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STEADY SPLASHING FLOWS

by

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Summary

We consider steady two-dimensional free-surface flows involving a jet or splash which rises then falls under gravity. First we examine a stream which is uniform far upstream in a channel of finite constant depth that ends abruptly with a barrier in the form of a vertical or inclined wall, which first forces the flow upward, then either lets it continue in the same direction or bounce back. The resulting splash then falls forever as if into a bottomless chasm. The second configuration examined may be described as a very wide ship bow in water of infinite depth. Namely, we consider a flow past a semi-infinite flat-bottomed body of finite draft, terminated by a plane front face inclined at a prescribed angle. There is a submerged stagnation point on the front face, to which is attached a bifurcating streamline originating far upsteam, such that all of the fluid lying above that streamline is drawn into the splash, whereas all of the fluid below it passes beneath the body. In each case, depending on input parameters such as the angle and height of the front face or barrier, and the value of the far-upstream Froude number, the splash may fall either before or beyond the barrier. The problem is solved via an integral equation formulation, and results are presented in the form of graphs and video images.

Mathematical formulation

To derive the integral equations, we transform the physical plane $z = x + \mathrm{i}y$ into the plane of the complex potential $f(z)$ and then map it into a lower half plane of an artificial variable $\zeta = \xi + \mathrm{i}\eta$. The relation between f and ζ is given by $f = -\log\zeta$ for the channel flow and $f = \zeta - \log\zeta$ for the bow flow.

We introduce for both flows a positive constant $a < 1$ such that $\zeta = a$ is the image of the point where separation of the splash from the barrier occurs. The intervals $-\infty < \xi < 0$ and $0 < \xi < a$ are the transformed free streamlines and $\zeta = 0$ corresponds to the jet far downstream. The point $\zeta = 1$ corresponds to the corner of the barrier for the channel flow, and to the submerged stagnation point for the bow flow. For the latter flow we also introduce a constant $b > 1$ to mark the image $\zeta = b$ of the point where the front face of the bow meets the flat bottom surface of the ship.

The flow problem is solved with the logarithmic hodograph

$$\Omega = \tau(\zeta) - \mathrm{i}\theta(\zeta) = \log f'(z)$$

as a dependent variable. In the numerical solution we reduce the problem to an integral equation for the unknown $\theta(\xi)$ on the interval $(-\infty, a)$, that is, on the free surface, observing that θ is wholly known on the interval $\xi > a$, i.e. on the body.

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The free-surface boundary condition is constancy of pressure, or from Bernoulli's equation,

$$\frac{1}{2}e^{2\tau} + gy = \frac{1}{2}, \tag{1}$$

where it is assumed that the far-upstream free-surface level is $y = 0$, and that the upstream uniform flow is of unit magnitude. Representing τ and y in terms of θ in equation (1), we turn it into an integral equation for $\theta(\xi)$. To do so, we note first that $\tau(\xi)$ is the Hilbert transform of $\theta(\xi)$. The expression for the $y-$coordinate is found from

$$\frac{dy}{d\xi} = \frac{df}{d\xi}e^{-\tau}\sin\theta, \tag{2}$$

valid for real $\zeta = \xi$, where $df/d\xi = -\xi^{-1}$ for the flow in the channel and $df/d\xi = (1 - \xi^{-1})$ for the flow past the ship. In both cases, equation (2) must be integrated separately in two intervals $\xi \in (-\infty, 0)$ and $\xi \in (0, a)$, as the singularity representing the ultimate fate of the jet does not permit integration through the origin $\zeta = 0$. The initial condition is $y(-\infty) = 0$ for the integration in the interval of negative ξ. Detachment at the junction point between body and free surface is in the present paper assumed to be smooth, with continuous slope and finite non-zero velocity. The appropriate initial condition at $\zeta = a$ for use in the interval $(0, a)$ must be found by integrating (2) from a to 1 for the flow in the channel and from a to b for the flow past the ship.

Results

The integral equations so obtained can be solved numerically by reduction to a finite system of nonlinear equations as in Tuck & Goh (1985) or Tuck (1987). An alternative approach based on a semicircular parametrization and series truncation was used in Vanden-Broeck & Keller (1987), Dias & Tuck (1991), Dias & Christodoulides (1991) and Dias & Vanden-Broeck (1993). For the channel flow, Dias and Christodoulides (1991) solved the limiting problem for a very high vertical barrier where the detachment is via a stagnation point, producing a splash falling before the barrier. Dias and Tuck (1991) reduced the height of the vertical barrier and allowed smooth detachment from its top edge as in the present work, causing the splash to fall beyond the barrier. Vanden-Broeck and Dias (1993) solved a similar problem for a flow past a high vertical ship bow, again with stagnant detachment. Wiryanto and Tuck (1996) recomputed Dias and Christodoulides' solution using the integral equation method, obtaining somewhat more accurate results.

Figures 1 and 2 are typical for the channel flow, showing two possible types of interaction of the stream with the barrier. A backward-diverted jet is observed when the barrier's angle to the horizontal is less than $\pi/2$, but not too small, and the barrier is reasonably high; otherwise the jet falls beyond the barrier. For high walls, the present program fails just before the height of the topmost point of the free surface reaches the stagnation level, with some indication that this point is moving toward the junction point, as in the solutions of Dias and Christodoulides (1991) and Wiryanto and Tuck (1996). In those cases where the jet falls beyond the barrier, the height of the topmost point of the upper free surface increases as we decrease the angle or increase the height of the wall. The program then fails just before this point reaches stagnation level, with some indication that there would as usual be a 120° angle at that point if the limiting solution could be reached.

Figures 3 and 4 exhibit computed flows past a ship bow and we can observe the same tendencies here as with the channel flow. Again there is a domain of parameter values where numerical solution was not achieved, and it may be that no solution exists with the assumed topology. For instance, by changing the slope of the front face, we move from a flow with a backward-diverted splash "into the stream", to a splash falling forward "into the ship", but are confronted in between by an interval of angles at which the algorithm does not converge. This interval may contain a sub-interval in which the splash rises to a stagnation level, where it divides into two streams, one backward and one forward in a fountain-like manner, as in Vanden-Broeck (1993), but we have not yet computed such flows.

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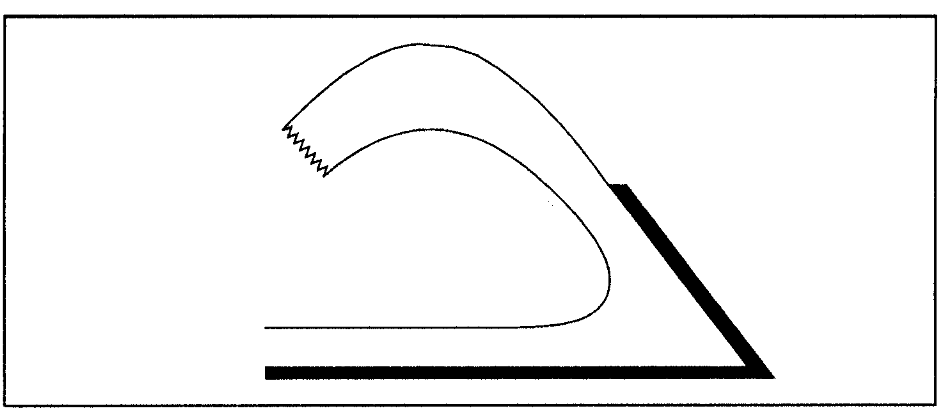

Figure 1:

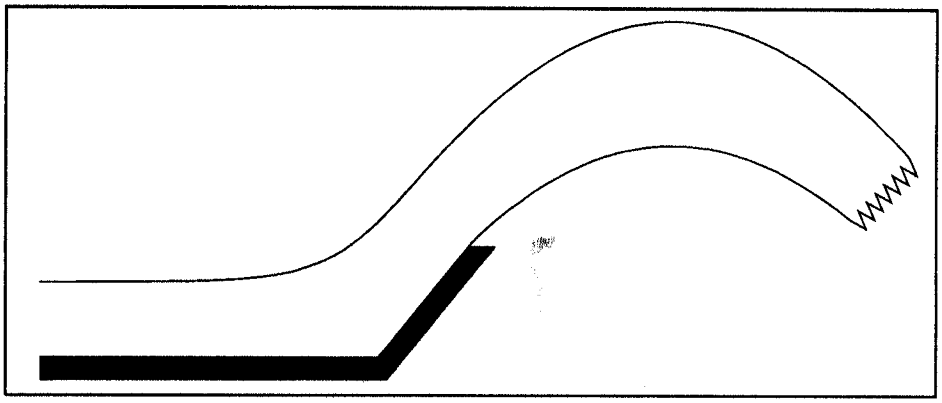

Figure 2:

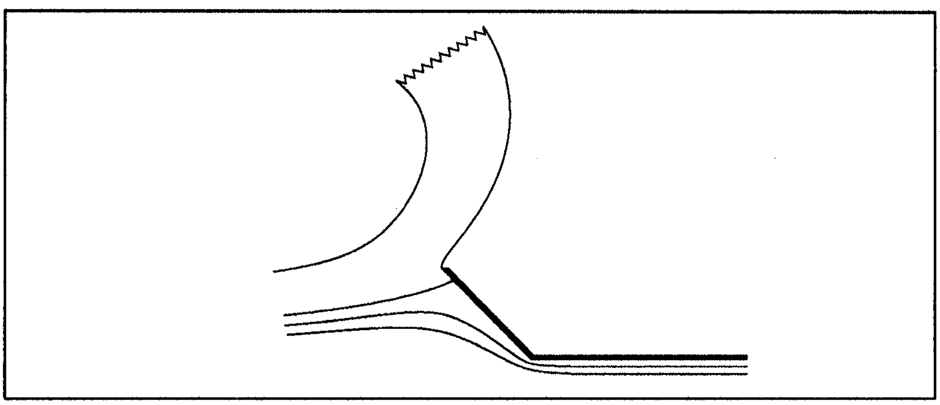

Figure 3:

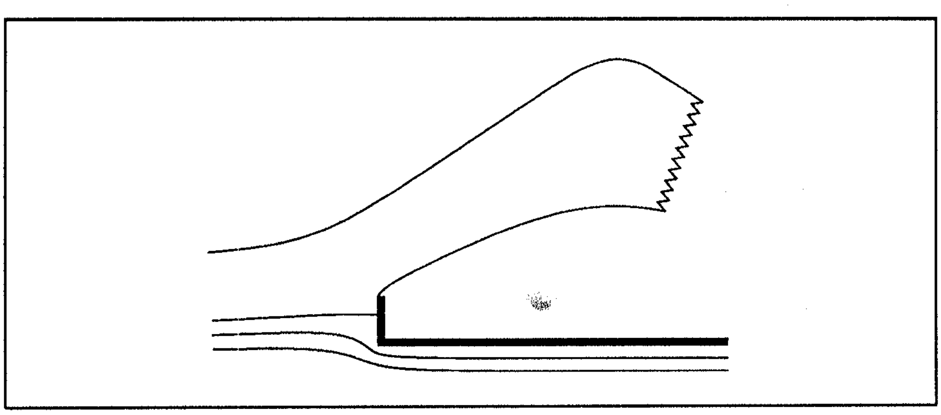

Figure 4:

DISCUSSION

Grilli S.: Did you calculate dynamic pressures induced by jets on solid boundaries?

Tuck E., Simakov S., Wiryanto L.: Not yet. We hope to do this later.