On Minimizing Wave Resistance or Drag

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This paper treats the problem of determining the hull form for a ship of constant volume and fixed draft moving with constant Froude number which minimizes either wave resistance or total drag (ignoring spraying and wave breaking). We choose one of these quantities as a constraint and minimize the other over a set of admissible hull forms. We treat this constrained problem by simultaneously determining the hull form and the velocity potential for that particular hull form. Unilke the traditional approach of minimizing the Michell integral for the wave resistance (e.g. Chapter 6 of [3]), the present paper goes a step further in applying modern shape optimization techniques to the Kelvin–Neumann integral equation by finding an optimal solution for the total (wave plus viscous) drag. The procedure is similar to that employed in shape optimization for zero forward speed [1],[2].

Consider a ship with wetted surface S enclosing (together with the water plane) a constant volume V_0 moving with a constant forward speed U_0 in the x-direction and employ the standard linearized free surface boundary condition. We choose to represent the velocity potential of the wave problem as a center plane source distribution [4],

$$\phi(\mathbf{r}) = \int_{S_0} M(\mathbf{r}')G(\mathbf{r},\mathbf{r}')ds' + U_0 x \tag{1}$$

where S_0 is the center plane, a planar region contained in the projection of S on the (x, z) plane, $G(\mathbf{r},\mathbf{r}')$ is the Green's function for the Kelvin-Neumann problem in the absence of the ship and M is the unknown source distribution. M is a solution of the first kind integral equation

$$\frac{\partial}{\partial n}\int_{S_0} M(\mathbf{r}')G(\mathbf{r},\mathbf{r}')ds' = -U_0\,\hat{\mathbf{n}}\cdot\hat{\mathbf{x}}, \quad \mathbf{r} \text{ on } S \tag{2}$$

and $\hat{\mathbf{n}}$ is taken to be the outward normal (into the fluid) on S.

In terms of the Havelock function $H(\mathbf{r},\mathbf{r}')$

$$G(\mathbf{r},\mathbf{r}') = -\frac{1}{|\mathbf{r}-\mathbf{r}'|} - \frac{1}{|\mathbf{r}+\mathbf{r}'|} + H(\mathbf{r},\mathbf{r}') \tag{3}$$

where $\mathbf{r}_1' = (x', y', -z')$, the wave resistance can be expressed as

$$D_w = \int_{S_0}\int_{S_0} M(\mathbf{r})M(\mathbf{r}')\frac{\partial}{\partial x}H(\mathbf{r},\mathbf{r}')dsds' \tag{4}$$

and the viscous drag, for simplicity, is assumed to be proportional to the surface area, i.e.,

$$D_v = \int_S ds. \tag{5}$$

We confine attention to a set of admissible surfaces, $\mathcal{A}_{v_0,\lambda_0}$, with constant volume, symmetric about the center plane with a rectangular shape of *fixed* draft/waterplane length as follows:

$$S \in \mathcal{A}_{v_0,\lambda_0} \text{ if } S = S^+ \cup S^-$$

where

$$S^+ = \{\mathbf{r}|y = f(x,z) \geq 0,\ |x| \leq x_0,\ \int_{-x_0}^{x_0}\int_0^{z_0} f(x,z)dzdx = \frac{V_0}{2},$$
$$0 \leq z \leq z_0,\ f \in C^2(\mathrm{supp} f),\ \mathrm{supp} f \subset [-x_0, x_0] \times [0, z_0],\ \frac{z_0}{x_0} = \lambda_0\}$$

and

$$S^- = \{\mathbf{r} = (x, -y, z)\ |(x,y,z) \in S^+\}$$

On $S^\pm$, we have $\hat{n} = \frac{(-f_x, \pm 1, -f_z)}{\sqrt{1+f_x^2+f_y^2}}$.

Then we may define the defect in satisfying the integral equation as

$$\|\frac{\partial}{\partial n}\int_{S_0} M(\mathbf{r}')G(\mathbf{r},\mathbf{r}')ds' + U_0\hat{\mathbf{n}}\cdot\hat{\mathbf{x}}\|^2_{L^2(S_0)}. \tag{6}$$

However S is not known. This quantity may be expressed in terms of integrals over planar regions in the (x, z) plane as

$$J = \int_{-\alpha x_0}^{\alpha x_0}\int_0^{\alpha z_0} |U_0 f_x + \left(f_x\frac{\partial}{\partial x} - \frac{\partial}{\partial y} + f_x\frac{\partial}{\partial z}\right)\int_{S_0} M(\mathbf{r}')G(\mathbf{r},\mathbf{r}')ds'\Big|_{y=f(x,z)}|^2$$
$$\cdot\sqrt{1+f_x^2+f_z^2}dzdx \tag{7}$$

when $0 < \alpha < 1$ is a constant and x_0 and z_0 are respectively the half-length and draft. This functional involves both the unknown source distribution as well as the unknown surface.

Similarly the wave resistance has the form

$$D_w = \int_{-\alpha x_0}^{\alpha x_0}\int_0^{\alpha z_0}\int_{-\alpha x_0}^{\alpha x_0}\int_0^{\alpha z_0} M(x,z)M(x',z')\frac{\partial H}{\partial x}(x,z,x',z')dxdzdx'dz'. \tag{8}$$

while the viscous drag can be expressed approximately in terms of the unknown surface as

$$D_v = \int_{-x_0}^{x_0} \int_{0}^{z_0} \sqrt{1 + f_x^2 + f_z^2}\, dz dx \tag{9}$$

Introducing new variables $L\tilde{x} = x$, $L\tilde{z} = z$, and defining $\tilde{f}(\tilde{x}, \tilde{z}) := \frac{1}{L} f(x, z)$, the functionals J and D_w may be rewritten so as to entail integration over the fixed domain $[-\alpha, \alpha] \times [0, \lambda_0 \alpha]$:

$$J = \int_{-\alpha}^{\alpha} \int_{0}^{\alpha\lambda_0} |U_0 \tilde{f}_{\tilde{x}} + \left(\tilde{f}_{\tilde{x}} \frac{\partial}{\partial \tilde{x}} - \frac{\partial}{\partial \tilde{y}} + \tilde{f}_{\tilde{z}} \frac{\partial}{\partial \tilde{z}} \right) \int_{-\alpha}^{\alpha} \int_{0}^{\alpha\lambda_0} M(\tilde{x}, \tilde{y}) G(L\mathbf{r}, L\mathbf{r}') \Big|_{\tilde{y} = \tilde{f}(\tilde{x}, \tilde{z})} |^2$$
$$\cdot \sqrt{1 + \tilde{f}_{\tilde{x}}^2 + \tilde{f}_{\tilde{z}}^2}\, (L^2)\, d\tilde{z} d\tilde{x}, \tag{10}$$

and

$$D_w = L^2 \int_{-\alpha}^{\alpha} \int_{-\alpha}^{\alpha\lambda_0} \int_{0}^{\alpha\lambda_0} \tilde{M}(\tilde{x}, \tilde{z}) \tilde{M}(\tilde{x}', \tilde{z}') \frac{\partial}{\partial \tilde{x}} H(\tilde{x}, \tilde{z}, \tilde{x}', \tilde{z}') d\tilde{x} d\tilde{z} d\tilde{x}' d\tilde{z}' \tag{11}$$

while D_v has the form

$$D_v = L^2 \int_{-1}^{1} \int_{0}^{\lambda_0} \sqrt{1 + \tilde{f}_{\tilde{x}}^2 + \tilde{f}_{\tilde{z}}^2 d\tilde{z} d\tilde{x}}. \tag{12}$$

We choose to study, here, optimization problems which can be formulated in terms of these expressions for fixed λ_0,

(PI) minimize $D_v + V_0 J$

over the class U_{λ_0} subject to the constraint

$$D_w \leq K$$

where K is some preassigned constant, and

(PII) minimize $D_w + V_0 J$

over the class U_{λ_0} subject to the constraint

$$D_v \leq K.$$

The new concept of shape optimization may be found useful in ship design.

References

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