VIOLENT SURFACE MOTION AROUND VERTICAL CYLINDERS IN LARGE, STEEP WAVES - IS IT THE RESULT OF THE STEP CHANGE IN RELATIVE ACCELERATION?

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Abstract
Violent surface motions are seen around fixed vertical cylinders in large, steep waves. They are also predicted, by the small-time expansion method, when a vertical cylinder moving horizontally in still water undergoes a step change in its horizontal acceleration. But a sharp (classically 120 degree) crest, which is the defining feature of steep waves, necessarily implies a step change in horizontal particle acceleration in the incident wave (since surface slope = $a_x/(g + a_z)$ where a_x and a_z are horizontal and vertical components of particle acceleration).This observation suggests that the violent surface motion is *the result of a step change in the relative cylinder acceleration.*

The leading-order small-time expansion result on the moving cylinder is derived and discussed below. Despite the violence of the surface motion, it is noteworthy that the hydrodynamic force per unit length on the cylinder actually *reduces* near the surface. In terms of slender body theory, the total effect of the free surface is of a higher order in slenderness than the other forces acting.

1. Background
There is considerable current interest in the oil industry in the violent motion of the water surface produced around a vertical cylinder by a wave which is relatively large (height/diameter = 2, say) and also very steep (height/length = 0.1, say). Figures 1, 2 and 3 illustrate the effect at realistic scales. Its importance is not only in the potentially damaging effect on the superstructures of offshore oil platforms, but also in the "ringing" vibration produced by the associated sudden load fluctuations. From a theoretical point of view, the interest in the phenomenon is that it is highly non-linear (the vibration can be at ten times the wave frequency), and that Stokes expansion methods widely employed (see e.g. Malenica & Molin, 1995) for calculating wave loads are probably irrelevant, see Chaplin, Rainey & Yemm, 1997.

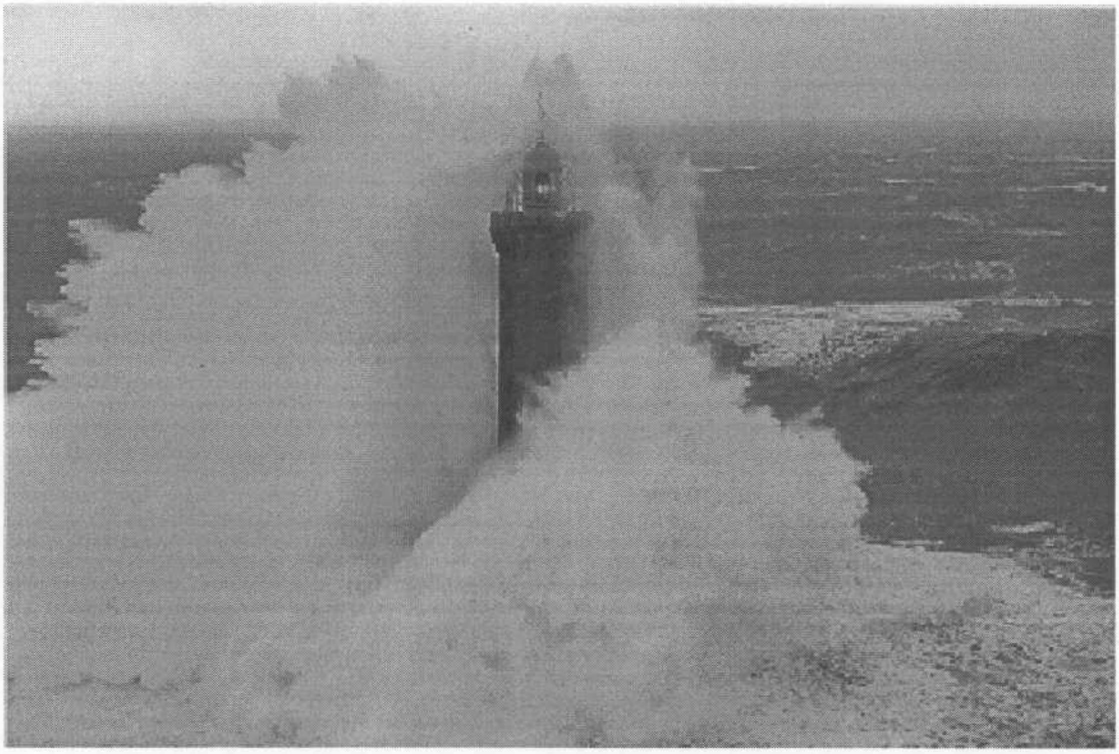

Figure 1. Large wave at the AR-MEN lighthouse, Brittany; diameter approximately 8m

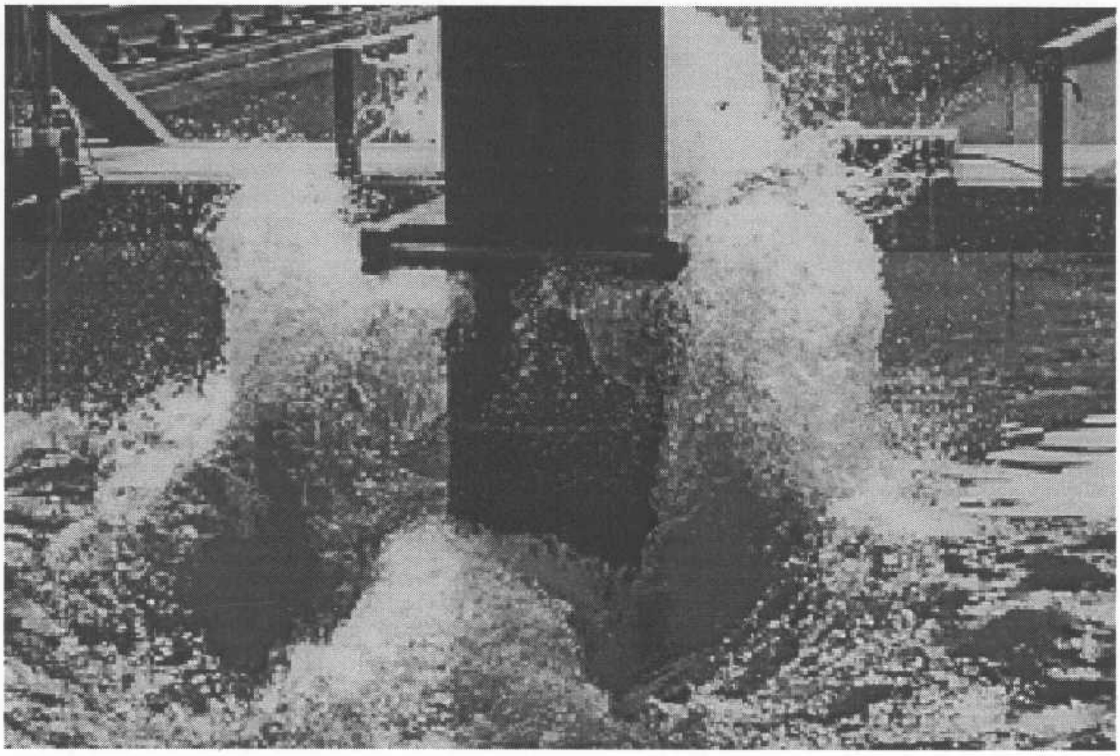

Figure 2. Focused wave at the large flume at De Voorst; cylinder diameter 0.5m

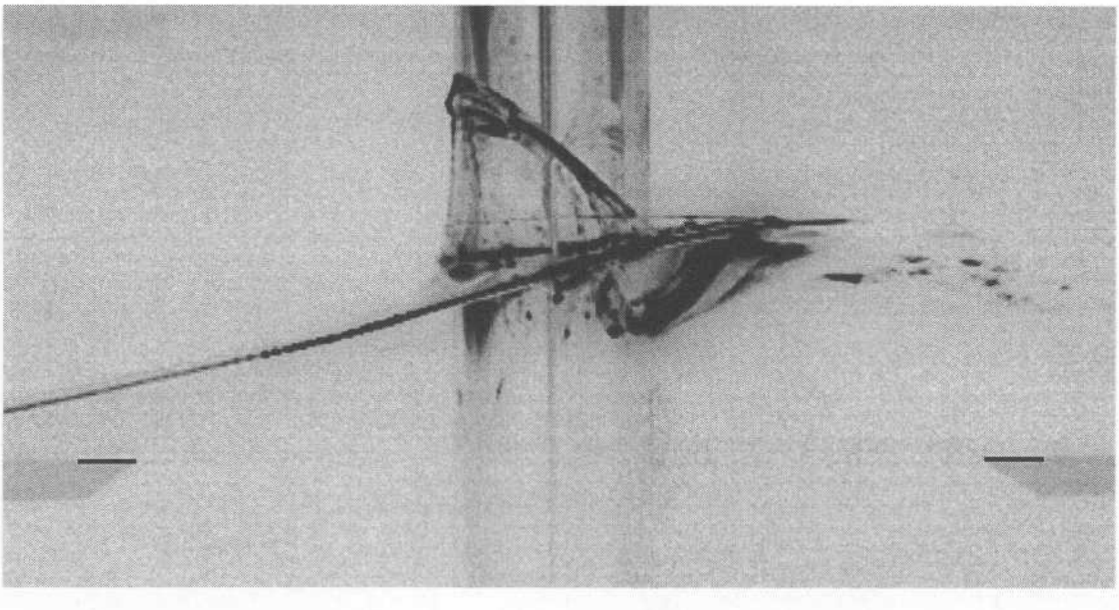

Figure 3. Focused wave passing a cylinder of diameter 0.1m in recent experiments on "ringing" (after Chaplin, Rainey & Yemm, 1997)

2. The small-time expansion solution for a vertical cylinder moving horizontally from rest in still water
In the well-known small-time expansion scheme, which has recently been applied to this problem by Wang and Chwang (1989), the velocity potential φ and surface elevation η are expanded as power series thus:

$$\phi = \phi_1 t + \phi_2 t^2 + \phi_3 t^3 + \ldots \qquad \eta = \eta_1 t + \eta_2 t^2 + \eta_3 t^3 + \ldots \qquad (1)$$

By Taylor series expansion of ϕ about the still-water position ($z = 0$), the free-surface boundary conditions can then be applied at $z = 0$, as a series of conditions of successively higher order in the time t. The process is closely analogous to Stokes' expansion, with t replacing the Stokes expansion parameter. The leading-order boundary conditions are:

$$\phi_1 = 0 \qquad\qquad \eta_1 = 0, \quad 2\eta_2 = \partial\phi_1 / \partial z \tag{2}$$

This potential-flow problem can be easily solved, for finite depth in the case of Wang and Chwang, and for infinite depth here. We simply observe (see e.g. Bland (1961) p.108-9) that there are solutions of the form:

$$\phi = \phi_1 t = at[kK_1'(kb)]^{-1} K_1(kr)\cos\theta \sin kz \tag{3}$$

where the cylinder velocity is at (i.e. a step change in acceleration from zero to a), b is its radius, and r, θ, z are the usual cylindrical co-ordinates, with $\theta = 0$ being the direction of motion and z being positive upwards. Rather than the required normal velocity on the cylinder surface of $at\cos\theta$, these solutions have a velocity of $at\cos\theta\sin kz$; they must therefore be combined by means of the Fourier sine transform:

$$\frac{2}{\pi}\int_0^\infty \frac{\sin kz}{k} dk = \mathrm{sgn}(z) \tag{4}$$

to give the required solution as:

$$\phi_1 = \frac{-2}{\pi}\int_0^\infty \frac{aK_1(kr)\cos\theta \sin kz}{k^2 K_1'(kb)} dk \tag{5}$$

so that the leading-order expression for the free surface elevation is:

$$\eta = \frac{-1}{\pi}\int_0^\infty \frac{at^2 K_1(kr)\cos\theta}{kK_1'(kb)} dk \tag{6}$$

This is readily plotted with MATHCAD and is shown (out to a distance $10b$ from the axis) in Figure 4 below.

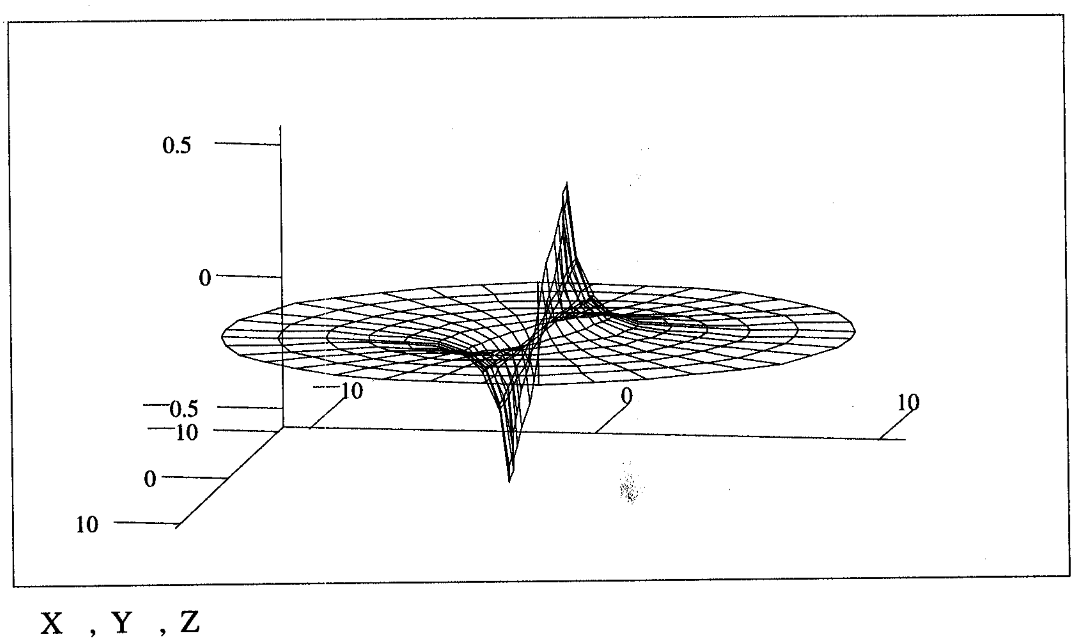

Figure 4. Free surface around a vertical cylinder accelerating from rest in still water

For present purposes the striking feature is the singularity at the cylinder surface, reminiscent of the flows in Figures 1-3. This feature has long been recognised in earthquake engineering, as occurring next to dams (Westergaard 1933) and cylindrical piers (Jacobsen 1949) undergoing impulsive motions. In this literature the surface boundary conditions (2) appear to be taken as axiomatic, without any formal justification by a small-time expansion. Indeed, the dam or pier motion is taken as a continuous vibration, from which the impulsive motion is later synthesised.

Perhaps no less interesting is the force per unit length on the cylinder, which is readily obtained to leading order, by integrating the transient pressure $-\rho\phi_t = -\rho\phi_1$ from (5), around the cylinder, to give:

$$2\rho b\int_0^\infty \frac{aK_1(kb)\sin kz}{k^2 K_1'(kb)}dk \tag{7}$$

in the sense opposing the acceleration *a* of the cylinder, i.e. as an "added mass force". This expression can also readily be evaluated with MATHCAD, as shown (from $z = -10b$ to $z = 0$) in Figure 5 below.

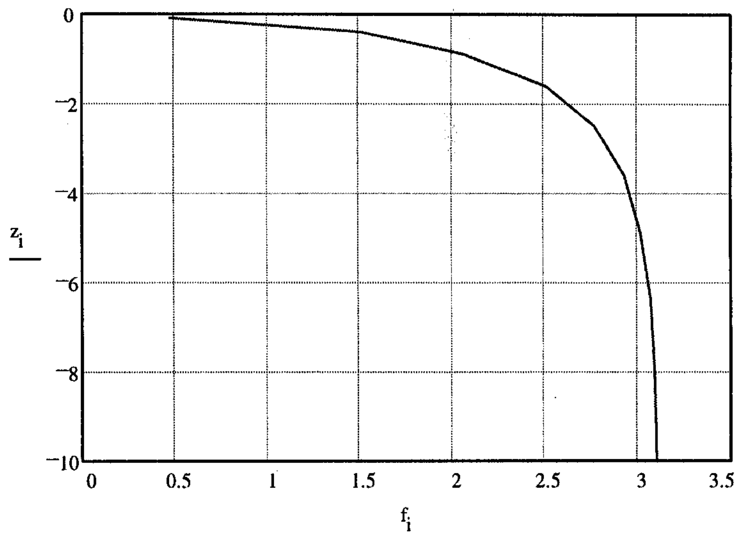

Figure 5. Variation of force per unit length *f* (normalised w.r.t. $\rho b^2 a$) with depth *z*

As expected, it converges to $\pi\rho b^2 a$ at large depth (i.e. the 2-D "Morison" result), but tends to *zero* as the surface is approached. Thus the violent surface motion of Figure 4 is associated, somewhat surprisingly, with a *reduction* of the hydrodynamic load, compared with the 2-D result. The difference in the total load can be obtained by integrating the above curve with depth - according to MATHCAD the result is about $\pi\rho b^3 a$. In terms of slender-body theory, the dependence on b^3 makes the effect of higher order in slenderness than other "end effects". There is thus no contradiction with the "end effect" discussed by the author at the 10th Workshop (Rainey, 1995) - that was of lower order in slenderness, but evidently only important when the flow remains smooth (i.e. the Stokes' expansion case of small wave steepness), as indeed suggested then.

This work was supported by EPSRC through MTD Ltd (grant GR/J23198), and jointly funded by the managed programme on uncertainties in loads on offshore structures.

References

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DISCUSSION

Schultz W.: The small time expansion is non-uniformly valid near the contact line (Joo, Schultz, Messitor 1991?; King & Needham 1995). A local expansion no longer has the $\phi_0 = 0$ free surface condition and modifies the *ln tanh* singularity. It is likely that this modification might have negligible effect on a slamming force, based on your results. Any comment?

Rainey R.C.T.: Indeed. In 1994, in the *Journal of Fluid Mechanics*, Vol 268 pp 89-101, King and Needham treat the 2-D "wavemaker" problem rigorously, and I understand from Howell Peregrine that their methods ought to be applicable in the present case of a vertical cylinder. Such a rigorous treatment might show that the surface motion remains violent, and the slamming force is little changed, as you suggest. If you, or they, were to investigate this matter further, I would be delighted!

Grilli S.: Two dimensional computations using fully nonlinear potential flow equations have shown that very large upward vertical accelerations can be created for certain types of so-called flip-through wave impacts on vertical structures. I would think that a similar phenomenon could occur for vertical cylinders, assuming some sort of wave focusing ensuring quasi-2D conditions. This hence could explain the violent upward motions you observed. We of course have to wait for the 3D codes to be efficient enough to get a more definite answer. Can you comment on this?

Rainey R.C.T.: I assume you mean the type of "flip-through" impact described in the paper by Peregrine and Cooker at the 1990 WWWFB in Manchester, and subsequently e.g. in *Coastal Engineering (1992)*. This is a very interesting suggestion, but actually I am not aware of it having been seen in model tests on cylinders. The phenomenon I am describing occurs well before wave breaking, and there is no vertical wall of water approaching the cylinder and "flipping through".