Computation of impulse response function using differential properties of the time-domain Green function.

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In this paper, three methods for evaluating the time-domain Green function during BEM computations of impulse response functions of floating bodies are compared, in terms of cpu time performances. The impulsive wave radiation problem is solved in the general frame of linear free-surface potential flow. The comparison is made by considering a single degree of freedom, say heave motion, without loss of generality.

A body of wetted surface $\boldsymbol{C}$, in equilibrium on the fluid at rest, is given at t=0 an impulsive motion defined by a step velocity. We shall work here on the heave motion: $\mathbf{V}_e = \mathbf{z}H(t)$. The resulting velocity potential is the solution of the Fredholm-Volterra integral equation :

$$\frac{\Phi(M,T)}{2} - \iint_C \Phi(M',T)\frac{\partial}{\partial n'}G_0(M',M)\ dC = -H(T)\iint_C G_0(M',M)n_z(M')\ dC$$
$$-\int_0^T dT' \iint_C F(M',T',M,0)n_z(M')dC + \int_0^T dT' \iint_C \Phi(M',T')\frac{\partial}{\partial n'}F(M',T',M,T)dC \qquad (1)$$

where the Green function is given by :

$$G(M',t',M,t) = -\frac{1}{4\pi}\left[\delta(t-t')G_0(M',M) + H(t-t')\mathcal{F}(M',t',M,t)\right] \ ; \quad G_0(M',M) = \left(\frac{1}{R} - \frac{1}{R_1}\right) \qquad (2\text{-a/b})$$

and : $$\mathcal{F}(M',t',M,t) = F(r,(z+z'),(t-t')) = 2\int_0^\infty J_0(Kr)e^{K(z+z')}\sqrt{K}\sin\sqrt{K}(t-t')dK \qquad (2\text{-c})$$

The term (2-b) is the impulsive part of the function, whereas (2-c) is generally referred to as the memory part.

Since the early eighties, several authors have proposed numerical algorithms for the solution of the above 3D problem, or some variant (Liapis 1986, King 1987, Kormeyer 1988, Magee 1991, Bingham 1994,...). In his program, Ferrant (1988) used a zeroth order direct BEM method. His computer code was used as a basis in the present study; it was adapted to take into account the new computational techniques for the Green function. Let us briefly summarize here the main features of the code. The body surface $\boldsymbol{C}$ is discretized into plane triangular or quadrilateral panels C_j. Sources of constant density $\sigma = \partial\Phi/\partial n$, and dipoles of constant density $\mu(=-\Phi)$ are distributed over the panels. The source strength σ_j on each panel is known through the body no-flux boundary condition. Denoting :

$$D_{ij} = \iint_{C_j} \frac{\partial}{\partial n_j}G_0(M_i,M_j)\ dC_j \qquad S_{ij} = \iint_{C_j} G_0(M_i,M_j)\ dC_j \qquad (3)$$

$$s_{ij}(T) = \iint_{C_j} F(M_j,0,M_i,T)dC_j \qquad d_{ij}(T) = \iint_{C_j} \frac{\partial}{\partial n_j}F(M_j,0,M_i,T)dC_j \qquad (4)$$

the continuous integral equation (1) degenerates into a discrete finite order algebraic system :

$$[D_{ij}][\mu_j(T)] = -H(T)[S_{ij}][\sigma_j(0)] - \int_0^T dT'[s_{ij}(T')][\sigma_j(0)] + \int_0^T dT'[d_{ij}(T-T')][\mu_j(T')] \qquad (5)$$

Coefficient matrices $[D_{ij}]$ and $[S_{ij}]$ (3) are computed by the classical Hess et Smith formula. Since they are time independent, they are evaluated once for all, and the first one is inverted at the onset by a gaussian procedure. The solution at each time step is then obtained by a simple matrix product after updating the RHS. It requires the computation of convolution products of the past solution with the Green function and its gradient [see (5)]. The matrices s_{ij} and d_{ij} must be evaluated at each time step, and stored in order to compute the kernel of the convolution integral from 0 to the current time t. This is the main burden in this kind of time-domain computations, and the most time consuming.

The original expression (2) is not well suited for numerical evaluation; then, in the eighties, several authors (Jami 1981, Liapis 1986, King 1987, Newman 1985-1992,...) developed numerical procedures based on: series or asymptotic expansions, Filon quadratures, recursive relations,... The first method to be implemented in our code was based on these formulas (King (1987)); it will be referred to as *"series expansions method"* in the sequel.

The second numerical method for the evaluation of the Green function memory term (2-c) is based on a bi-linear interpolation in a pre-computed table. (Ferrant 1988b, Magee & Beck 1989). This is made possible by the change of variable $\lambda = KR_1$ in (2-c) yielding:

$$F(r,z,t) = 2R_1^{-3/2}\int_0^\infty J_0(\lambda\sqrt{1-\mu^2})e^{-\lambda\mu}\sqrt{\lambda}\sin\left(\sqrt{\lambda}\tau\right)d\lambda \ , \qquad (6)$$

where the integral is a function of only two "natural variables": $\mu = -z/R_1$, $\tau = t/\sqrt{R_1}$. This second method, which was already implemented in the original code, will be denoted: *"tabulation method"* in the following.

At the last Workshop in Marseille (Clément 1997), we gave a third alternative way to compute the time-domain Green function and its gradient. It is based on a general lemma (Clément 1998) which establishes that these functions satisfy very simple fourth order Ordinary Differential Equation (ODE). It was shown for instance that $F(r,Z,t)$, is a solution of:

$$(r^2+Z^2)\frac{\partial^4 F}{\partial t^4} - Zt\frac{\partial^3 F}{\partial t^3} + \left(\frac{t^2}{4} - 4Z\right)\frac{\partial^2 F}{\partial t^2} + \frac{7t}{4}\frac{\partial F}{\partial t} + \frac{9}{4}F = 0 \qquad (7)$$

The spatial derivatives of F satisfy similar ODE, differing only by their numerical coefficients. Then, since the time grows monotonously in the time stepping methods considered, these functions may be computed "in-line" by integrating the ODE with respect to the time variable using standard algorithms like Runge-Kutta or predictor-corrector. In the present study, second and fourth order Runge-Kutta methods where tested. (see Fig.3).
The integrals over the panels (4) may then be computed by Gauss point quadratures rules. Both the one point and the four points rules were tested; it was found that, for a given final accuracy, the single point algorithm which requires a finer meshing, was relatively more efficient than the four points, due to a better approximation of the Rankine part of the solution. We therefore kept this one point integration rule in all the computations reported herein.
The impulsive hydrodynamic forces are finally computed as:

$$M_{ij}(T) = \iint_C \Phi_i(M',T)n_j(M')dC \qquad L_{ij}(T) = \iint_C \frac{\partial}{\partial T}\Phi_i(M',T)n_j(M')dC \qquad (8)$$

IRF of a heaving hemisphere.

As a test case to compare the three above mentioned methods, we began with the simple problem of the heaving hemisphere.

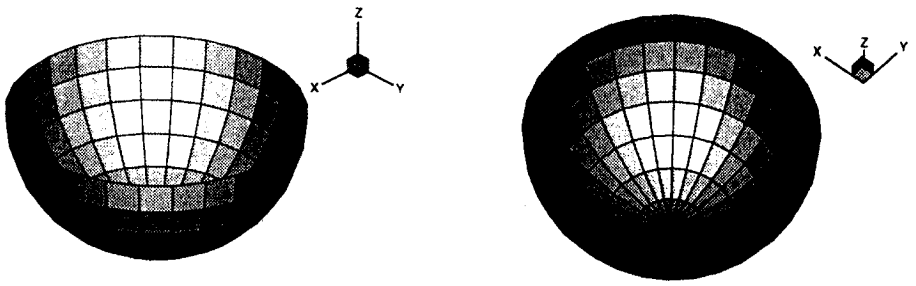

Fig 1: Floating hemisphere : a 4x49 panels mesh.

The responses functions $M_{33}(T)$ and $L_{33}(T)$ resulting from a step vertical velocity are computed. A typical result for 0<t<20 is plotted in figure 2. Convergence with the mesh size was investigated by varying systematically the number of panels. It was found that the mesh illustrated by Figure 1 above (*i.e* 4x7x7 panels) gave results converged to within 1% in the vicinity of t=1.475, where $L_{33}(T)$ reaches its maximum value.

The oscillations observed in the tail of the response, for t>6, are the time-domain counterpart of the well known *"irregular frequencies"*, and arise from the same origin. Since we were mainly interested here in *cpu* time statistics, we did not tried to suppress this phenomenon by the help of the usual dedicated methods. This point is left for a further study.

Let us now compare the *cpu* time required by the three methods of Green function evaluation (Fig.3). The numerical process of the convolution integral suggests a quadratic growth of the computing time. Such a behaviour was observed with both the tabulation and the ODE methods, whereas the series expansion method presents two different regimes. For t<18, we observe a quasi-linear growth of the cpu-time. In this area, the Green function is evaluated by series expansion or by Filon quadrature, according to the relative position of source and field points. These methods are far more time consuming than the asymptotic expansion which is used when $\tau > 14$. When all the couples of points satisfy this conditions, the program speeds up and a quadratic behaviour is recovered. The benefit of using the ODE method is clearly illustrated by Figure 3. One should notice that these curves correspond to quite long simulations. In the present case of the heaving hemisphere, a simulation up to t=20 should be sufficient from a practical point of view (see Fig.2). It would result in the computing time shown in the table below, when using a *DEC Alpha 500* workstation, at 330MHz.

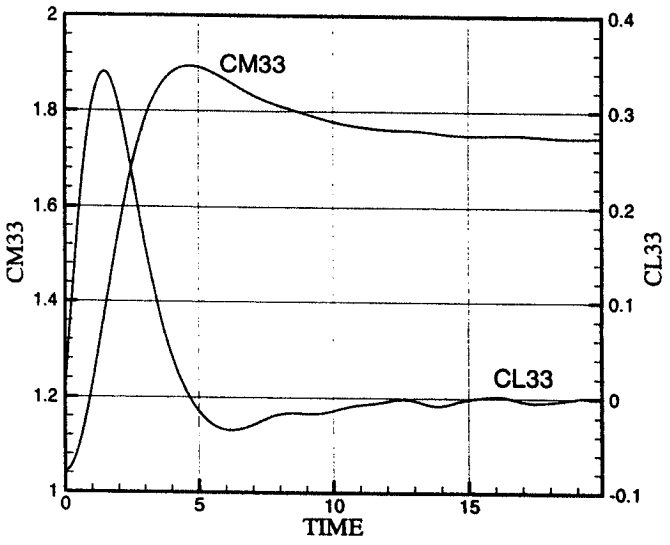

Fig.2: time-domain *"added mass"* $M_{33}(T)$, and impulse response function $L_{33}(T)$. dt=0.025, 4x49 panels.

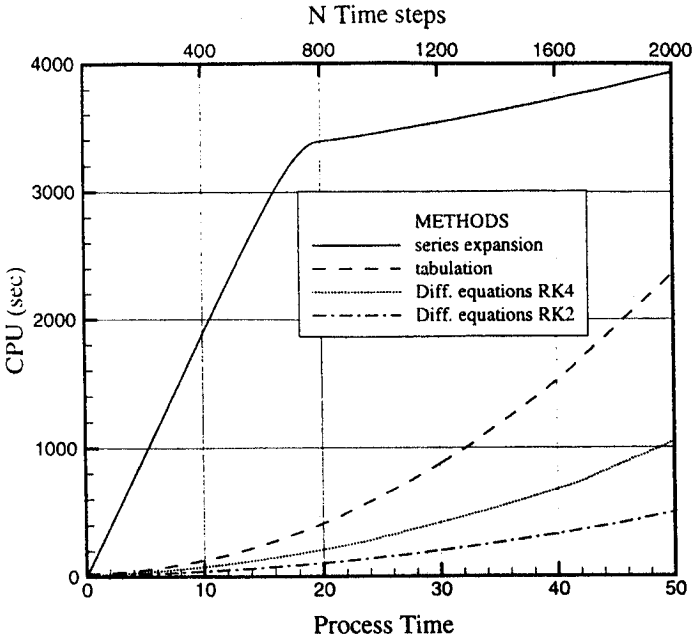

Fig.3 : Floating hemisphere. (4x49 panels): *cpu* time requirement

Method	*cpu*-time (sec.)	
series	3398	
tabulation	413	(≈1/8)
ODE RK4	211	(≈1/16)
ODE RK2	104	(≈1/33)

Further results for other different floating bodies, including the ISSC TLP platform, will be given at the conference.

This first application of the differential properties of the time-domain Green function is very encouraging, in a numerical point of view, and we recommend to use this powerful approach whenever a numerical evaluation of this function, or its spatial derivatives, is needed.

Other applications of the above mentioned Green function ODE, in the time-domain and in the frequency domain as well, are under consideration and will be presented in further communications.

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