarans for Froude number larger than 0.6. The additional nonlinear contribution is not much dependent on the selected Fn_{loc}^{min}, since the design speed(Froude number) of ships with low block coefficient are usually around or large than 0.3, and the n_1 component of the normal vector on the body surface is small along the midbody. The pressure on the body surface is

$$P = -\rho U\frac{\partial\phi}{\partial x} - \rho\frac{1}{2}[(\frac{\partial\phi}{\partial y})^2 + (\frac{\partial\phi}{\partial z})^2] - \rho g z \qquad (10)$$

In the linear and second order $2\frac{1}{2}D$ method the pressure is integrated in a similar way as the linear and second order 3-D problem.

The total nonlinear contribution R_w^{non} can be obtained by a following equation

$$R_w^{non} = R_{non}^{2\frac{1}{2}D} - R_{lin}^{2\frac{1}{2}D} \qquad (11)$$

where $R_{non}^{2\frac{1}{2}D}$ and $R_{lin}^{2\frac{1}{2}}D$ are wave resistances based on the solution of nonlinear and linear and second order $2\frac{1}{2}D$ methods.

VERIFICATIONS AND VALIDATION

The numerical program of the linear 3D solution has been verified by comparing with analytical and semi-analytical solutions. The first case is the flow past a sphere in infinite fluid.

	M2257	M2258	M2259	M2275	M2175B	M2175C
Length on waterline (m)	2.355	2.355	2.355	2.355	5.891	5.965
L_{PP} (m)	2.267	2.267	2.267	2.267	5.926	6.000
Breadth waterline (m)	0.373	0.373	0.373	0.373	0.889	0.889
Draught (m)	0.093	0.093	0.093	0.093	0.299	0.299
Volume of displacement	0.043	0.043	0.043	0.043	0.884	0.881
Prismatic-coefficient	0.641	0.605	0.550	0.584	0.577	0.568
Block-coefficient	0.545	0.545	0.545	0.545	0.561	0.553
Midship section coefficient	0.850	0.898	0.990	0.934	0.973	0.973
Longitudinal C.B. (LCB %)	-3.69	-3.73	-3.74	-3.71	-4.46	-4.69
Wetted surface (m^2)	0.921	0.924	0.944	0.929	6.219	6.362

Table 1: List of principal hull data of a number of passenger ships and car ferries, and two container ships.

Both the velocity potential and the velocity on the body surface is checked. Satisfactory agreement is obtained.

The second case is to test the numerical solution against the Green's function(source) with forward speed. The Green's function satisfies the linear free-surface condition(eq.(5)), the 3D Laplace equation and the radiation condition. The test is done in the following way. A small sphere near the free surface surrounding the source is used as a body surface. The body boundary condition is $\frac{\partial\phi}{\partial n} = C$, where C is a constant. C is dependent on the radius of the sphere. The total mass flux is constant, which is equal to the mass flux due to a source(Green's function). The free-surface condition is the classical free-surface condition (5), which is same as the Green's function satisfies. Then the velocity potentials on the free surface are compared with each other. Good agreement is obtained.

The method has been verified by thin ship theory for the wave resistance of the Wigley hull. Good agreement between the numerical and analytical solutions is obtained.

The theory has been validated by model tests for a number of passenger ships and car ferries, which have been carried out at MARINTEK. Here one chooses the models without appendices. The models M2257, M2258, M2259, M2275 have been investigated. The principal hull data are given in Table 1. In addition two container ships with model M2175B and M2175C have been used in the validation. The principal hull data are also given in Table 1.

For optimization of the hull, the parameter midship section coefficient C_M is studied experimentally. Figure 1 shows the theoretical and experimental results for four models with different C_M. In the model tests the total resistance is measured. Since the model tests have not been done for small Froude numbers, the form factor can not be predicted from the model tests. Before one can compare with wave resistance, one must estimate the viscous resistance. The empirical formula of Holtrop(1984) is used here for estimating viscous resistance. MARINTEK uses a different expression for the form factor k_1, which is given as

$$k_1 = 0.6C_1 + 145.0{C_1}^{3.5} \qquad (12)$$

$$where \;\; C_1 = \frac{C_B}{L}((T_{AP} + T_{FP})B)^{0.5} \qquad (13)$$

Here T_{AP} is draught at AP, T_{FP} draught at FP, L length on waterline, B breadth and C_B block-coefficient. The viscous force due to flow separation is not included in the form factor of MARINTEK. That means that the viscous drag force is part of the residuary resistance. In the following comparison the residuary resistance is defined by MARINTEK standard. That means that the difference between the form factors of Holtrop(1984) and MARINTEK has been corrected. It seems that the theory predict well the residuary resistance for models with difference C_M coefficients.
Figure 2 shows the residuary resistance coefficients of models M2175B and M2175C. Both theoretical and experimental results have been presented. The geometries of M2175B and M2175C are identical except the model M2175C with bulb and the model M2175B without bulb. The residuary resistances are significaltly reduced for the model with bulb both in the experimental and theoretical results. The numerical results agree well with experimental results.

CONCLUSION

A nonlinear theory to predict wave resistance of ship with low block coefficients is developed. The numerical program is validated by a series of passenger ships and two container ships with and without bulb. The residual resistance from model tests have been compared with theoretical results. For a series of passenger ships and car ferries with different C_M values, good agreement between the theoretical and experimental results is obtained. The program predicts well the resistance of container ships with and without bulb.

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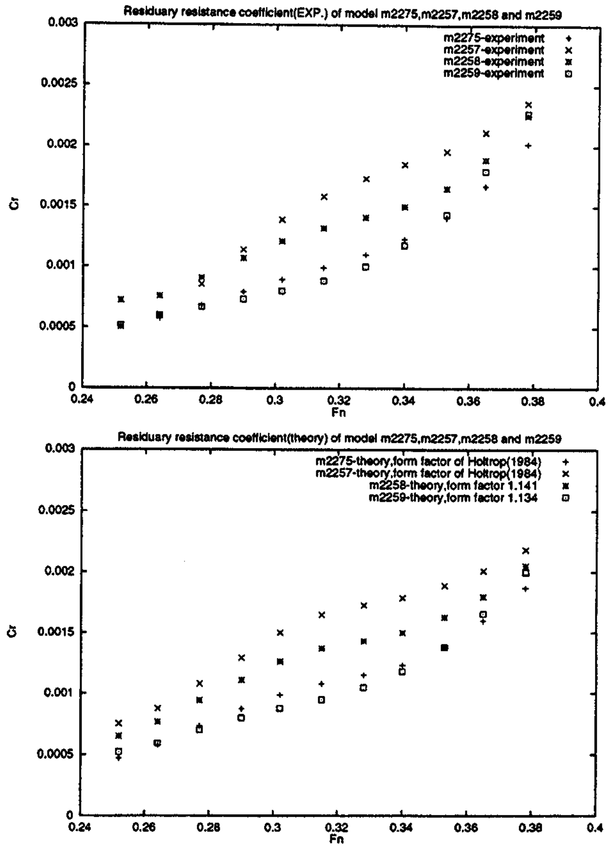

Figure 1: Residual resistance coefficient of model M2275, M2257 and M2258 and M2259 with different C_M values. The theoretical and experimental results are presented.

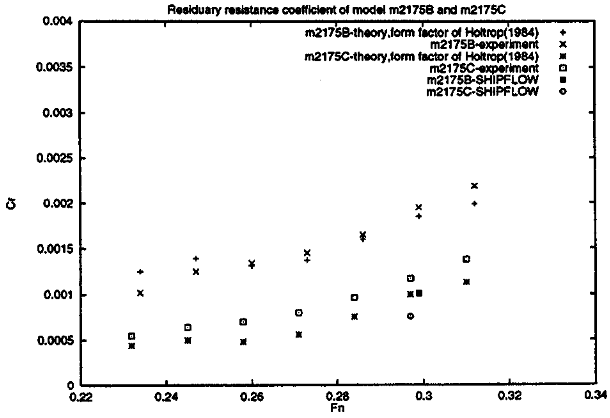

Figure 2: Residuary resistance coefficients of model M2175B and M2175C. The theoretical and experimental results are presented.