Wave pattern analysis applied to nonlinear ship wave calculations

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Calculation methods for the steady wave pattern of a ship in still water usually suppose potential flow, and today often impose nonlinear free surface boundary conditions. Advantages of including all nonlinear effects have been demonstrated before [1]. One of these is that an accurate prediction of the wave *resistance* in principle becomes possible: Unlike Dawson-type linearisations, the full nonlinear free-surface conditions theoretically ensure full agreement between the resistance found from a far-field momentum balance and that from pressure integration over the hull. Thereby they rule out the occurrence of negative wave resistance predictions and similar problems.

Even so, computing wave resistance by hull pressure integration remains less attractive, having significant sensitivity to the hull panelling, in particular at low Froude numbers. The predicted wave *pattern*, however, is much less sensitive to the hull panelling. This suggests to evaluate the wave resistance from the calculated wave pattern, using a *wave pattern analysis* technique as developed in the past for towing tank experiments. For use in a computational method a transverse cut technique is most suitable, and has been applied before. Nakos [2] used it succesfully for a linearised wave pattern calculation. Busch [3] used wave pattern analysis in a nonlinear method, obtained good results for a very slender hull form but did not pursue the application to other cases.

The RAPID method

The method used in the present study is RAPID [4, 5]. It solves the steady potential flow problem with fully nonlinear free-surface boundary conditions by an iterative technique. Each iteration linearises with respect to the free-surface shape and velocity field found in the previous iteration. The solution is generated by source distributions on the hull surface and on a plane at a small distance above the free surface. The free-surface collocation points are on the wave surface itself, and are distributed over a finite domain around the ship. The method is quite efficient, converging usually in 8 to 15 iterations; each iteration asks some 6 sec. at 2500 panels, 50 sec. at 8000 panels, on a CRAY 916 vector computer. It is in routine use at MARIN since 1994, several hundreds of calculations being made each year; and besides is used at a few shipyards.

Transverse cut analysis

Transverse cut analysis (see e.g. [6]) uses an expression for the wave resistance in terms of the amplitude of the wave components proceeding in all directions:

$$Rw_{pat} = \frac{1}{8\pi}\int_0^{\infty}(F^2(u)+G^2(u))\frac{\sqrt{1+4u^2}}{1+\sqrt{1+4u^2}}du \ , \qquad (1)$$

for a wave pattern represented by :

$$\eta(x,z) = \frac{1}{4\pi}\int_{-\infty}^{+\infty}[F(u)\sin(sx+uz)+G(u)\cos(sx+uz)]du \ , \qquad (2)$$

where the dispersion relation is $s = \sqrt{(1+\sqrt{1+4u^2})/2}$. The amplitudes F and G can be found by applying a particular Fourier analysis to the wave height distribution along two transverse lines far behind the ship. In [3] it is shown that a larger number of wave cuts is preferable to provide redundancy and increase the accuracy of the result. In our study 8 cuts appeared to work well. The overdetermined system is solved in a least-squares sense.

The transverse cuts must be located far enough aft of the hull, they must extend laterally to outside the Kelvin wedge, they must be free of reflection effects, and the free-surface panel width must be small enough to resolve all wave components of interest. With some adjustments the ensuing requirements to the free-surface panelling can easily be met.

Dependence on longitudinal position

Evidently, the wave pattern resistance is supposed to be independent of the longitudinal position of the cuts. But without further precautions it is not, as Fig.1 illustrates for the case of a slender vessel at $Fn = 0.33$. In general we observe that:

- Close to the stern, there is a rather quick variation of the resistance found;
- The resistance displays wavy variations around a mean line, with a slowly decreasing amplitude;
- The wave pattern resistance gradually decreases with distance of the cuts behind the stern;
- All values found are significantly lower than the pressure integration resistance level.

The first point is an expected symptom of the near-field disturbance around the hull, which violates the basic assumptions of the analysis. As far as our experience goes, further than 0.3 to 0.5 L aft of the stern this effect is mostly negligible. The second effect was unexpected and is studied below. The last two points are discussed in the last section.

Effect of nonlinearities on transverse cut analysis

To detect the cause of the wavy variations we go back to the derivation of transverse cut analysis. The starting point is a momentum or energy balance for a control volume surrounding the ship hull, bounded by the wave surface, an inlet plane and lateral boundaries at infinity, and a transverse outlet plane at any distance behind the stern. This provides the wave resistance in terms of an integral over the outlet plane, plus a line integral along its intersection with the wave surface:

$$Rw_{far} = \frac{1}{2}\int_D\int(-u^2+v^2+w^2)dS + \frac{1}{2}\int_{l_D}\eta^2 dz. \tag{3}$$

Here, u, v and w are disturbance velocity components relative to the undisturbed flow.

The first term is not easily evaluated accurately and therefore is recast in an expression in terms of wave heights only. To this purpose, the far-field wave pattern is assumed to have the form (2), i.e. to be a superposition of simple sinusoidal wave components that satisfy the dispersion relation for the steady wave pattern. Substituting the corresponding potential field into (3) provides the expression (1) for the wave resistance in terms of the amplitude of the wave components.

If fully nonlinear boundary conditions are imposed, the resistance from (3) of course is independent of the position of the outlet plane, as there is no energy flux through the wave surface. Therefore, the wavy variations of the resistance must be due to an incorrect approximation of the integral over the outlet plane, in particular the assumed linearity of the wave components. This is confirmed by the fact that for a Neumann-Kelvin result (obtained with the same numerical method) the wavy variations of the wave pattern resistance are almost absent, as Fig. 1 shows: in that case the expression (2) is consistent with the linear free-surface boundary condition imposed, so the approximation of the outlet plane energy flux is exact. However, using a full Bernoulli expression for the wave height in the Neumann-Kelvin result disturbs this consistency and already introduces a pronounced waviness. We conclude that the wavy variations in the wave pattern resistance are a consequence of the inconsistency of applying a linear analysis to a wave pattern that satisfies nonlinear boundary conditions.

While the amplitude of the resistance variations decays, they persist up to 2 ship lengths aft of the stern in the cases studied. The nonlinear effects therefore extend over a much larger distance than has always been assumed, and this will apply to experimental transverse cut analysis as well.

In order to eliminate the resistance variations, we have tried to replace the wave pattern at the location of the transverse cuts by a 'corresponding' linear wave pattern. One attempt was to use the linear (Kelvin) expression for η , using the free-surface velocities computed by the nonlinear method; but this turned out to increase the amplitude of variations even further. A second attempt was to use again the Kelvin expression, but in terms of the velocities at the still water level. In the case studied this eliminated most of the resistance variations (Fig. 2). However, this procedure is less suitable for routine application and seems to lack a theoretical justification.

A more obvious alternative, and the procedure adopted now, is to spread out the transverse cuts over a longer area. In all cases tried, the waviness in the resistance had a length quite close to the fundamental wave length $\lambda = 2\pi Fn^2$. If the 8 transverse cuts we use are distributed over an area of this length, the wavy variations are "averaged out", and a much more stable result is obtained, as Fig.2 shows.

To get some confirmation that this average value agrees with the true wave resistance as would be obtained from (3), a simple analysis has been carried out. As suggested by the appearance of the fundamental wave length, we represent the wave shape by a single 2D component and make a perturbation expansion in the wave height. For the first order wave $\eta = \frac{1}{2}H_1 \sin x$, substitution in (1) yields:

$$Rw_{pat} = \frac{1}{16}H_1^2 ,$$

so for a linear wave component the resistance from wave pattern analysis is independent of x. Moreover, substituting the associated potential field into (3) produces the same expression for the resistance to leading order. However, for a second-order wave,

$$\eta = \frac{1}{2}H_1 \sin x + \frac{1}{2}H_2 \sin x - \frac{1}{8}H_1^2 \cos x,$$

the resistance from the wave pattern analysis is

$$Rw_{pat} = \frac{1}{16}H_1^2 + \frac{1}{8}H_1 H_2 + \frac{1}{32}H_1^3 \sin x\,,$$

while the control volume integration produces the same but without the $\sin x$ term. Therefore, eliminating the wavy part of Rw_{pat} by averaging over a wavelength brings it in agreement with the energy balance and yields the correct result in this case.

Effect of numerical damping

We now come back to the last two features noted above: the gradual decrease of the average resistance level with x, and the fact that Rw_{pat} is less than the resistance from pressure integration. Both are obvious consequences of the numerical damping inherent to the method. While this damping has little effect in the near field, it causes a slightly too quick decay of the wave amplitude and affects the far field resistance evaluation (which is quadratic in wave amplitudes).

The numerical damping has been analysed theoretically in [5], and indeed there appears to be a direct correspondence between the results of that analysis and the slope of the $Rw(x)$−line. E.g. for the raised-panel method used, the analysis tells that the damping increases when the distance of the panels above the free surface is reduced, and this is reflected in a steeper decrease of the resistance. This resistance decrease may be supposed to be an exponential function of the distance from the point where the waves have been generated. If, as a rough approximation, this virtual origin is chosen at e.g. half the ship length, as a sort of average for bow and stern wave systems, the $Rw_{pat}(x)$ line can be extrapolated backwards to that origin. This largely compensates the dependence of the wave pattern resistance on numerical parameters that affect the damping.

In principle, a better approach is to reduce the slope of the resistance line by reducing the numerical damping. This can be done by optimising the difference scheme in the free surface boundary conditions. The theoretical analysis of the dispersion and damping in [5] indicates how to design a special-purpose difference scheme that virtually eliminates the damping for panellings as coarse as 10 panels per wavelength. Experience with such schemes is now being collected.

Conclusions

- As a result of the inconsistency of applying linear wave pattern analysis to the wave pattern computed with a nonlinear method, the wave pattern resistance has a wavy dependence on the distance aft of the stern.
- The nonlinear effects causing these resistance variations may persist to as far as 2 ship lengths astern; the same will also be true for experimental wave pattern analysis.
- The variations can and may be largely eliminated by spreading the wave cuts over a fundamental wave length.
- While the wave pattern resistance is much less sensitive to the hull panelling than the hull pressure integration, it is affected by cumulative numerical damping effects and thus poses some additional demands to the numerics. At present its use is for comparative rather than absolute predictions.

Acknowledgment

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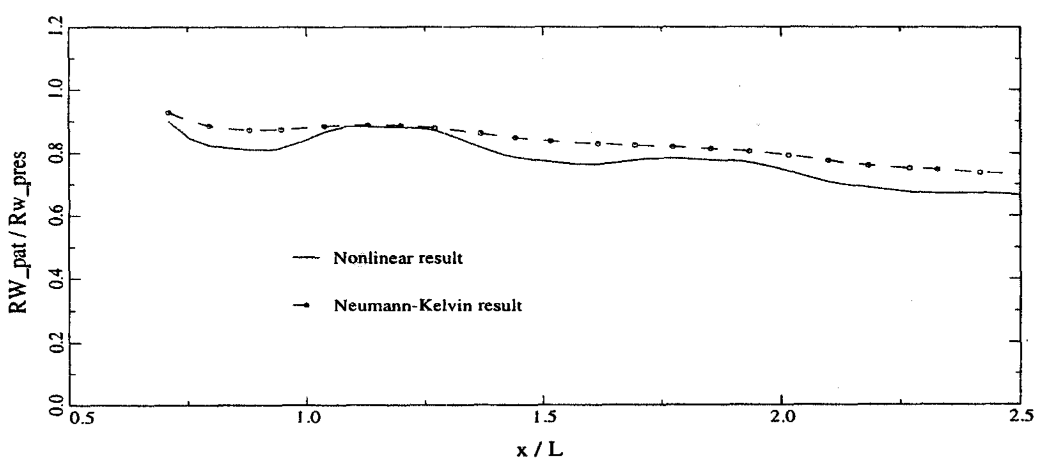

Figure 1: Wave pattern resistance as a function of position of transverse cuts, for nonlinear and linearised method.

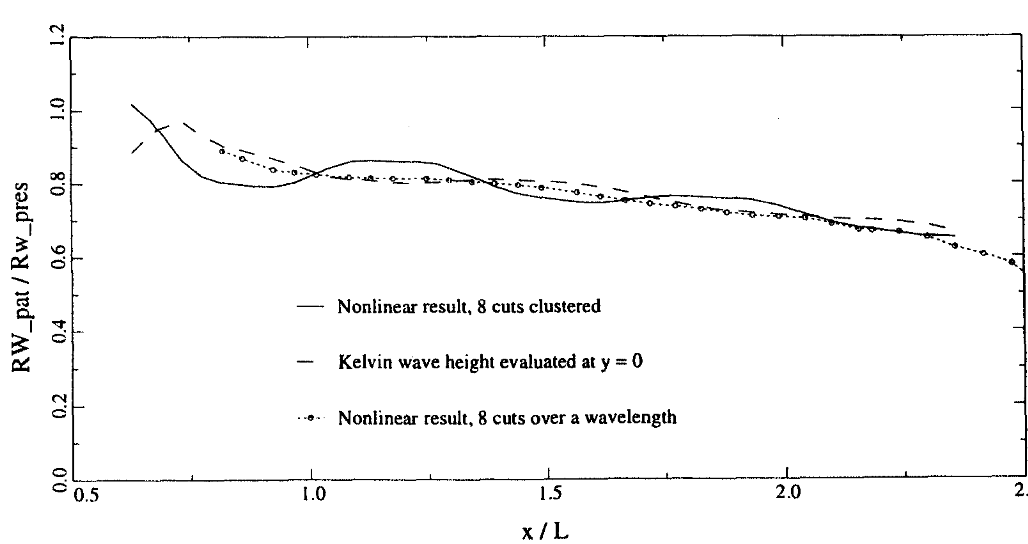

Figure 2: Wave pattern resistance as a function of position of transverse cuts, for nonlinear method. Two ways to eliminate waviness.