WATER WAVES BENEATH A FLOATING ELASTIC PLATE

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1. Introduction

Recently the estimation of elastic motion of a very large floating structure (VLFS) has been carried out for the Mega-Float project in Japan. Dimensions of the floating structure in this project is 5,000 m length and 1,500 m width. The typical wave period of the installation point is 6 seconds. Several reliable numerical works have been completed, however it was very tough job to obtain the reliable result since the length of incident waves is very short compaired to the dimensions of VLFS. Following the numerical results, it is found that the elastic motion of VLFS is seems like a propagation of water waves beneath a thin elastic-plate. However, since those numerical works are based on the modal analysis, it is difficult to imagine a image of the motion of VLFS as propagating waves before summing up each modes. Therefore, another approach is needed to make simple image of the motion of VLFS. Ohkus and Nanba [1996] treated this problem as a wave propagation beneath a thin elastic-plate and presented a free surface condition which is imposed on the region covered with the plate. Helmans [1997] also presented a similar treatment in which he applied the assumption of very short wave length. In the present paper, a similar free surface condition for the region covered with the plate is applied and a Green function of that problem is derived. The eigen function expansion method is evolved from Green's second identity.

2. Free Surface Condition

Suppose a flat floating platform of draft d ($d = 1 \sim 2m$ in Mega-Float project) located in the x-y plane which coincides with the still water surface. Following previous works, the assumption of $d/\lambda \ll 1$ is applied. Since the motion of the fluid is supposed to be invicid, irrotational and incompressible. The velocity potential satisfying Laplace's equation is introduced. Further assumption is that the motion is sinusoidal with the angular frequency ω.

Thin elastic plate theory gives the equation of the vertical displacement ζ of the plate.

$$m\frac{\partial^2\zeta}{\partial x^2} = -D\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2\zeta - \rho g\zeta - i\rho\omega\phi(x,y,0) \tag{1}$$

Where, m is the mass of unit area of the plate, ρ the density of the water and g the gravitational acceleration. D is the flexural rigidity of the platform given by $D = ET^3/12(1-\nu^2)$. Where T is the thickness, E the equivalent Young's modulus of the plate and ν Poisson's ratio. Since the mass of plate is uniformly distributed, it is obvious that the left hand side of (1) is negligible.

Substituting the body boundary condition of the plate into equation (1), the free surface condition is obtained.

$$-K\phi + \left[\hat{\beta}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 + 1\right]\frac{\partial\phi}{\partial z} = 0 \qquad on \quad z = 0 \tag{2}$$

Where, $\hat{\beta} = D/(\rho g)$ and $K = \omega^2/g$. In the two-dimensional case the free surface condition is reduced as follows:

$$-K\phi + \left(1 + \beta\frac{\partial^4}{\partial x^4}\right)\frac{\partial\phi}{\partial z} = 0 \qquad on \qquad z = 0 \tag{3}$$

Where, $\beta = \frac{EI}{\rho g}$ and EI presents the bending rigidity. Suppose the plane progressive waves on the plate floating on the water of depth h. Where the water depth is assumed to be as same order as the wave length. The following dispersion relation is obtained.

$$K = \alpha(1 + \beta\alpha^4)\tanh\alpha h \tag{4}$$

It is apparent that two roots of equation (4) are located on the positive and negative real axis and innumerable roots are located on the imaginary axis. Other four roots are also found on each quarter planes.

3. Two-Dimensional Green Function

A two-dimensional Green function which satisfies Laplace's equation, the free surface condition (3) and the bottom condition is obtained as follows:

$$G(x, z, \acute{x}, \acute{z}) = \sum_{n=0}^{\infty} \frac{e^{-i\alpha_n|x-\acute{x}|}\{K\sinh\alpha_n\acute{z} + (1+\beta\alpha_n^4)\alpha_n\cosh\alpha_n\acute{z}\}\cosh\alpha_n(z+h)}{\alpha_n\{\frac{Kh}{\sinh\alpha_n h} + (1+5\beta\alpha_n^4)\sinh\alpha_n h\}}. \tag{5}$$

Where, $\alpha_n(n = 0, 1, 2\cdots)$ denote the roots of equation (4) located in the lower half plane. Applying Green's second identity to the region $x \leq 0$, the following integral equation is obtained.

$$\begin{aligned}\phi &= i\omega\frac{\beta}{K}\left(\frac{\partial G}{\partial z}\frac{d^3\zeta}{dx^3} - \frac{\partial^2 G}{\partial x\partial z}\frac{d^2\zeta}{dx^2} + \frac{\partial^3 G}{\partial x^2\partial z}\frac{d\zeta}{dx} - \frac{\partial^4 G}{\partial x^3\partial z}\zeta\right)_{x=0} \\ &\quad - \int_{-h}^{0}(\phi\frac{\partial G}{\partial \acute{x}} - \frac{\partial\phi}{\partial\acute{x}}G)d\acute{z}\end{aligned} \tag{6}$$

Where, it is assumed that the plate is infinitely long and covers all left half plane. The regular radiation condition is imposed on the left end boundary.

If the boundary condition at $x = 0$ and the end conditions of the plate are given, above integral equation would be solved.

4. Eigen Function Expansion

If the integral appeared in equation (6) can be carried out in advance, the following series expansion would be obtained.

$$\phi = \sum_{n=0}^{\infty} iT_n\frac{\omega}{\alpha_n}\frac{\cosh\alpha_n(z+h)}{\sinh\alpha_n h}e^{i\alpha_n x} \tag{7}$$

Orthogonality

The eigen functions appeared in equation (7) are not orthogonal. However, the following relations are obtained.

$$\hat{I_n} = \int_{-h}^{0}\phi\cosh\alpha_n(z+h)dz = \hat{q_n}T_n - \frac{d^2\zeta}{dx^2}\hat{S_n} + \zeta\alpha_n{}^2\hat{S_n} \tag{8}$$

$$\tilde{I_n} = \int_{-h}^{0}\frac{\partial\phi}{\partial x}\cosh\alpha_n(z+h)dz = \tilde{q_n}T_n - \frac{d^3\zeta}{dx^3}\hat{S_n} + \frac{d\zeta}{dx}\alpha_n{}^2\hat{S_n} \tag{9}$$

Where,

$$\hat{q_n} = \frac{i}{2}\frac{\omega}{\alpha_n}\left(\frac{h}{\sinh \alpha_n h} + \frac{1}{K}(1 + 5\beta{\alpha_n}^4)\sinh \alpha_n h\right) \tag{10}$$

$$\tilde{q_n} = -i\alpha_n \hat{q_n} \tag{11}$$

$$\hat{S_n} = -i\omega\frac{\beta}{K}\alpha_n \sinh \alpha_n h \tag{12}$$

5. Examples of 2-D Problem

Transmission and Reflection of Incident Waves

When plane waves incident on the elastic plate, same are reflected at the edge of the plate and others are transmitted into the region covered with the elastic plate. The velocity potential is represented by the series of eigen functions in the region $x \geq 0$.

$$\phi = i\frac{\omega}{k_0}\frac{\cosh k_0(z+h)}{\sinh k_0 h}e^{ik_0 x} + \sum_{n=0}^{\infty} iR_n \frac{\omega}{k_n}\frac{\cosh k_n(z+h)}{\sinh k_n h}e^{-ik_n x} \tag{13}$$

Where, k_n denotes the roots of the disparsion relation of water waves.

$$K = k_n \tanh k_n h \tag{14}$$

The velocity potential in the region $x \leq 0$ is represented by the series expansion (7).

It is well known that the eigen functions appeared in equation (13) are orthogonal. Therefore, employing the condition that the velocity potential and the horizontal velocity are continuos at the matching boundary, we can get the same number of equations as the number of coefficients R_n and T_n. However, we also have other unknowns ζ_{xx} and ζ. The end condition of the plate i.e. the shearing force and the moment at the end are free gives the following two equations.

$$\sum_{n=0}^{\infty} \alpha_n^3 T_n = 0, \qquad \sum_{n=0}^{\infty} \alpha_n^2 T_n = 0. \tag{15}$$

Then, the problem can be solved, since the number of equations is as same as the number of unknowns.

Reduction of Transmitted Waves

It is strongly required that the motion of VLFS must be very small. However, it was found in the previous works that the motion of VLFS is not negligible. Then, some ideas for the reduction of the motion is proposed. One simple method is attaching a plate or block at the tip of the VLFS to block the transmission of incident waves.

Suppose a block of draft $\hat{d}$ and bredth $2b$ attaching at the edge of the plate, the velocity potential under the block is represented as follows:

$$\begin{aligned}\phi &= \frac{i\omega}{2l}Z(-x^2 + z^2 + 2hz) + \frac{i\omega}{6l}\Theta(-x^3 + 3xz^2 + 6hxz) \\ &+ \sum_{n=1}^{\infty}\left(S_n \frac{\cosh \frac{n\pi}{l}x}{\cosh \frac{n\pi}{l}b} + A_n \frac{\sinh \frac{n\pi}{l}x}{\sinh \frac{n\pi}{l}b}\right)\cos\frac{n\pi}{l}(z+b)(-1)^n.\end{aligned} \tag{16}$$

Where, $l = h - \hat{d}$, Z the heave amplitude and Θ the roll amplitude. The equation of motion of the block is given by

$$\left.\begin{aligned} Z(2bg - \omega^2\tfrac{W}{g}) &= F_f + EI\tfrac{\partial^3\zeta}{\partial x^3} \\ \Theta(W\cdot GM - \omega^2 I_{\theta\theta}) &= M_f + -EI\tfrac{\partial^2\zeta}{\partial x^2} - b\cdot EI\tfrac{\partial^3\zeta}{\partial x^3} \end{aligned}\right\} \tag{17}$$

Where, F_f and M_f are fluid dynamic forces, W weight of the block, GM metacentric height and $I_{\theta\theta}$ moment of inertia of the block. End conditions of the plate are given by

$$Z - b\Theta = \zeta \quad , \quad \Theta = \frac{d\zeta}{dx}. \tag{18}$$

Now the number of equations is as same as the number of unknowns. We can get the solution.

6. Conclusions

The treatment of the motion of VLFS as a propagation of water waves beneath a elastic plate is presented in this paper. Some examples of solution for the two-dimensional problem are shown. Further results will be demonstrated at the workshop.

References

Hermans, A. J. (1997) "The Excitation of Waves in a Very Large Floating Flexible Platform by Short Free-Surface Water Waves", Proc. of 12th Int. Workshop on Water Waves and Floating Bodies", Carry-le-Rouet, France

Ohkusu, M. and Nanba, Y. (1996) "Analysis of Hydroelastic Behavior of a Large Floating Platform of Thin Plate Configuration in Waves", Proc. of Int. Workshop on VLFS, Hayama, Japan

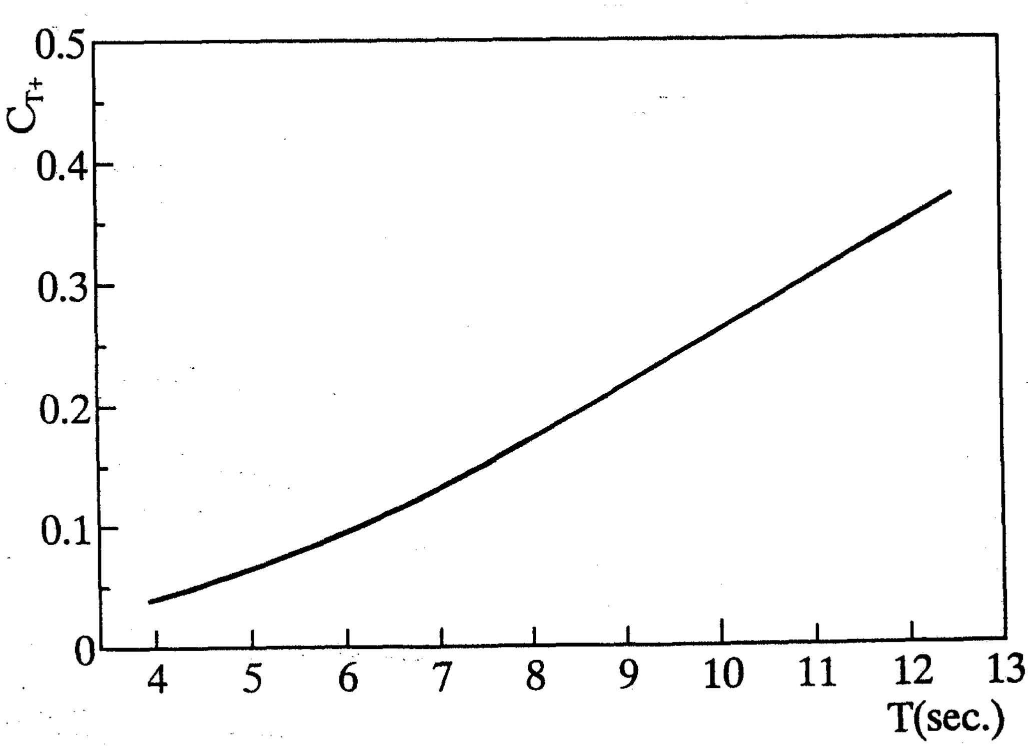

Fig.1 Transmission coefficient at the edge of infinitely long elastic plate.