One-side inequalities in the problem of wave impact

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The plane unsteady problem of wave impact onto an elastic beam is considered. Initially a wave crest touches the horizontal and beam at its left edge and hits the beam from below thereafter at a constant velocity. The problem is coupled, the beam deflection is determined by the hydrodynamic loads, which themselves depend on velocities of beam elements. The wetted part of the beam, where the hydrodynamic loads are applied, is unknown and has to be found together with the liquid flow and the beam deflection. The original formulation of the impact problem contains not only equations of motion and boundary conditions but one-side inequalities as well. The first inequality implies that the liquid particles cannot penetrate the plate and the second one that the pressure in the contact region cannot be less than a limiting value p_{adh}, which depends on adhesive forces between liquid and the plate surface. The additional limitations make us to control both the pressure distribution along the wetted area and the free surface elevation at every time step and to change the boundary conditions when and where the mentioned one-side inequalities fail.

If the pressure on the plate drops down to the limiting value p_{adh}, a new 'inner' free surface has to be introduced. After that we shall consider the wave impact onto the plate with attached cavity. Within the framework of incompressible liquid we do not know any approach to describe the asymptotic behaviour of the 'inner' free surface just after its appearance. Separation of compressible liquid from the surface of a rigid plate under its impact onto the liquid was described by Korobkin (1994). In this case the formation of the 'inner' free surface is due to relief wave interaction, which come from the periphery of the plate. For elastic-plate impact the pressure in the contact region can drop down to the limiting value owing to the plate flexibility, which reduce the local impact velocities. The numerical codes available to deal with the elastic plate impact is mainly to evaluate the plate deflection and the stress distribution in the plate but not the pressure distribution along the wetted area. Moreover, experiments on elastic plate impact indicate great scattering of the measured pressure which does not encourage us to treat the problem in a deterministic way. In order to evaluate the pressure on the contact region, it is suggested to proceed as follows: (i) determine the beam deflection and its velocity; (ii) evaluate the pressure from the hydrodynamic part of the problem taking the beam deformation as given and taking into account the singularity of the pressure close to the contact points. This approach makes it possible to distinguish low-pressure zones, where liquid can separate from the beam surface (see Korobkin (1996)). But a model to describe the initial stage of the liquid separation and the cavity evolution thereafter is still not available. This is a reason why at present the one-side inequality for the pressure cannot be incorporated into computer codes.

The one-side inequality for the surface elevation implies that the shape of the free surface has to be evaluated at every time step together with the beam deflection and we need to check that the free surface does not intersect the surface of the entering body. The moment t_*, when the free boundary of the liquid touches the body surface outside the contact region, has to be distinguished, and the scheme of the flow has to be changed at this instant of time. At this instant a new part of the contact region appears, which is started from the point of the first contact. These two parts of the contact region are separated by the cavity filled with air. At stage where $t > t_*$ the interaction between the body and the liquid continues but in presence of the cavity. This effect is similar to the air-cushion effect well-known in the impact theory but it has also its own peculiarities. Namely, at the moment of time, when the cavity has been formed, a part of body is already submerged in the liquid and the value t_* is determined by both the body deformations and the liquid flow at $t < t_*$.

This effect was discovered in the problem of wave impact onto an elastic beam at its edge. Parameters of the beam and the wave responsible for this effect were distinguished. In particular, the cavity formation was detected for impact of the wave with the initial radius of curvature at its top $R = 10$m at the velocity $V = 3,5$m onto the elastic plate of mild steel with its length $2L$ and thickness h being $1m$ and 1cm respectively. The wave hits the plate from below at its left-hand side edge. The plate is assumed simply supported at its edges. Numerical calculations were performed within the framework of the Wagner approach with 5 and 10 "dry" modes of the beam taken into account. The numerical method was described by Khabakhpasheva and Korobkin (1997). In the case under consideration the shape of the free surface was controlled and it was revealed that the free surface touches the right edge of the plate when only about 75cm of the plate is wetted. Therefore the initial dimension of the cavity is about 15cm. The formation of the cavity is due to the strong interaction of the plate with the liquid. The plate deformations are not great but they are sufficient to decrease the rate of the contact region expansion so much that the beam edge touches the liquid surface before the whole plate is wetted.

There are two dimensionless parameters α, β, which determine the peculiarities of elastic plate impact. Two regions in the plane of the parameters (α, β), where the Wagner approach fails, were distinguished without taking into account the one-side inequalities. In the first region, the size of the contact region is not monotonic function of time, which is prohibited by the classical Wagner theory. In the second region, the rate of the contact region expansion is unlimited, which indicates very high hydrodynamic loads. These loads cannot be described correctly with the incompressible liquid model and acoustic effects have to be taken into account. Within both regions the numerical calculations were performed using the numerical code described by Khabakhpasheva and Korobkin (1997). The second region of the parameters α, β is of great interest because it distinguishes the impact conditions with very high loads. It should be noted that these loads are much higher than those for a rigid plate and are due to the plate flexibility. The high hydrodynamic loads increase the beam deflection and make it possible that the beam edge touches the liquid free surface well before the beam is totally wetted or the Wagner approach fails.

1 Formulation of the problem

Until the time moment t_* the problem of wave impact onto an elastic plate at its edge is solved within the framework of classical Wagner approach. At the next stage, $t > t_*$, the liquid flow and the beam deflection are governed by the following equations

$$\varphi_{xx} + \varphi_{yy} = 0 \qquad (y < 0), \tag{1}$$

$$\varphi_y = -1 + w_t(x,t) \qquad (y = 0,\ \ 0 < x < c(t),\ \ d(t) < x < 2), \tag{2}$$

$$\varphi = 0 \qquad (y = 0,\ \ x < 0,\ \ x > 2,\ \ c(t) < x < d(t)), \tag{3}$$

$$\varphi \to 0 \qquad (x^2 + y^2 \to \infty), \tag{4}$$

$$p(x,y,t) = -\varphi_t(x,y,t), \tag{5}$$

$$\alpha\frac{\partial^2 w}{\partial t^2}+\beta\frac{\partial^4 w}{\partial x^4}=p(x,0,t) \qquad (0<x<2,\ \ t>t_*), \tag{6}$$

$$w=0, \qquad w_{xx}=0 \qquad (x=0,\ \ x=2), \tag{7}$$

$$w(x,t_*)=w_0(x),\ \ w_t(x,t_*)=w_1(x) \qquad (0<x<2, t=t_*). \tag{8}$$

Here c(t) indicates the position of the left end of the cavity and d(t) the position of its right end. It is assumed that the air in the cavity is absent, which leads to the boundary condition $\varphi=0$, where $y=0,\ c(t)<x<d(t)$. The function $w_0(x)$ and $w_1(x)$ are determined by the solution of the problem at the initial stage, $0<t<t_*$. The shape of the elastic surface $y_b(x,t)$ is given by $y_b(x,t)=x^2/2-t+w(x,t)$ in the both parts of the contact region, $0<x<c(t)$ and $d(t)<x<2$. Condition (3) shows that outside the contact region, $y=0,\ x<0,\ x>2$ and $c(t)<x<d(t)$, the liquid particle can move only vertically. The function $c(t)$ and $d(t)$ are unknown in advance and have to be determined together with the liquid flow.

2 Hydrodynamic problem

The hydrodynamic part of the problem (1)-(4) provides the deformation of the free surface. In particular,

$$Y(x,0,t)=\frac{1}{\pi W(x)}\left[\int_0^c \frac{y_b(\tau,t)W(\tau)}{\tau-x}d\tau-\int_d^2 \frac{y_b(\tau,t)W(\tau)}{\tau-x}d\tau\right]+\frac{D(t)}{\pi W(x)}, \tag{9}$$

where $c(t)<x<d(t)$, $W(\tau)=\sqrt{\tau(c-\tau)(2-\tau)(d-\tau)}$ for $0<\tau<c(t)$ and $d(t)<x<2$, $W(x)=\sqrt{x(x-c)(2-x)(d-x)}$, $D(t)$ is an arbitrary function of time, $X-iY$ is the analytical function of the complex variable $z=x+iy$ in the lower half-plane, $X(x,y,t)$ is the horizontal displacement of a liquid particle and $Y(x,y,t)$ is its vertical displacement. The one-side inequality

$$Y(x,0,t)<y_b(x,t) \quad (c(t)<x<d(t))$$

leads to the following two equations with respect to the unknown functions $c(t)$ and $d(t)$

$$\int_0^c y_b(\tau,t)\sqrt{\frac{\tau(2-\tau)}{(c-\tau)(d-\tau)}}d\tau-\int_d^2 y_b(\tau,t)\sqrt{\frac{\tau(2-\tau)}{(\tau-c)(\tau-d)}}d\tau=0,$$

$$\int_0^c y_b(\tau,t)\sqrt{\frac{\tau(2-\tau)(d-\tau)}{c-\tau}}d\tau+\int_d^2 y_b(\tau,t)\sqrt{\frac{\tau(2-\tau)(\tau-d)}{\tau-c}}d\tau=D(t).$$

Differentiation of the last equations in time provides the system

$$a_{11}(c,d,t)\dot c+a_{12}(c,d,t)\dot d=b_1(c,d,t), \tag{10}$$

$$a_{21}(c,d,t)\dot c+a_{22}(c,d,t)\dot d=b_2(c,d,t)+\dot D(t). \tag{11}$$

Far away the contact region, $|z|\to\infty$, the asymptotic behaviours of the displacements and the liquid velocity are given as

$$X-iY\sim\frac{iD(t)}{\pi W(z)},\quad \varphi_x-i\varphi_y\sim\frac{iC(t)}{\pi W(z)}$$

The asymptotic formulae imply

$$\dot D=C(t) \tag{12}$$

where dot stands for derivative in time. The value $C(t)$ as function of c and d is determined by the condition that $\varphi(x,0,t)=0$ where $c(t)<x<d(t)$.

3 Elastic problem

The beam deflection $w(x,t)$ is sought in the form

$$w(x,t) = \sum_{n=1}^{\infty} a_n(t) \sin(\pi n x/2),$$

Where the coefficients $a_n(t)$ are satisfied the system, which follows from (5) and (6),

$$\frac{d\vec{a}}{dt} = (\alpha I + \kappa S)^{-1}(\beta D\vec{q} + \vec{f}), \qquad \frac{d\vec{q}}{dt} = -\vec{a}. \tag{13}$$

Here $\vec{a} = (a_1, a_2, a_3, \ldots)^T$, $\vec{q}$ is the vector $\vec{q} = (q_1, q_2, q_3, \ldots)^T$, $q_n = (\beta\lambda_n^4)^{-1}(\alpha\dot{a}_n + b_n)$, $\vec{f} = (f_1(c,d), f_2(c,d), f_3(c,d), \ldots)^T$, I is the unit matrix, D is the diagonal matrix, $D = \mathrm{diag}\{\lambda_1^4, \lambda_2^4, \lambda_3^4, \ldots\}$. $S = (S_{nm})_{n,m=1}^{\infty}$ is the matrix with the elements

$$S_{nm}(c,d) = \int_0^c \varphi_n(x,0,c,d)\sin(\pi m x/2)dx + \int_d^2 \varphi_n(x,0,c,d)\sin(\pi n x/2)dx,$$

where φ_n is the solution of the following boundary value problem

$$\Delta\varphi_n = 0 \quad (y<0) \qquad \varphi_n = 0 \quad (y=0: \; x<0, \; c<x<d, \; x>2)$$

$$\frac{\partial\varphi_n}{\partial y} = \sin(\pi n x/2) \quad (y=0: \; 0<x<c, \; d<x<2)$$

The initial conditions for the sistem (10)-(13) are

$$\vec{a} = 0, \quad \vec{q} = 0, \quad c = c_*, \quad d = 0 \qquad (t = t_*). \tag{14}$$

The initial-value problem (10)-(14) is solved numerically by the fourth-order Runge-Kutta method with uniform step Δc. The variable c was chosen as independent variable instead of time t.

4 Numerical results

The calculations were performed for $\alpha = 0.157, \beta = 0.03$, with 2, 5 and 10 modes. The presence of air in the cavity is not taken into account. It was found that the cavity is very thin and localized near the right edge of the plate. The hydrodynamic pressures during the cavity collapse are very high but are of short duration. Main effect of the high pressures is on the stresses, which grow significantly close to the edge. It was revealed that the value t_* has to be evaluated very precisely to make the numerical scheme stable. This condition is not easy to fulfil. But it was proved that system (10), (11) provides $\dot{d}(t_*) = 0$. Therefore, in order to make the scheme stable, we can keep $\dot{d}(t) = 0$ at several initial steps in time. The equality $\dot{d}(t_*) = 0$ demonstrates that we cannot consider the impact of the right edge of the beam onto free surface independently on the total geometry of the beam even just after the impact occurs. This conclusion shows that "local effects" have to be treated very carefully and that our intuition can give us wrong ideas.

References

Korobkin A. A. Low-pressure zones under a liquid-solid impact // Bubble Dynamics and Interface Phenomena, Kluwer Academic Publishers. Netherlands. 1994. P. 375-381.

Korobkin A. A. Wave impact on the centre of an Euler beam // J.Appl.Mech.Tech.Phys. 1997 (In press).

Khabakhpasheva T.I., Korobkin A.A. Wave impact on elastic plates // Proc.12th Intern. Workshop on Water Waves and Floating Bodies, Carry-le-Rouet, France, 1997. P. 135-138.