On the piston mode in moonpools

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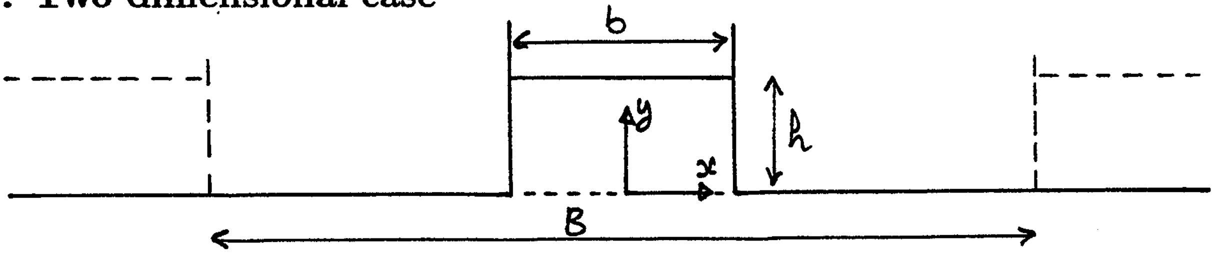
Introduction

Barges with large rectangular moonpools housing the upper ends of the risers are now being considered as floating production systems in mild seas like the Gulf of Guinea. A related hydrodynamic problem is how much water motion takes place in the moonpool, under barge motion and wave induced fluctuating pressures, and whether there are some risks of water flowing over the deck. This water motion mostly takes place at the natural modes of the moonpool: the sloshing modes, back and forth inbetween the vertical walls (like in a tank), and the 'piston' mode, where the water inside the moonpool heaves up and down more or less like a solid body. This piston mode usually is the most bothering one and its effects must be taken into account in the design process.
Here we are just concerned with the first step of the analysis, that is estimating the natural frequency of the piston mode. Another problem that we will consider is how flat the free surface is when the water heaves up and down. In the process we will also obtain the natural frequencies and eigen vectors of the sloshing modes but these will be given no further attention.
The barge is assumed to be a rectangular box of length L, beam B and draft h. The moonpool is located at mid-deck, and is also rectangular with length l and width b. The waterdepth is taken to be infinite.
The problem is tackled via potential flow theory. We consider it successively in two and three dimensions. Two dimensions means a rather elongated moonpool, the hydrodynamic problem being solved in a transverse plane. This case also applies to twin hull geometries. The simplifying assumption that we make in three dimensions is that the length and beam of the barge are much larger than the moonpool's, so we can take them to be infinite. In two dimensions we grossly account for the actual beam of the barge, which nevertheless must be large when compared to the moonpool width.

1. Two dimensional case



Let us first assume the beam of the barge to be infinite so that the geometry looks as sketched above. The fluid domain can be decomposed into two subdomains, by introducing a ficticious bottom to the moonpool: inside the moonpool, and the infinite half-plane $y < 0$.
In this latter domain the velocity potential $\Phi(x, y, t)$ obeys the Laplace equation, the no-flow condition $\Phi_y = 0$ on the horizontal axis for $|x| > b/2$, and matching conditions with the potential inside the moonpool on the ficticious bottom $|x| < b/2$, $y = 0$. If $\Phi_y(x, 0, t)$ is the vertical flow velocity on this segment, then the complex velocity potential in the lower half

plane is simply given by

$$f(z,t) = -\frac{1}{\pi} \int_{-b/2}^{b/2} \Phi_y(\zeta,0,t) \, \ln(z-\zeta) \, d\zeta \tag{1}$$

(e.g. see Newman, 1977, ch 5.7).
So far we have given no consideration to conditions at infinity in the lower domain. As the water heaves up and down inside the moonpool, there is, at given time, a net flux across the cut. Then, according to (1), the pressure is infinite at infinity. This means that our idealization is unphysical. What happens practically is that the water flows around the keel of the barge and radiates waves. This effect can be very crudely modeled by putting two sinks at $x = \pm B/2$, so that the complex velocity potential now writes

$$f(z,t) = -\frac{1}{\pi} \int_{-b/2}^{b/2} \Phi_y(\zeta,0,t) \left[\ln(z-\zeta) - \frac{1}{2}\ln(z-B/2) - \frac{1}{2}\ln(z+B/2)\right] d\zeta \tag{2}$$

Φ on the cut is given by

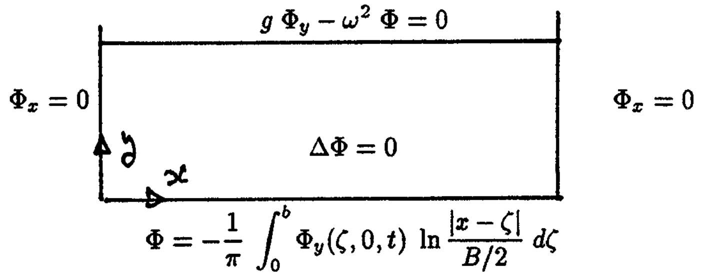
$$\Phi(x,0,t) = -\frac{1}{\pi} \int_{-b/2}^{b/2} \Phi_y(\zeta,0,t) \left[\ln|x-\zeta| - \frac{1}{2}\ln(B/2-x) - \frac{1}{2}\ln(B/2+x)\right] d\zeta \tag{3}$$

or, more simply, with an error $O(b^2/B^2)$

$$\Phi(x,0,t) = -\frac{1}{\pi} \int_{-b/2}^{b/2} \Phi_y(\zeta,0,t) \, \ln\frac{|x-\zeta|}{B/2} \, d\zeta \tag{4}$$

This is the boundary condition that we take to solve the flow inside the moonpool.

The boundary value problem that we must solve is now



with the coordinate system shifted to the corner. We look for Φ under the form

$$\begin{aligned}\Phi(x,y,t) &= \Re\left\{\varphi(x,y)\, e^{-i\omega t}\right\} \\ \varphi(x,y) &= A_0 + B_0\, y/h + \sum_{n=1}^{N}(A_n \cosh\lambda_n y + B_n \sinh\lambda_n y) \cos\lambda_n x\end{aligned}$$

(with $\lambda_n = n\,\pi/b$) so that the Laplace equation and the no-flow conditions on the vertical walls are fulfilled. The boundary condition on the ficticious bottom gives

$$A_0 + \sum_{n=1}^{N} A_n \cos\lambda_n x = -\frac{1}{\pi}\int_0^b \left(B_0/h + \sum_{n=1}^{N} \lambda_n B_n \cos\lambda_n \zeta\right) \ln\frac{|x-\zeta|}{B/2}\, d\zeta$$

Integrating each side in x from 0 to b gives

$$A_0 = \frac{1}{\pi}\frac{b}{h}\,B_0\,\left(\frac{3}{2} + \ln\frac{B}{2\,b}\right) - \frac{b}{\pi^3}\sum_{n=1}^{N}\lambda_n\,B_n\int_0^{\pi}\int_0^{\pi}\cos n\,v\;\ln|u-v|\;du\;dv \qquad (5)$$

Similarly multiplying both sides with $\cos\lambda_m\,x$ and integrating gives

$$A_m = -\frac{2\,b}{\pi^3\,h}B_0\int_0^{\pi}\int_0^{\pi}\cos m\,u\;\ln|u-v|\;du\;dv - \frac{2\,b}{\pi^3}\sum_{n=1}^{N}\lambda_n B_n\int_0^{\pi}\int_0^{\pi}\cos m\,u\;\cos n\,v\;\ln|u-v|\;du\;dv$$

or, in vector form

$$\vec{A} = \mathbf{AB}\cdot\vec{B} \qquad (6)$$

with $\vec{A} = (A_0,\ldots,A_N)$ $\vec{B} = (B_0,\ldots,B_N)$.

Considering then the free surface condition we obtain the following equations

$$\frac{g}{h}\,B_0 = \omega^2\,(A_0 + B_0) \qquad (7)$$

$$g\,\lambda_n\,(A_n\;\tanh\lambda_n\,h + B_n) = \omega^2\,(A_n + B_n\;\tanh\lambda_n\,h) \qquad (8)$$

which, upon combination with (6), gives the eigen value problem

$$[\mathbf{D_1}\cdot\mathbf{AB} + \mathbf{D_2}]\,\vec{B} = \omega^2\,[\mathbf{AB} + \mathbf{D_3}]\,\vec{B} \qquad (9)$$

where $\mathbf{D_1}$, $\mathbf{D_2}$, $\mathbf{D_3}$ are diagonal matrices. This problem is solved by a standard method and the eigen frequencies ω_i and eigen vectors $\vec{B}_i$ are obtained.

The natural frequency of the piston mode can be estimated by neglecting its coupling with the (even) geometric sloshing modes. From (5) and (7), setting all B_n to zero for $n \geq 1$, we obtain

$$\omega_0 = \sqrt{\frac{g}{h}\,\frac{1}{1 + \frac{b}{\pi\,h}\,\left(\frac{3}{2} + \ln\frac{B}{2\,b}\right)}} \qquad (10)$$

This has been found to yield values always within 1 % of the ones obtained by solving the eigen value problem (9), whatever the ratio h/b.
The next figure shows the shapes of the free surface at three different values of the h/b ratio: 0.01, 0.10 and 1.00, in the case $B = 2\,b$. All curves are normalized so that the area underneath (the value of $A_0 + B_0$) is equal. It can be seen that assuming the free surface to be flat yields conservative results at (very) small h/b ratios.

2. Three dimensional case

In the 3D case there is no problem with the pressure behaving badly at infinity. The boundary condition on the bottom of the moonpool becomes

$$\Phi(x,y,0,t) = \frac{1}{2\pi}\int_0^b\int_0^l \frac{\Phi_z(x',y',0,t)}{\sqrt{(x-x')^2 + (y-y')^2}}\;dx'\;dy'$$

The velocity potential inside the moonpool is expressed as

$$\Phi(x,y,z,t) = \Re\left\{\varphi(x,y,z)\;e^{-i\,\omega\,t}\right\}$$

$$\varphi = \sum_{n=0}^{N} \sum_{q=0}^{Q} \cos \lambda_n x \; \cos \mu_q y \; (A_{nq} \; \cosh \nu_{nq} z + B_{nq} \; \sinh \nu_{nq} z)$$

with $\lambda_n = n\,\pi/l$, $\mu_q = q\,\pi/b$, $\nu_{nq}^2 = \lambda_n^2 + \mu_q^2$
and, when $n = q = 0$, the hyperbolic functions are to be replaced with $A_{00} + B_{00}\, z/h$.
The same Galerkin procedure as in the 2D case can be used, resulting in integrals like

$$\int_0^l dx \int_0^l dx' \int_0^b dy \int_0^b dy' \; \frac{\cos \lambda_m x \; \cos \lambda_n x' \; \cos \mu_p y \; \cos \mu_q y'}{\sqrt{(x-x')^2 + (y-y')^2}}$$

to be evaluated.
Again an approximate value of the natural frequency of the piston mode can be obtained by neglecting coupling with the (even) geometric sloshing modes. One obtains

$$\omega_{00} = \sqrt{\frac{g}{h\,(1+C)}} \tag{11}$$

with

$$C = \frac{1}{2\pi} \, \frac{1}{b\,l\,h} \left(b^2\, l \; \mathrm{arg\,sinh} \, \frac{l}{b} + b\, l^2 \; \mathrm{arg\,sinh} \, \frac{b}{l} + \frac{1}{3}\,(b^3 + l^3) - \frac{1}{3}\,(b^2 + l^2)^{3/2} \right) \tag{12}$$

For a square pontoon ($l = b$), one has

$$\omega_{00} = \sqrt{\frac{g}{h + 0.24\, b}} \tag{13}$$

Further results will be given at the workshop.

Reference

NEWMAN, J.N. 1977. *Marine Hydrodynamics*, MIT Press.