Two-Dimensional Inviscid Transom Stern Flow

S. M. Scorpio and R. F. Beck
University of Michigan, USA

Introduction

Two-dimensional fully nonlinear transom stern flow is investigated using the Desingularized Euler - Lagrange Time-domain Approach or DELTA method. Mixed Euler-Lagrange time stepping is due to Longuet-Higgins and Cokelet (1976). The field equation is solved using the desingularized boundary integral method described in Beck et al. (1994). The flow is unsteady in that the problem is started from rest and accelerated up to steady forward speed. The purpose of this study is to compare with previous steady calculations and to provide a starting point for extending to unsteady fully nonlinear three-dimensional transom stern flows.

The cases studied herein correspond to those in Vanden-Broeck and Tuck (1977) and Vanden-Broeck (1980). They compute nonlinear waves behind a transom stern using a series expansion in the Froude number. The problem is solved in an inverse manner in which the coordinates x and y are the dependent variables and the velocity potential and stream function ϕ and ψ are the independent variables. The series expansions in x and y are everywhere divergent but can be summed by standard methods. Integro-differential equations with nonlinear boundary conditions are solved in the inverse space to obtain the expansion coefficients.

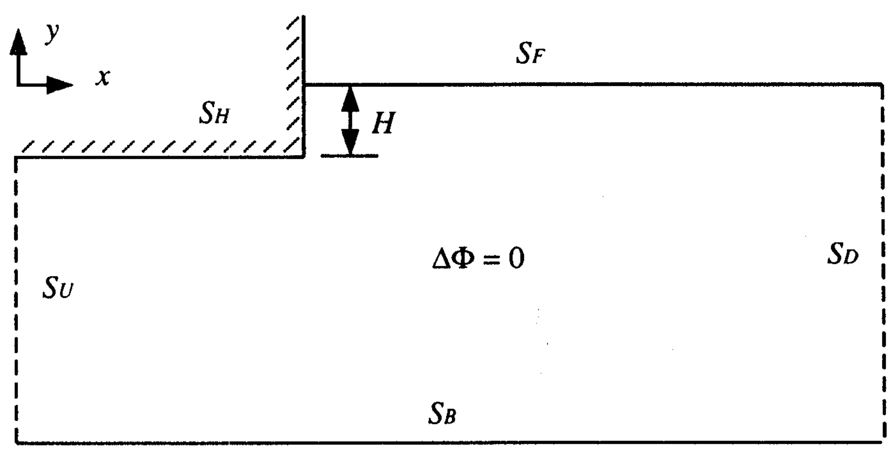

Figure 1: Problem configuration

Problem Formulation

Figure 1 shows the problem configuration. The $x - y$ coordinate system is translating with speed U_b in the negative x direction. Laplace's equation governs in the fluid domain and the velocity potential is $\Phi = U_b x + \phi$. The surfaces which bound the fluid are: S_F = Free Surface; S_H = Body Surface; S_U = Upstream Truncation Surface; S_D = Downstream Truncation Surface; S_B = Bottom Surface.

The boundary conditions are:

$$\left.\begin{aligned} \frac{D\eta}{Dt} &= \frac{\partial\phi}{\partial z} \\ \frac{D\phi}{Dt} &= -g\eta + \tfrac{1}{2}|\nabla\phi|^2 \end{aligned}\right\} \qquad \vec{x} \in S_F$$

$$\frac{\partial\phi}{\partial n} = -U_b n_1 \qquad \vec{x} \in S_H$$

$$\nabla\phi \to 0 \quad \text{as} \quad y \to -\infty \qquad \vec{x} \in S_B$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + \nabla\Phi \cdot \nabla$ is the material or Lagrangian derivative, $\vec{n} = (n_1, n_2, n_3)$ is the unit normal on the body pointing out of the fluid, g is the acceleration of gravity, η is the free surface elevation, and ϕ is the perturbation potential. The boundaries S_U and S_D are unspecified. We have run cases with S_U and S_D prescribed to satisfy continuity and saw very little difference in the results as long as S_U and S_D are far enough up and downstream respectively. We placed the trunction boundaries about twelve wavelengths away from the transom for these calculations.

Results

Vanden-Broeck (1980) suggested that two realistic solutions exist for the steady flow behind a transom stern. For small values of the Froude number, the flow rises up the transom to a stagnation point. The free surface separates from the transom at the stagnation point creating waves downstream which increase in steepness with increasing Froude number. We'll call this solution A. This solution is physically unreasonable for large values of Froude number because the ratio of stagnation height to transom depth goes to infinity as the Froude number goes to infinity. For large Froude numbers a second, more physically realizable solution exists in which the flow separates cleanly from the bottom of the transom. We'll call this solution B. This solution reduces to the uniform stream as Froude number tends to infinity and the downstream waves steepen as Froude number becomes small. In fact, Vanden-Broeck (1980) found a minimum Froude number ($= 2.26$) below which the downstream waves would exceed the theoretical breaking wave steepness limit ($2A/\lambda = 0.141$).

The problem is started from rest and the hull is accelerated up to steady forward speed. Using the DELTA method, the inviscid solution *always tends towards configuration A* as the hull reaches steady forward speed, regardless of the Froude number. In a viscous fluid, we know that the flow behaves like solution B for high Froude numbers. As the hull speed increases from rest, the flow separating from the bottom of the transom becomes turbulent, resulting in the "dead water" region commonly observed behind transom sterns. Consequently the pressure behind the transom is lowered. Eventually the falling pressure causes the free surface to drop to the bottom of the transom resulting in the solution B flow. Once the flow is separating cleanly from the transom, the turbulence is confined to the thin boundary layer (for high speeds) and viscous wake. Using an inviscid flow model, it appears to be impossible to proceed from transom wetted to transom dry. However, we did find two techniques which resulted in solution B.

The first was to start the problem at steady forward speed with the transom out of the water. The hull is then lowered slowly into the water. As the hull is lowered, the free surface remains separated from the bottom of the transom and solution B results. This technique will not work for a problem starting from rest with the transom immersed. In order to obtain solution B for the problem starting from rest we tried a second technique in which we attempt to mimic the effect of the dead water region by artificially lowering the stagnation pressure on the transom. This pressure drop can be modeled in the inviscid flow code by modifying the boundary condition on the transom. The condition,

$$\frac{\partial \phi}{\partial n} = -U_b n_1$$

causes the stagnation pressure. We reduce the stagnation pressure by modifying the transom boundary condition to:

$$\frac{\partial \phi}{\partial n} = -U_b n_1 \left(2e^{-\beta t^2} - 1\right)$$

As the hull accelerates up to speed, the pressure on the transom drops until the free surface drops down to the bottom of the transom. When the hull reaches steady speed, solution B is recovered.

The general numerical details are similar to those outlined in Beck et al. (1994). There is a double node where the free surface meets the transom in the solution A flow. One node satisfies the body boundary condition while the other satisfies the free surface boundary condition. Treating the intersection in this manner has consistently worked well in the desingularized method. There is one additional constraint (or Kutta condition) at the bottom of the transom in the solution B flow. The free surface nodes are allowed to move downstream with the fluid velocity during the intermediate time steps (we're using 4^{th} order Runge-Kutta). At the end of a major time step the free surface nodes are regrided back to their original positions by interpolating elevations and potentials. The Kutta condition is imposed by regriding the first free surface node back to the bottom of the transom. The potential at this point is computed from the source strengths.

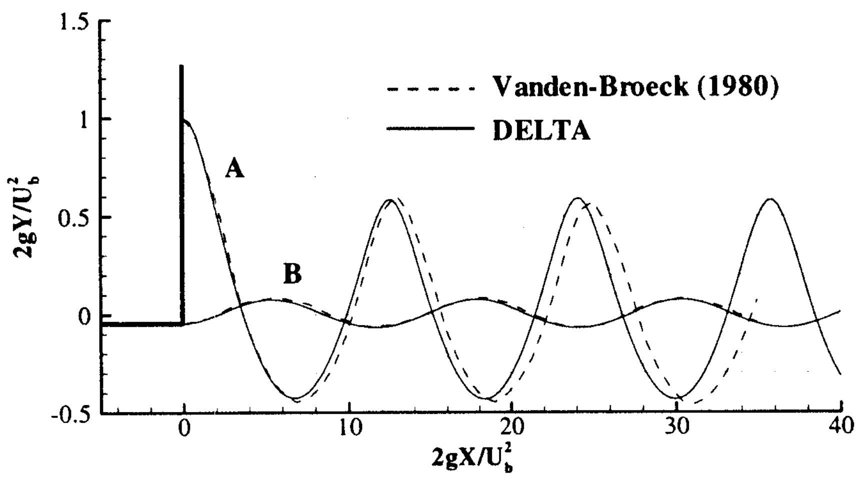

Figure 2: Solutions A and B at $F_H = 6.3$

Figure 2 shows the waves generated by the transom stern at Froude number based on transom depth of $F_H = U_b/\sqrt{gH} = 6.3$. The fully nonlinear solution starting from rest is compared with Vanden-Broeck's 1980 results which are also fully nonlinear. Both steady state solutions A and B are compared. The solutions agree quite well except there is a noticeable difference in wavelength for solution A.

	a^*	λ^*	$2a^*/\lambda^*$	λ_0^*	λ_4^*
Vanden-Broeck	0.53	12.1	0.088	12.6	11.5
DELTA	0.51	11.6	0.088	12.6	11.6

Table 1: Comparison of downstream wave characteristics for solution A

Table 1 shows downstream wave characteristics for the Vanden-Broeck (1980) and DELTA solution A. Here, $a^* = a2g/U_b^2$ is the nondimensional wave amplitude found by subtracting the minimum wave elevation from the maximum and dividing by two for the downstream waves and $\lambda^* = \lambda 2g/U_b^2$ is the nondimensional wavelength. Since the phase speed of the waves equals U_b, we can use the deep water dispersion relation to estimate the wavelength. The linear wavelength is $\lambda_0^* = \lambda_0 2g/U_b^2 = 4\pi = 12.6$. Using the 5^{th} order dispersion relation for deep water Stokes waves ($U_b^2 = g/k(1 + (ka)^2 + 5/4(ka)^4)$) and the computed wave amplitude (a) we can solve for the wave number (k) and get an estimate for the nonlinear wavelength ($\lambda_4^* = \lambda_4 2g/U_b^2$). Although both computations show waves with the same steepness, Vanden-Broeck's waves do not satisfy 5^{th} order dispersion.

Conclusions

For two-dimensional transom stern flow, the transition from transom wetted to transom dry at high Froude number is accomplished in the inviscid flow model by modifying the transom boundary condition. Perhaps a more appropriate transom boundary condition could be contrived which allows solution A for low Froude numbers and transitions appropriately to solution B as the Froude number increases through the critical value ($F_H = 2.26$). Presumably this technique may be applied to the unsteady three-dimensional problem. Of course flow behind a three-dimensional transom is much more complex and requires further study.

Acknowledgments

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References

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DISCUSSION

Tuck E.O.: Can you explain why the particular form of the body boundary condition was chosen, with the property that the RHS exactly changes sign as t goes from 0 to ∞. As $t \to \infty$, since no part of this boundary is wet, it surely doesn't matter what the limiting boundary condition is.

Scorpio S., Beck R.: The boundary conditions was: $\frac{\partial \phi}{\partial n} = -U_b n_1 \left(2e^{-\beta t^2} - 1\right)$.

At $t = 0$, $\frac{\partial \phi}{\partial n} = -U_b n_1$, as $t \to \infty$, $\frac{\partial \phi}{\partial n} \to +U_b n_1$.

Initially we tried $\frac{\partial \phi}{\partial n} = -U_b n_1 \, e^{-\beta t^2}$ but $\frac{\partial \phi}{\partial n} \to 0$ as $t \to \infty$ was not strong enough to suck the free surface down to the bottom of the transom. $\frac{\partial \phi}{\partial n}$ had to change sign in order to generate the necessary drop in pressure. Professor Tuck is absolutely correct in that there is no significance in $\frac{\partial \phi}{\partial n} \to +U_b n_1$ as $t \to \infty$.

In fact, the form of the boundary condition was arbirarily chosen to provide a smooth transition from transom wetted to dry. Surely there are many choices of boundary condition which would produce the same result.

Yeung R.W.: In a recent work (Yeung, 1991, Math. Approaches to Hydrodynamics, SIAM Publ.) a number of "time-dependent" solutions were worked out in the context of "solution A". There was basic agreement with Van-den-Broeck's results. In the same article (see also Yeung & Ananthakrishnan, 1992, 19th ONR Symposium), a solution with viscosity is given to illustrate how an entrained vortex is first formed at the "sharp stern", leading eventually to its "sheering off". Presumably, at sufficiently large time, the drop in water level in the stern will approach the keel point. These references may serve to explain what is happening at the stern physically.

Scorpio S., Beck R.: We would like to thank Professor Yeung for his comments and for the references that we are sure will be most helpful. The mechanisms that cause the transom stern flow to proceed from wetted to dry as the ship accelerates from rest are very interesting. We think there is a good qualitative understanding of this process already. Perhaps some careful physical experiments, or numerical experiments as cited by Professor Yeung, can give us a better quantitative understanding of this process.