Water Impact Model with Horizontal Velocity

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High-speed planing boats have gained wide popularity in many areas of boating, including the military, pleasure boaters, and high speed ferries. Yet planing craft are known to suffer from unexpected behavior at their operational speeds. Research at the University of Michigan intending to understand dynamic instability has used a water impact model to determine the flow over a cross section of the hull. The impact model takes a two-dimensional section of the hull and predicts how the flow moves over the bottom as the hull section enters the water. Based upon a low order strip theory, the planing hull is viewed as a series of cross sections at different points of impact (near the bow the hull is just starting to enter the water, near the transom the hull has mostly entered the water) this model determines the transverse flow characteristics over the entire hull. The resulting boundary value problem can be numerically solved using a two-dimensional vortex distribution.

Xu [2] developed a theory based on this impact model which allowed for asymmetric hulls and asymmetric vertical impact. Xu built on Vorus' [1] work which included a "flat" cylinder theory to allow arbitrary sectional contour impact, reordering the variables in the first order in a physically consistent manner. Xu introduced two types of vertical impact due to asymmetry. Type A flow is when there is small asymmetry and on both sides the zero-pressure points (C_1 and C_2)and jet-spray roots (B_1 and B_2)advance out towards the chine. Type B flow occurs when there is large asymmetry and the flow is forced to separate at the keel so that only on one side does the zero-pressure point (C_1) advance out towards the chine.

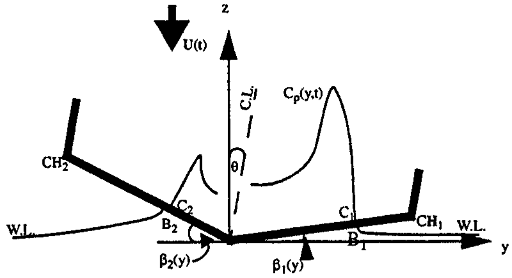

Figure 1: Type A Flow

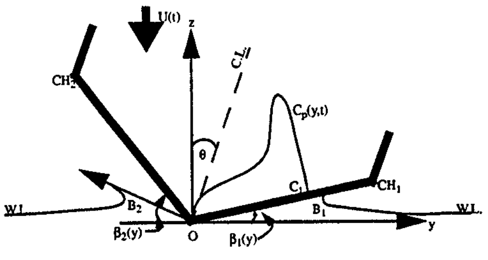

Figure 2: Type B Flow

Xu investigated the limits of asymmetry in his thesis. As the heel angle, θ (which equals one-half the difference between β_1 and β_2 and represents the asymmetry), increased from zero, the wetted point, C_2, (on the left side of the hull) in Type A flow moved backward until it reached the keel. When C_2 reached the keel the flow was forced to separate and the impact became Type B. The conditions were obtained from the basic solutions of flat-sided contours with constant impact velocity. By holding β_1 constant and determining the angle θ at which Type B flow occurred, β_2 was determined. The limiting angle of β_2 versus the corresponding β_1 is plotted in Figure 3 and suggests that the critical value of β_2 is relatively insensitive to β_1.

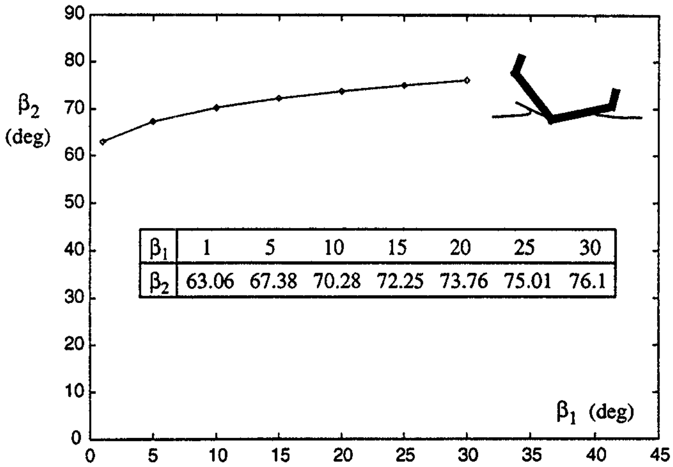

Figure 3: For each β_1, the value of β_2 which causes Type B flow [3]

Asymmetry is very important when considering transverse plane motions. Transverse stability is also affected by horizontal velocity. In order to predict stability in the transverse plane, horizontal velocity during impact needs to be taken into consideration. Ideally, both asymmetry and horizontal velocity would be included, but as a first attempt consider a symmetric hull that has both a vertical and a horizontal velocity during impact.

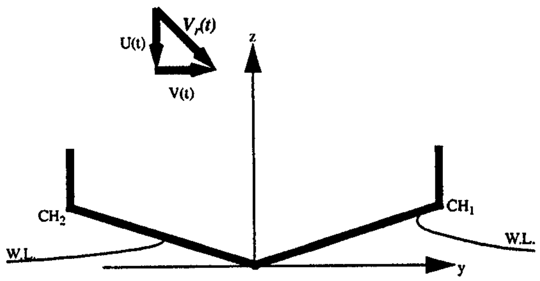

Figure 4: Symmetric Impact with Horizontal Velocity

Such a hull, when roll is constrained, would produce impact flows similar to asymmetric flow where the zero-pressure point moves out toward the chine faster on one side than the other. In other words, such an impact would produce a Type A flow. The question posed here is under what conditions Type B flow can be produced, i.e. what is the ratio of horizontal velocity to vertical velocity at which the flow separates off the keel at impact. A symmetric case is first considered to reveal flow characteristics that are significant contributions towards solving the transverse stability problem.

The model is defined by the dynamic boundary condition, the kinematic boundary condition, the kutta conditions, and the displacement continuity condition. The dynamic boundary conditions result in the equations for the velocities of the jet roots.

$$Y_{B1t} = \frac{v_s^2(b_1, t) + v_n^2(b_1, t)}{2v_s(b_1, t)}$$

$$Y_{B2t} = \frac{v_s^2(-b_2, t) + v_n^2(-b_2, t)}{2v_s(-b_2, t)}$$

where Y_{B1t} and Y_{B2t} are the jet velocities and v_s and v_n are the perturbation velocities. The jet spray root and zero-pressure point locations are nondimensionalized by the right hand side zero-pressure point and are b_1, b_2, 1, and c_2 respectively. The dynamic boundary conditions are found assuming $v_s = -\frac{\gamma}{2}$ and by using the Biot-Savart law.

$$\frac{1}{2}\gamma(\xi, t)\sin\beta(\xi) + \frac{1}{2\pi}\int_{-b_2(t)}^{b_1(t)} \frac{\gamma(s, t)}{s-\xi}ds = -\frac{1}{2}V\sin(2\beta(\xi)) - \cos^2\beta(\xi)$$

The solution to this equation is

$$\gamma_c(\xi, t) = 2\cos\tilde{\beta}(\xi)\Bigg(\sin\tilde{\beta}(\xi)\left\{-\frac{1}{2}V(t)\sin[2\beta(\xi)] - \cos^2\beta(\xi)\right\}$$

$$- \frac{\chi(\xi, t)}{2\pi}\Bigg\{2\int_{-c_2(t)}^{1}\frac{\cos\tilde{\beta}(s)}{\chi(s, t)(s-\xi)}\left[-\frac{1}{2}V(t)\sin[2\beta(s)] - \cos^2\beta(s)\right]ds$$

$$+ \int_1^{b_1(t)}\gamma_s(s, t)ds + \int_{-b_2(t)}^{-c_2(t)}\gamma_s(s, t)ds - \int_1^{b_1(t)}\frac{\gamma_s(s, t)}{\chi(s, t)(s-\xi)}ds + \int_{-b_2(t)}^{-c_2(t)}\frac{\gamma_s(s, t)}{\chi(s, t)(s-\xi)}ds\Bigg\}\Bigg)$$

where $\tilde{\beta}(\xi) = \tan^{-1}[\sin\beta(\xi)]$ and $\chi(\xi, t) = \frac{\kappa(\xi, t)}{\sqrt{|(\xi + c_2(t))(1-\xi)|}}$ with $\kappa(\xi, t) = \kappa_1(\xi, t)\kappa_2(\xi, t)$, here

$\kappa_1(\xi, t) = \lim_{K_1 \to \infty} \Pi_{k=1}^{K_1}\left|\frac{\xi_{1k,1} - \xi}{\xi_{1k,0} - \xi}\right|^{\frac{\bar{\beta}_{1k}(t)}{\pi}}$ and $\kappa_2(\xi, t) = \lim_{K_2 \to \infty} \Pi_{k=1}^{K_2}\left|\frac{\xi_{2k,1} - \xi}{\xi_{2k,0} - \xi}\right|^{\frac{\bar{\beta}_{2k}(t)}{\pi}}$. The Kutta conditions must be satisfied at the wetted points C_1 and C_2 to guarantee velocity continuity. In other words, the singularities in $\gamma_c(\xi, t)$ at $\xi = 1$ and $\xi = -c_2$ must be removed. That is,

$$0 = \sin\tilde{\beta}(1)\left(-\frac{V(t)}{2}\sin[2\beta(1)] - \cos^2\beta(1)\right) + \frac{\chi(1, t)}{2\pi}\Bigg\{V(t)\int_{-c_2(t)}^{1}\frac{\sin[2\beta(s)]\cos\tilde{\beta}(s)}{\chi(s, t)(s-1)}ds$$

$$+ 2\int_{-c_2(t)}^{1}\frac{\cos^2\beta(s)\cos\tilde{\beta}(s)}{\chi(s, t)(s-1)}ds + \int_1^{b_1(t)}\gamma_s(s, t)ds + \int_{-b_2(t)}^{-c_2(t)}\gamma_s(s, t)ds$$

$$- \int_1^{b_1(t)}\frac{\gamma_s(s, t)}{\chi(s, t)(s-1)}ds + \int_{-b_2(t)}^{-c_2(t)}\frac{\gamma_s(s, t)}{\chi(s, t)(s-1)}ds\Bigg\}$$

and

$$0 = \sin\tilde{\beta}(-c_2(t))\left(\frac{V(t)}{2}\sin[2\beta(-c_2(t))] - \cos^2\beta(-c_2(t))\right)$$

$$+ \frac{\chi(-c_2((t),t))}{2\pi}\left\{-V(t)\int_{-c_2(t)}^{1}\frac{\sin[2\beta(s)]\cos\tilde{\beta}(s)}{\chi(s,t)(s+c_2(t))}ds + 2\int_{-c_2(t)}^{1}\frac{\cos^2\beta(s)\cos\tilde{\beta}(s)}{\chi(s,t)(s+c_2(t))}ds\right.$$

$$\left.-\int_{1}^{b_1(t)}\gamma_s(s,t)ds + \int_{-b_2(t)}^{-c_2(t)}\gamma_s(s,t)ds - \int_{1}^{b_1(t)}\frac{\gamma_s(s,t)}{\chi(s,t)(s+c_2(t))}ds + \int_{-b_2(t)}^{-c_2(t)}\frac{\gamma_s(s,t)}{\chi(s,t)(s+c_2(t))}ds\right\}$$

The displacement continuity condition is a conservation of mass requirement. It requires that the displacement of the cylinder and the free surface contours combine to be a continuous nontrivial function of y to the second order. Similar to the Kutta conditions these are found by removing singularities at the points of discontinuity.

$$0 = Z_{WL}(t)\int_{-c_2^*(t)}^{1}\frac{\cos^2\beta(s)\cos\tilde{\beta}(s)}{\chi^*(s,t)(s-1)}ds - \int_{-c_2^*(t)}^{1}\frac{\cos^2\beta(s)H_c(s,t)\cos\tilde{\beta}(s)}{\chi^*(s,t)(s-1)}ds$$

$$+ Y_H(t)\int_{-c_2^*(t)}^{1}\frac{\sin[2\beta(s)]\cos\tilde{\beta}(s)}{\chi^*(s,t)(s-1)}ds$$

and

$$0 = Z_{WL}(t)\int_{-c_2^*(t)}^{1}\frac{\cos^2\beta(s)\cos\tilde{\beta}(s)}{\chi^*(s,t)(s+c_2^*(t))}ds - \int_{-c_2^*(t)}^{1}\frac{\cos^2\beta(s)H_c(s,t)\cos\tilde{\beta}(s)}{\chi^*(s,t)(s+c_2^*(t))}ds$$

$$+ Y_H(t)\int_{-c_2^*(t)}^{1}\frac{\sin[2\beta(s)]\cos\tilde{\beta}(s)}{\chi^*(s,t)(s+c_2^*(t))}ds$$

The results of an impact model that included both asymmetry and horizontal velocity could be incorporated into a nonlinear motion simulator in order to provide an analytical transverse stability tool. The results would also allow a dynamic righting arm curve to be developed for high speed planing craft. Horizontal velocity is a significant component of transverse planing stability and as such needs to be addressed. This is the goal of solving the problem presented here.

References

[1] Vorus, W.S. 1996 A flat cylinder theory for vessel impact and steady planing resistance. *Journal of Ship Research*, **40**, 2

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[3] Xu, L., Troesch, A.W., and Vorus, W.S. 1998 Asymmetric Vessel Impact and Planing Hydrodynamics, *Journal of Ship Research*, **42**, 3