Uniform solution for water wave diffraction-radiation at small forward speed in water of finite depth

Zalar M.[1] & Malenica Š.[2]

(1) BRODARSKI INSTITUT, Av. V.Holjevca 20, 10020 Zagreb, Croatie
(2) BUREAU VERITAS - DTA, 17bis Place des Reflets, 92400 Courbevoie, France

Wave diffraction radiation at small forward speed has received much attention in the recent past, essentially for two reasons : the first one is the need to evaluate the influence of current on the first order quantities (forces, air gap, ...) and the second one is the calculation of the wave drift damping coefficients which are important in the studies of the slow drift motions of off-shore structures. Several methods of calculations were proposed [6, 11, 10, 3]. The most powerful methods are based on the use of the Kelvin type Green functions because they permit the resolution of the small forward speed problem in the same way as the zero forward speed one, the only essential difference being the Green function. Unfortunately the direct evaluation of the Green function is not trivial [4, 1] and usually we take a profit of the small forward speed assumption by introducing the additional perturbation with respect to the forward speed parameter $\tau = U\omega/g$. The inconvenient of this approach is the secularity (unphysical growth for $R \to \infty$) of the solution. As far as the infinite water depth is concerned, the first attempts to overcome this difficulty were made in [6] where the authors operate on the wave part (residues) and found the necessary corrections in the far-field. A more direct approach was performed in [9] where the nonsecular solution was obtained by operating directly on the integral representation of the Green function and very simple expressions were found. The same solution was reproduced in [7] by completely different method based on the multiple scale technique. In the case of finite depth the situation is more complicated because of the more complex expressions for the corresponding Green function and, up to now, the only practical numerical code is based on the secular solution [5]. It is however important to note the work presented in [1] where the uniform solution for the Green function was presented based on the direct use of the integral representation, but the numerical difficulties are numerous so that no practical numerical code was presented. On the other hand the method presented in [9] for the infinite water depth has no equivalent in the finite depth and it is not clear that the simple expressions could be obtained. In summary, we can say that the efficient uniform solution for finite water depth doesn't exists up to now. The purpose of the present paper is to offer a simple practical way to solve this problem.

BVP and integral equations

The assumptions of the potential flow ($\Delta\phi = 0$ in the fluid) lead to the following linearised free surface condition in the frequency domain [$\phi(\mathrm{x}, t) = \Re\{\varphi(\mathrm{x})e^{-i\omega t}\}$]:

$$-\nu\varphi + 2i\tau\frac{\partial\varphi}{\partial x} + \frac{\partial\varphi}{\partial z} - 2i\tau\nabla\bar{\phi}\nabla\varphi + i\tau\varphi\frac{\partial^2\bar{\phi}}{\partial z^2} = 0 \tag{1}$$

where $\nu = \omega^2/g$, $\omega = \omega_0 - k_0 U\cos\beta$ is the encounter frequency, ω_0 is the fundamental frequency of the incomming wave, β is its incident angle, k_0 is the wavenumber ($\omega_0^2 = gk_0\tanh k_0 h$), h is the waterdepth and $\bar{\phi}$ is the double body potential.
The decomposition ($\varphi = \varphi_I + \varphi_L + \tau\varphi_N$) proposed in [3] is adopted here so that the following free surface conditions are deduced :

$$-\nu\varphi_L + 2i\tau\frac{\partial\varphi_L}{\partial x} + \frac{\partial\varphi_L}{\partial z} = 0 \quad ; \quad -\nu\varphi_N + 2i\tau\frac{\partial\varphi_N}{\partial x} + \frac{\partial\varphi_N}{\partial z} = Q \tag{2}$$

with $Q = 2i\tau\nabla\bar{\phi}\nabla(\varphi_I + \varphi_L) - i\tau(\varphi_I + \varphi_L)\frac{\partial^2\bar{\phi}}{\partial z^2}$, and φ_I denoting the incident wave potential.
This decomposition leads to the following integral representation for the potential and integral equations

for the unknown source strength σ :

$$\varphi_L = \iint_{S_B} \sigma_L G dS \quad , \quad \left\{ -\frac{1}{2}\sigma_L + \iint_{S_B} \sigma_L \frac{\partial G}{\partial n} dS = V_n \right\}_{S_B} \tag{3}$$

$$\varphi_N = \iint_{S_B} \sigma_N G dS - \iint_{S_F} Q G dS \quad , \quad \left\{ -\frac{1}{2}\sigma_N + \iint_{S_B} \sigma_N \frac{\partial G}{\partial n} dS = \iint_{S_F} Q \frac{\partial G}{\partial n} dS + i\frac{m_j}{\nu} \right\}_{S_B} \tag{4}$$

where $G(\mathbf{x};\boldsymbol{\xi})$ is the Green function, $V_n = -\partial\varphi_I/\partial n$ for diffraction, $V_n = n_j$ for radiation, and m_j are the well known terms accounting for the interactions of the double body flow with the body motions. As we can see the integral equations are very similar to the zero forward speed case except for the free surface integral which can be evaluated relatively easily because of its local character. So the main difficulty is to find the efficient way to evaluate the Green function.

Green function

According to the above discussions, the small forward speed Green function $G(\mathbf{x};\boldsymbol{\xi})$ should satisfy the following set of equations :

$$\left.\begin{array}{ll} \Delta G = \delta(\mathbf{x}-\boldsymbol{\xi}) & 0 \geq z \geq -h \\ -\nu G + 2i\tau\dfrac{\partial G}{\partial x} + \dfrac{\partial G}{\partial z} = 0 & z = 0 \\ \dfrac{\partial G}{\partial z} = 0 & z = -h \\ G \rightarrow \dfrac{f(\xi,\theta,z)}{\sqrt{kR}} e^{ikR(1+2\tau\partial k/\partial\nu\cos\theta)} & R \rightarrow \infty \end{array}\right\} \tag{5}$$

with $\nu = k\tanh kh$, and δ denoting the Dirac delta function.
The direct perturbation $G = G_0 + \tau G_1$ leads to the following solution for G_1 :

$$G_1 = -2i\frac{\partial^2 G_0}{\partial\nu\partial x} \tag{6}$$

This solution for is secular and G_1 behave as $G_1 \approx f_1(\xi,\theta,z)(x-\xi)(kR)^{-1/2}e^{ikR}$ for $R \rightarrow \infty$.
We propose now to use multiscale pertubation and we write :

$$G(\mathbf{x};\boldsymbol{\xi}) = g_0(\gamma;\mathbf{x};\boldsymbol{\xi}) + \tau g_1(\gamma;\mathbf{x};\boldsymbol{\xi}) \tag{7}$$

where $\gamma = \tau(x-\xi)$.
The two functions g_0 and g_1 should satisfy the following equations :

$$\left.\begin{array}{llll} \Delta g_0 = \delta(\mathbf{x}-\boldsymbol{\xi}) & , & \Delta g_1 = -2\dfrac{\partial^2 g_0}{\partial\gamma\partial x} & 0 \geq z \geq -h \\ -\nu g_0 + \dfrac{\partial g_0}{\partial z} = 0 & , & -\nu g_1 + \dfrac{\partial g_1}{\partial z} = -2i\dfrac{\partial g_0}{\partial x} & z = 0 \\ \dfrac{\partial g_0}{\partial z} = 0 & , & \dfrac{\partial g_1}{\partial z} = 0 & z = -h \\ g_0 \rightarrow \dfrac{f_0(\gamma,\xi,\theta,z)}{\sqrt{kR}} e^{ikR} & , & (g_0 + \tau g_1) \rightarrow \dfrac{f(\gamma,\xi,\theta,z)}{\sqrt{kR}} e^{ikR(1+2\tau\partial k/\partial\nu\cos\theta)} & R \rightarrow \infty \end{array}\right\} \tag{8}$$

The most general solution for g_0 is :

$$g_0 = F(\gamma)G_0 \tag{9}$$

where G_0 is the classical zero forward speed Green function (with the encounter frequency).
On the other hand the particular solution for g_1 can be written in the form :

$$g_1 = F(\gamma)[-2i\frac{\partial^2 G_0}{\partial\nu\partial x} + C(x-\xi)G_0] \tag{10}$$

where C is the unknown constant to be determined by requiring the nonsecularity of the solution. The Poisson equation (8) for g_1 gives :

$$2\frac{\partial G_0}{\partial x}[CF(\gamma) + \frac{\partial F(\gamma)}{\partial\gamma}] = 0 \tag{11}$$

from which we deduce :

$$F(\gamma) = e^{-C\gamma} \tag{12}$$

Knowing the behaviour of G_0 for large R, the radiation condition for g_1 (8) gives the following value for the unknown constant C :

$$C = -2ik\frac{\partial k}{\partial \nu} \tag{13}$$

so that the final solution for the Green function becomes :

$$G = e^{2i\tau\frac{\partial k}{\partial \nu}k(x-\xi)}\Big\{G_0 - 2i\tau[\frac{\partial^2 G_0}{\partial \nu \partial x} + \frac{\partial k}{\partial \nu}k(x-\xi)G_0]\Big\} \tag{14}$$

If we compare the present expression to the secular one (6) we can see that very small modifications are necessary to obtain the uniformly valid solution. The most difficult term to calculate remains the double derivative of the zero forward speed Green function with respect to the wavenumber and the x coordinate. In the case of infinite depth, where the final expressions for the uniform solution are essentially the same [7], this can be done relatively easily because the zero forward speed Green function G_0 has simple form and derivatives can be obtained in an analytical way so that only things we need are zero forward speed Green function and its first radial derivative [3]. The finite water depth case is more complicated because of more complex representation of the zero forward speed Green function and the higher order derivatives should be calculated directly. In this case the efficiency of the calculations depends on the approximations used for the zero forward speed Green function. The most convenient way to approximate the G_0 in finite or infinite depth is the use of the Chebychev polynomial approximations [8, 2]. In this case the higher order derivatives can be obtained by direct derivation of the Chebychev polynomials provided that the coefficients for these polynomials were obtained with sufficient accuracy (10^{-6} seems to be sufficient). This is the case in our in house code HYDROSTAR, so that the higher order derivatives of G_0 can be calculated accurately without important numerical difficulties.

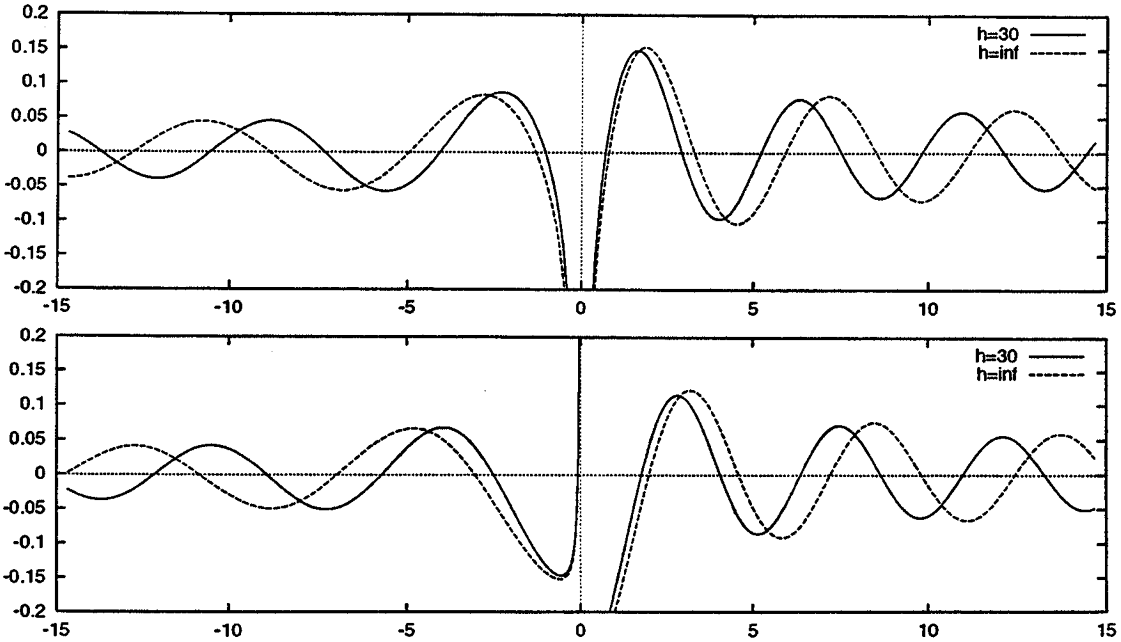

Figure 1: *The real (top) and imaginary (bottom) parts of $4\pi G$ vs. νx for infinite and finite water depth. The parameters are $\omega = 0.6$, $z = 0.$, $\zeta = 0.$, $y = 0.$, $\tau = 0.1$.*

Numerical results

On figure 1 we present first the results for the non-secular solution of the Green function in finite and infinite water depth in order to appreciate the influence of the water depth. In order to show the difference in the behaviour of the secular and nonsecular solutions, on figure 2 are presented the results for both cases. As we can see the non-secular solution becomes invalid very quickly when we leave the source.

Also the non-secular solution doesn't account for the Doppler effect and the wave lengths behind and in front of the source are the same. However, as it was pointed out in [7], the final solution for the potential should remain the same on the body up to the order $O(\tau)$ for both secular and nonsecular solutions. So we think that even secular solution can be used for the calculation of the forces by pressure integration, but not, for exemple, for the calculations of the wave elevations near the body as was demonstrated in [3].

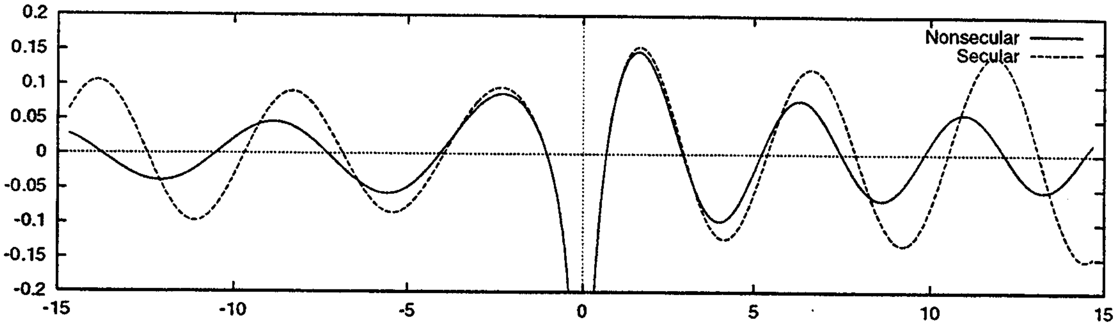

Figure 2: *Real part of the secular and nonsecular solution for $4\pi G$ vs. νx for the water depth $h = 30.$ and for the following parameters $\omega = 0.6$, $z = 0.$, $\zeta = 0.$, $y = 0.$, $\tau = 0.1$.*

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