On the Use of Free-Surface Distributions of Havelock Singularities

Carl A. Scragg

Science Applications International Corporation
San Diego, California

The use of Havelock singularity distributions in the solution of the Dawson free-surface problem was addressed by Scragg and Talcott in 1990 [1]. Distributing Havelock singularities on the hull and free-surface, the authors were able to generate numerical solutions to Dawson's linearized free-surface problem which combined the near-field accuracy of the Rankine/Dawson codes with the far-field accuracy of Havelock codes. Unfortunately, the associated increase in computational requirements relative to the Rankine/Dawson codes was not insignificant. Subsequently, we have adopted an hybrid approach to the solution of Dawson's problem in which Havelock singularities are distributed only over the undisturbed free surface $Z(x,y) = 0$, while the hull surface $S(x,z)$ is now modeled with distributions of Rankine singularities. This hybrid approach reduces the number of Havelock panels by roughly 50%. But more importantly, the unique properties of free-surface distributions of Havelock singularities allow us to analytically determine the dominant terms in the solution matrix. This leads to significant improvements in both numerical stability and computational economy.

The properties of Havelock singularities are well known. Any distribution of Havelock singularities will inherently satisfy the Laplace equation throughout the fluid domain and the Kelvin linearized free-surface condition,

$$\phi_z + {k_0}^{-1}\phi_{xx} = 0\,, \quad \text{on } z = 0\,,$$

where k_0 is the characteristic wave number g/U^2. In addition, since Havelock singularities satisfy the appropriate radiation condition in the far-field, there will be no problems with upstream radiating waves nor should we experience any reflections at computational boundaries. The Havelock singularity can be expressed as the sum of a simple Rankine singularity $(1/r)$ and its negative image above the free surface $(1/r^*)$, plus a regular wave term H:

$$G(x,y,z;\xi,\eta,\zeta) = -\frac{1}{r} + \frac{1}{r^*} + H(x,y,z;\xi,\eta,\zeta)\,.$$

A Havelock singularity distribution on the undisturbed free surface can be viewed as the limiting case of a submerged panel located at depth $\zeta = \epsilon$, as ϵ approaches zero. Immediately above the panel, at $z = \epsilon^+$, the Kelvin free-surface condition will be satisfied. However, we wish to impose the free-surface boundary condition on the fluid side of the panel, *i.e.* on $z = \epsilon^-$. Since both the image term and the wave term are continuous throughout the fluid domain, the only discontinuity which can exist across the panel will be due to the $1/r$ term. For a constant density panel, ϕ_{xx} will be continuous across the panel, but there will be a jump discontinuity in the normal velocity given by

$$\Delta\phi_z = 4\pi\sigma\,,$$

where the source density is given by σ. Therefore, at the panel's collocation point,

$$\phi_z + {k_0}^{-1}\phi_{xx} = -4\pi\sigma\,, \quad \text{at } z = \zeta^-\,.$$

If we distribute N Rankine panels over the surface of the hull $S(x,z)$, and M Havelock panels over a near-field region of the free surface $Z(x,y)$, then the determination of the source strengths σ^S and σ^Z satisfying the boundary conditions will involve solving a matrix equation of the form

$$\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \sigma^S \\ \sigma^Z \end{bmatrix} = \begin{bmatrix} \mathbf{B}^S \\ \mathbf{B}^Z \end{bmatrix}.$$

$\mathbf{A}_{11} = N \times N$ matrix containing the normal velocities on the hull generated by the Rankine hull panels. Note that the diagonal elements of $\mathbf{A}_{11}$ will be equal to 2π.

$\mathbf{A}_{12} = N \times M$ matrix containing the normal velocities on the hull generated by the panels on the free surface.

$\mathbf{A}_{21} = M \times N$ matrix containing the contribution to the free surface boundary condition due to the Rankine panels on the hull.

$\mathbf{A}_{22} = M \times M$ matrix containing the contribution to the free surface boundary condition due to the panels on the free surface.

The vectors $\mathbf{B}^S$ and $\mathbf{B}^Z$ specify the boundary condition to be satisfied on the hull and free surface respectively.

For many slender hull forms, numerical studies indicate that the choice between using a Kelvin free-surface linearization and a Dawson free-surface linearization does not lead to significant differences in the predicted wave fields [2,3]. However, when using free-surface Havelock singularities, a major computational savings can be realized by solving the simpler Neumann-Kelvin problem for such hull forms since all of the elements of $\mathbf{A}_{22}$ corresponding to the Kelvin free-surface boundary condition are known *a priori*:

$$a_{ij} = \left\{ \begin{array}{cc} -4\pi\,, & \text{if } i = j \\ 0\,, & \text{otherwise} \end{array} \right\}.$$

Even if we ignore for the moment the differences in the number of free-surface panels which may be necessary, the computational effort required to solve the Neumann-Kelvin problem using Havelock singularities on the free surface can actually be comparable to the effort required when using a standard Rankine method. For both methods, the calculation of the contributions due to Rankine elements on the hull contained in $\mathbf{A}_{11}$ and $\mathbf{A}_{21}$ will be identical. A tradeoff can occur between the effort required for the $N \times M$ Havelock calculations in $\mathbf{A}_{12}$ versus the $(N+M) \times M$ calculations in $\mathbf{A}_{12}$ and $\mathbf{A}_{22}$ which would be necessary for a standard Rankine method.

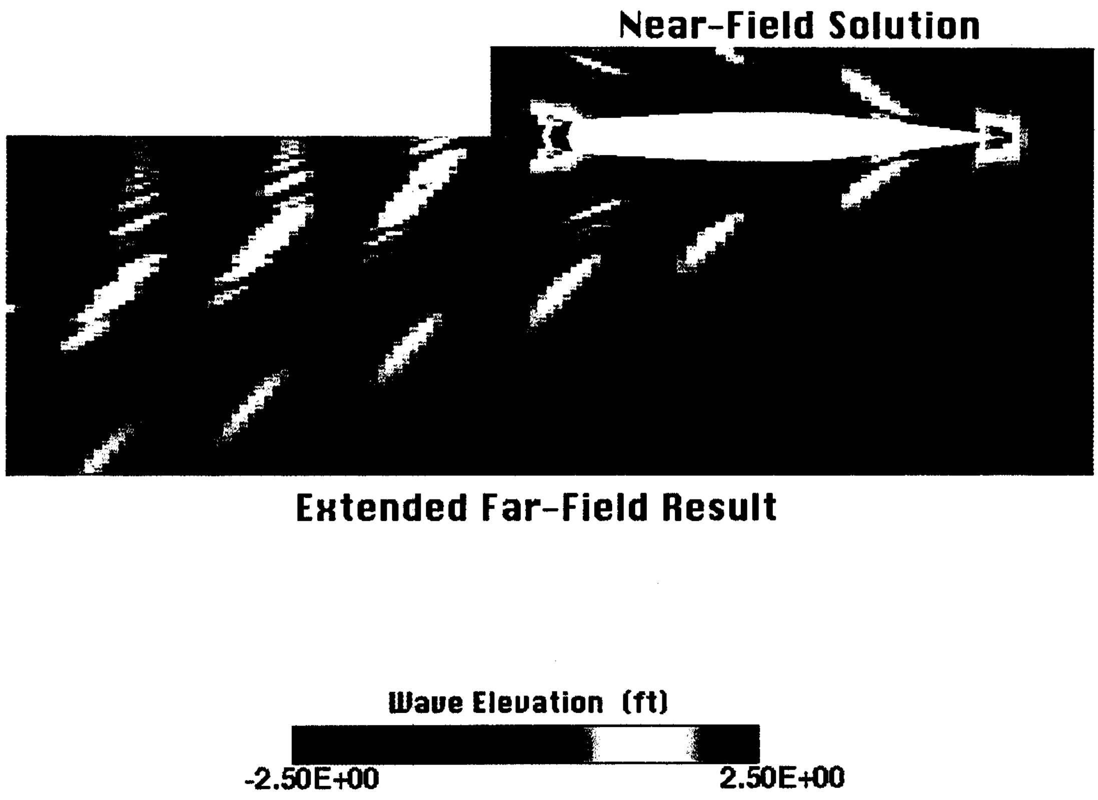
The influence matrix is diagonally dominant, and with our current Havelock method all of the diagonal elements are determined analytically. We have found that this approach leads to stable solutions that are free of the numerical noise which can be the result of small errors occurring in the calculation of diagonal elements.

For full hull forms, solving the Dawson problem can yield greater accuracy, and in this case the benefits of using Havelock free-surface panels are less obvious. The Dawson free-surface boundary condition contains several terms in addition to the two terms occurring in the Kelvin condition, and these additional terms must be calculated numerically. Consequently, the elements of $\mathbf{A}_{22}$ can no longer be specified simply from analytical considerations. It becomes necessary to perform $(N+M) \times M$ Havelock calculations, which will certainly be much more computationally intensive than performing the corresponding $(N+M) \times M$ Rankine calculations required in standard Rankine methods.

Where the Havelock/Dawson approach may have an advantage is in the calculation of the far-field Kelvin wake. Since the Havelock singularities naturally satisfy the proper far-field conditions, there is no need to increase the number of free-surface panels in order to extend the results to include a larger domain. We have found that the Havelock singularity strengths required to satisfy the Dawson free-surface condition decrease rapidly with distance from the hull, and therefore, reasonable accuracy can be achieved by panelizing a rather limited near-field region of the free surface. As an example, we present results for a surface combatant (lwl=540 feet) traveling at 18 knots. The panelized region extends roughly 100 feet forward, 100 feet aft, and 100 feet athwartships. Relative to the peak Havelock singularity strengths in the near-field, the calculated strengths have decreased by two orders of magnitude near the edges of this domain. We have found that the size of the near-field region which must be panelized depends primarily on the double-body solution and is not sensitive to Froude number. We note that for this 18 knot example, our domain extends little more than one half of a Kelvin wavelength from the hull.

The extension of the Havelock/Dawson solution to the calculation of the far-field Kelvin wake is straightforward. The contributions to the free-wave spectrum from each of the free surface Havelock panels can be readily calculated, and then the Kelvin wave elevation at any point in the far field can be economically calculated from the spectrum. As an example, we used our Havelock/Dawson code to calculate a relatively high resolution near-field solution using 1560 panels on the free surface. Then to capture the details of the Kelvin wake over the region extending four transverse wavelengths astern shown in the figure, we performed a far-field calculation using a free-wave spectral approach. Running on a 180 mHz PowerPC, the basic Havelock/Dawson solution required about 30 minutes. The extension to this far-field region took an addition 5 minutes. If we were to simply increase the number of panels, expanding the near-field domain to include this entire region, we would end up with over 10^4 panels, severely taxing our computational resources. Of course the near-field results of the better Rankine methods can also be extended to the far-field provided that the computational domain is large enough to enable accurate calculation of the free-wave spectrum. This would likely require less than 10^4 panels, but still many more than the 1560 panels required for the Havelock/Dawson method.

By using free-surface distributions of Havelock singularities we are able to exploit analytical expressions for the dominant terms in the influence matrix, reduce the number of panels required on the free surface, and economically extend the near-field results to the calculation of the far-field Kelvin wake. Although Rankine methods will remain attractive for near-field Dawson computations, we can expect to encounter other applications for which the unique properties of Havelock singularities are especially well suited.



[1] Scragg, C.A. and J.Talcott "Numerical Solution of the 'Dawson' Free-Surface Problem Using Havelock Singularities," 18th Sym. on Naval Hydrodynamics, Ann Arbor, 1991. pp 259-71

[2] Nakos,D. and P.Sclavounos "Ship Motions by a Three-Dimensional Rankine Panel Method," 18th Sym. on Naval Hydrodynamics, Ann Arbor, 1991. pp 21-40

[3] Raven, H. "Adequacy of Free Surface Conditions for the Wave Resistance Problem," 18th Sym. on Naval Hydrodynamics, Ann Arbor, 1991. pp 375-95