Finite Element Analysis of Non-linear Transient Waves in a Three Dimensional Long Tank

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Numerical simulation of propagating waves in a tank has been mainly based on the boundary element method (BEM), see for example Cointe[1], Contento *et al* [2], Dold & Peregrine[3].Grilli *et al* [4] and Lin *et al*[5]. Although BEM has many known advantages, its storage and CPU requirement increase at a rate proportional to the square of the number of nodes. Because of this, most analyses based on BEM provided results only for a short tank. Wang *at al* [6] combined a multi-subdomain approach with BEM and dealt with the problem of two dimensional waves in a long tank. It appears, however, that the effectiveness of this approach in three dimensions is less certain.

The finite element method (FEM) has recently been used in the nonlinear transient water wave problem [7,8]. It has been observed that FEM has several advantages for this problem. In particular, its matrix is banded and its influence coefficients can be obtained from the volumes of the elements when the linear shape function is used. It has been noticed [7,8] that FEM normally requires far less CPU and memory than BEM. However when the computational domain increases, these requirements of FEM are still excessive as reported by the authors [8], who adopted domain decomposition to reduce the memory requirement. Since that work, the authors have invested considerable efforts to improve the methodology and the CFD code. Changes are made mainly in two areas. Firstly we have replaced the Gaussian elimination method for solving the matrix equation with an iterative method. In the former method, all the coefficients within the band width have to be stored even if they are zero. In the latter method, however, only those non-zero terms have to be kept. The number of non-zero terms is usually less than one tenth of the band width. Thus the iterative method requires far less memory. Furthermore, zero operations have been eliminated in the iterative method, which makes the computation far more efficient. Also when the iteration is used in the time domain, the initial solution can be taken from that at the last step. All these have improved the efficiency of the computation significantly.

The second change we have made is based on the fact that the wave created by the wavemaker propagates towards the far end gradually. This means that the computational domain can be divided into disturbed and undisturbed domains as shown in Figure 1. In the undisturbed region, the free surface is considered as unchanged and as a result the coefficients corresponding to the nodes in this region can be kept constant.

When these changes were made, the CPU and memory requirement were significantly reduced. We calculated a case with about 203,520 elements and 1,000 time steps in the last ONR conference [8]. It took about 193 hours CPU time. The same job now takes about 10 hours on the same machine.

For the cases considered below, various parameters have been nondimensionalised as follows

$$(x,y,z) \rightarrow d(x,y,z) \qquad f \rightarrow \left(\rho g R_0^2 d\right) f$$
$$t \rightarrow (d/g)^{1/2} t \qquad \omega \rightarrow (g/d)^{1/2} \omega$$

where d is the water depth, ρ is the density, g is the acceleration due to gravity, ω is the frequency, R_0 is the radius of the cylinder and f is the force acting on the cylinder

The first case considered is the transient wave generated by a wavemaker. The length of the tank is $L = 100d$. The motion of the wave maker is governed by

$$U(t) = a\omega \sin(\omega t)$$

with the frequency $\omega = 1.45$ and the amplitude a=0.016. The total number of elements used in the 3D model is 2,127,943 and the calculation is over 14,400 steps ($\Delta t = 0.021666$). A numerical beach is applied at the far end. The total CPU for this case is about 147 hours on a DEC ALPHA 255^{233}. Figure 2 gives the wave history at the centre point of the tank ($x=L/2$). It can be seen that the calculated wave remains steady over a long period of time, which means that the reflection has no significant effect on the result yet. Figure 3 provides the wave profiles at two different time steps. The solid line corresponds to t=59T and the dashed line to t=72T (where T is the period of the wavemaker). They coincide with each other very well. However, as can be seen, the amplitude becomes smaller and smaller away from the wavemaker. This may be due to numerical dissipation but it requires further investigation.

The second case considered is a vertical cylinder in a wave tank with length $L = 40.5d$ while the water depth and the motion of the wavemaker are the same as above. The cylinder is placed at the centre of the tank and its diameter is $.05d$. 649,728 elements are used and the calculation is made over 10,000 time steps. The total CPU is 103 hours. Figure 4 gives the horizontal force on the cylinder.

Figure 5 shows the nonlinear effects on the force acting on the cylinder. All parameters are the same as those in figure 4 except the length of the tank has now been taken as 16.5d. The results correspond to three different motion amplitudes of the wavemaker: $a = 0.004, 0.016, 0.032$. The nondimensionalised forces defined above have been here divided by the nondimensionalised amplitude. The figure shows that when the amplitude is 0.032, the non-linear effects on the force become evident. Further results and discussions will be given in the workshop.

Acknowledgements

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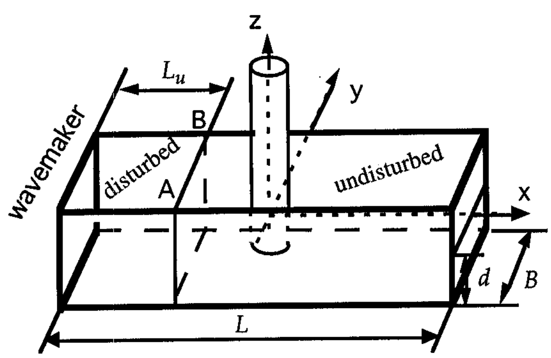

Figure 1 The division of disturbed and undisturbed regions

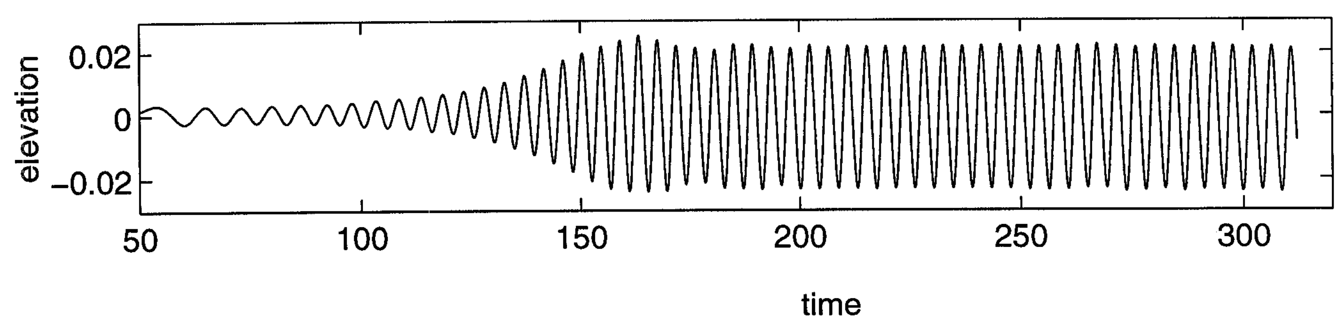

Figure 2. The wave history at the middle point of the tank

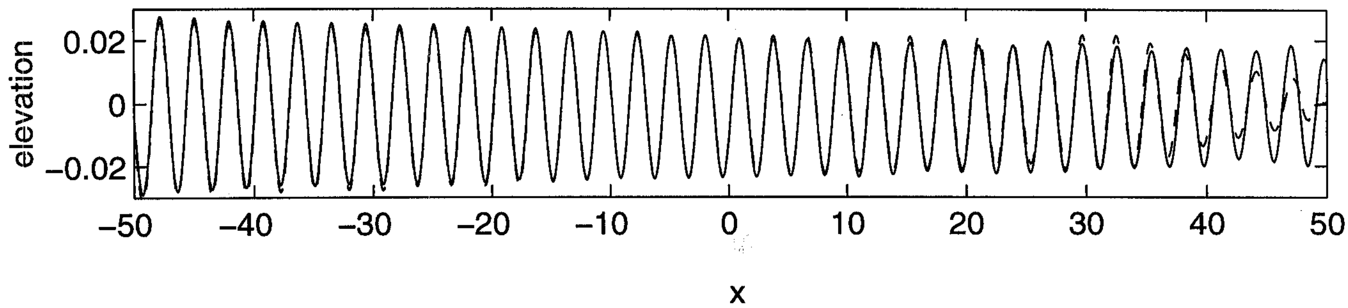

Figure 3 The wave profiles in the longitudinal plane y=0 at two particular time steps
(dashed line: t=59T ; Solid line: t=72T)

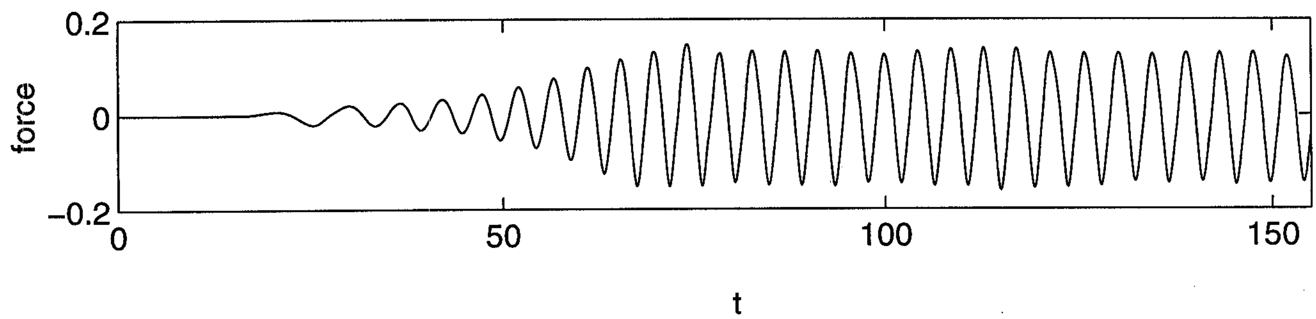

Figure 4 The force history acting on a cylinder in a tank with length=40.5d

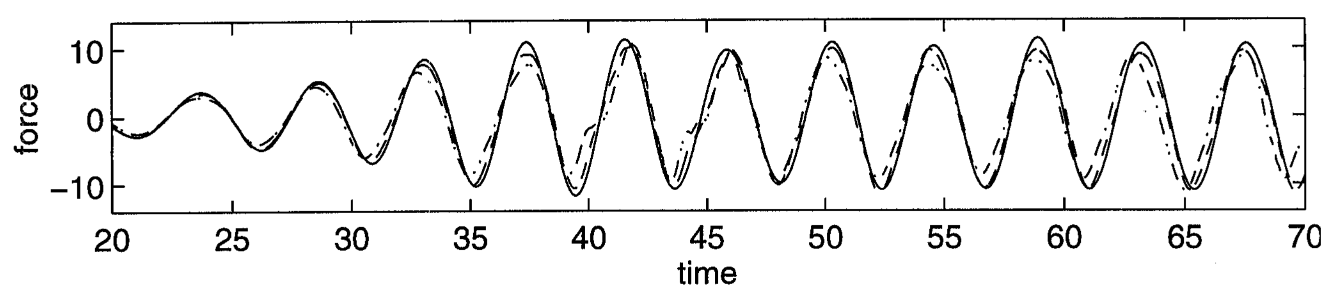

Figure 5 The force history on a cylinder at different motion amplitudes

of the wavemaker(———— a=.004; — — — a=.016; —·—·— a=.032)

DISCUSSION

Molin B.: Aren't you concerned by wall effects in your numerical tank, which seems to be rather narrow?

Ma Q.W., Wu G.X., Eatock Taylor R.: Yes, we would be concerned if these results were intended to simulate the open sea. The wall effects can be reduced by using a wider tank or by applying appropriate lateral beaches (the beach at the far end has been implemented in our work). Our future work will be investigating simulations of the open sea. At this stage we are more concerned by the other challenges of long time simulations. Our current set up is of course similar to what is done in many physical experiments.

Schultz W.: You indicate that regridding is quite dissipative. Have you tried various interpolating schemes during this process?

Ma Q.W., Wu G.X., Eatock Taylor R.: Yes, we have also tried higher order interpolation but the improvement did not meet our expectations, particularly in the case of steep waves. Reducing the amount of remeshing does reduce the dissipation but it may cause numerical instability. Further investigation is needed to find the optimum between remeshing and dissipation.

Grilli S.: I am surprised by your preliminary observation that FEM solutions are more efficient than BEM solutions for a Laplace's equation in 3D. It is my experience and there are results in the literature showing just the opposite, i.e., when comparing similarly accurate solution of a benchmark problem, using two similarly optimized FEM and BEM codes, the BEM method is always faster, sometimes by up to an order of magnitude, than the FEM method. And I am not even talking about multipole expansions that may be used in BEM.
Hence I think you might not have used a properly optimized BEM code in your comparisons (In particular, just dividing the domain in sub-regions with artificial matching boundaries may speed up BEM solutions by a factor equal to the number of sub-regions). Another advantage of the BEM vs FEM is that it is exact inside the domain, due to Green's identity. Can you comment on this?

Ma Q.W., Wu G.X., Eatock Taylor R.: We made our conclusion based on our own experience with BEM and FEM. We made clear that our BEM is not optimised and there might be better ways of programming the BEM code. To the best of our knowledge, however, it has yet to be shown that BEM can achieve the same efficiency as we achieved here. The only work we have seen, which used the multi-subdomain approach, is by Wang, Yao and Tulin (1995) for the 2D long wave tank problem. When the domain decomposition is optimised, the total number of coefficients is about $6M(2N+1)$, where M and N are total numbers of nodes on the free surface and the wave maker, respectively. This is comparable with the 2D version of our FEM, which has about $7MN$ coefficients. There are, however, several important points here:

(1) In our FEM code, the total number of coefficients is always linearly proportional to the total number of nodes, no matter how complicated the fluid domain is. In BEM, it very much depends on the shape of the domain. For example, it will be interesting to see how optimisation can be applied to a square shape or indeed to an arbitrary shape. Similarly, the efficiency of this technique is also uncertain for an arbitrary 3D domain, even for a 3D square wave tank.

(2) Even when the number of coefficients of BEM and FEM becomes comparable, the calculation of FEM coefficients is far more efficient. This is particularly important when millions of nodes are used.

(3) This work is part of our research effort in CFD. We are currently also undertaking research into viscous flows with a free surface. BEM is not applicable in this case.