Fully nonlinear properties of shoaling periodic waves calculated in a numerical wave tank

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In this work, nonlinear properties of finite amplitude shoaling periodic waves are calculated over mildly sloping bottom topographies, using a *numerical wave tank* which combines :

(i) a Boundary Element Model (BEM) solving Fully Nonlinear Potential Flow (FNPF) equations in a domain of arbitrary shape Ω (i.e., in a so-called physical space), up to and including wave overturning (Grilli *et al.*, 1989; Grilli and Subramanya, 1996; Fig. 1);

(ii) a generation of *zero-mass-flux Streamfunction Waves* at the deep water extremity of the tank, Γ_{r1} (i.e., exact periodic wave solutions of FNPF equations superimposed to a mean current equal and opposite to the wave mass transport velocity; Grilli and Horrillo, 1996a); and

(iii) an *Absorbing Beach* (AB) at the far end of the tank, which features both free surface absorption (through applying an external pressure; Cointe, 1990) and active absorption at the tank extremity, Γ_{r2} (using a piston-like condition; Clement, 1996). The beach depth is gradually increased to induce wave de-shoaling and a feedback mechanism adaptively calibrates the absorption coefficient, as a function of time, for the beach to absorb the period-averaged energy of incident waves, computed at the AB entrance, $x = x_l$ (Grilli and Horrillo, 1996a).

Incident waves of various heights H_o and periods T are modeled (covering the range k_oH_o = [0.028 ,0.105]), first over plane slopes s (1:35, 1:50, and 1:70; Fig. 1) and then over "natural beaches" of similar mean slope; in all cases both the AB location and characteristics are adjusted for the waves to shoal up to very close to their breaking point (BP). Due to the low reflection from such mild slopes and from the AB, a quasi-steady state is soon reached in the tank for which both *local and integral properties* of shoaling waves are calculated as a function of depth $h(x)$. These are the shoaling coefficient $K_s = H/H_o$, the phase velocity c, the wave relative height H/h (i.e. a measure of nonlinearity in classical shallow water wave models), the wave steepness $kH = 2\pi H/L$, the mean water level η_m, the radiation stress S_{xx}, the mean Eulerian current U_m, and the energy flux E_f.

For a shallow enough normalized depth ($k_oh < 0.5$ or $kh < 0.77$), significant differences are observed between FNPF results and 1st (LWT), 3rd (CWT), and higher-order steady wave (FSWT; Sobey and Bando, 1991) theories (Fig. 2). For the first two theories, low-order nonlinearity is clearly the main reason for the observed differences in a region where $H/h = \mathcal{O}(1)$; with the latter theory, the lack of skewness in the wave shape and the representation of the bottom by horizontal steps likely explain the observed differences. Despite the significant influence of actual bottom shape on the results, however, for the range of tested mild slopes, FNPF results are found to be

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fairly similar for the same wave taken at the same normalized depth ($k_o h$ or kh). This is also found true for a mildly sloping bottom with geometry corresponding to a "natural beach" and average slope 1:50. [This "natural beach" has a depth variation defined according to Dean's equilibrium beach profile, $h = A(x^* - x)^{2/3}$, with x^* denoting a constant, function of the location of the toe of the slope in depth h_o, and A depending on the specified average beach slope.] This, hence, allows us to use kh as the *unique parameter* describing a mild bottom variation and to compute additional results on a unique mild slope (1:50). Among these results, when taking all tested waves simultaneously, the normalized wave steepness kH/k_oH_o shows an almost one-to-one relationship with kh in the shoaling region (Fig. 3). Quite surprisingly, due to a partial compensation of nonlinear effects for the wave height and celerity, LWT is found to be quite a good predictor of this parameter (maximum difference is 11%), whereas discrepancies for H and c reach 55 and 85%, respectively.

For the tested waves, the wave set-down (Fig. 4a) is quite well predicted by the first-order perturbation of LWT, except in the shallower region, where it is smaller, following the steep drop in radiation stresses (Fig. 4b). [This could also partly be due to the mean undertow current. More work remains to be done about this.]. Radiation stresses are overpredicted by the first-order theory in the region where wave left/right asymmetry (i.e., skewness) becomes large, confirming the sensitivity of this parameter to wave shape. Otherwise, agreement with the theory is quite good. A Fourier analysis of surface profiles shows, as expected, a continuous transfer of energy from the fundamental to higher-order harmonics in the shoaling region (Fig. 4c); this illustrates nonlinear interactions in the shoaling wave field. The 3rd-harmonic amplitude a_3 is found to be strongly correlated with wave asymmetry/skewness.

More results will be presented at the workshop, including some for barred beaches. Further discussions can also be found in Grilli and Horrillo (1996b).

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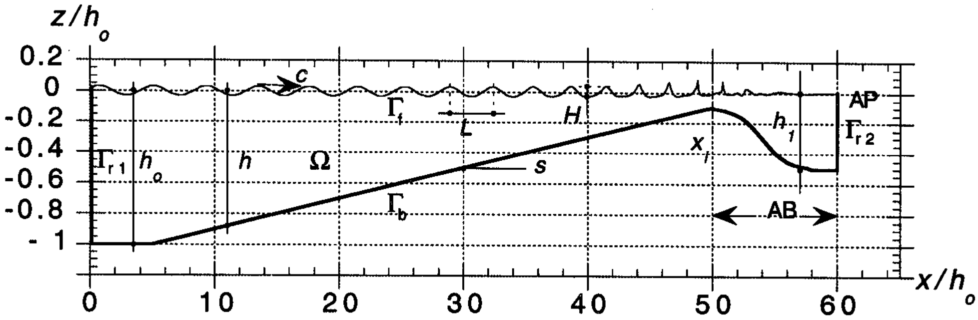

Figure 1: Sketch of "numerical wave tank" for FNPF computations of periodic waves shoaling over a plane slope s.

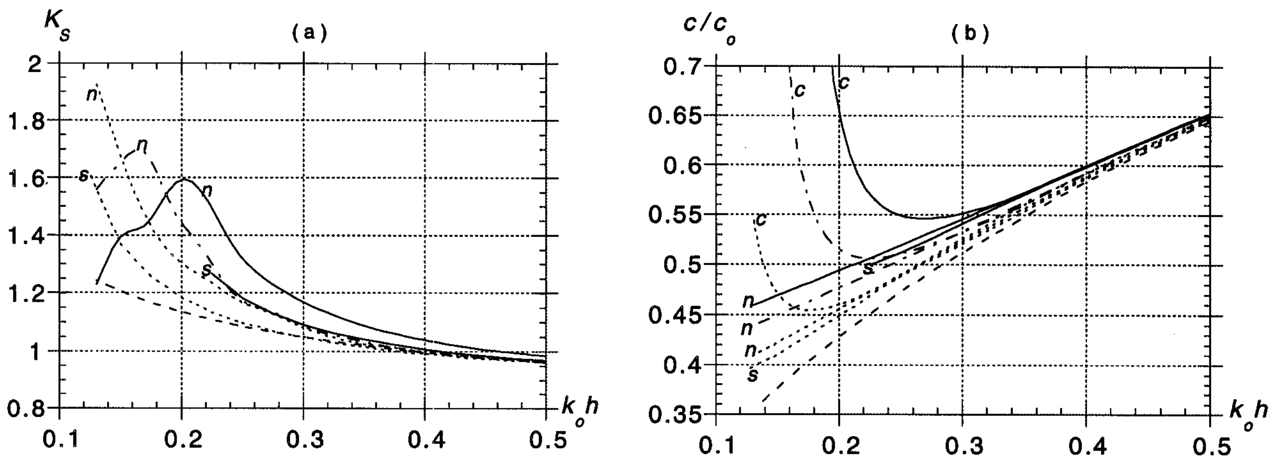

Figure 2: (a) shoaling coefficient $K_s = H/H_o$; and (b) celerity c, for periodic waves shoaling over a 1:50 plane slope, with $H'_o = H_o/h_o$ = (- - - -) 0.04, (— - —) 0.06, and (——) 0.08, and $T' = T\sqrt{g/h_o} = 5.5$: (n) FNPF results; (s) Sobey and Bando's (1991) FSWT results; (– – –) LWT results; (c) CWT results. $c_o = gT/(2\pi)$ is the (linear) deep water celerity.

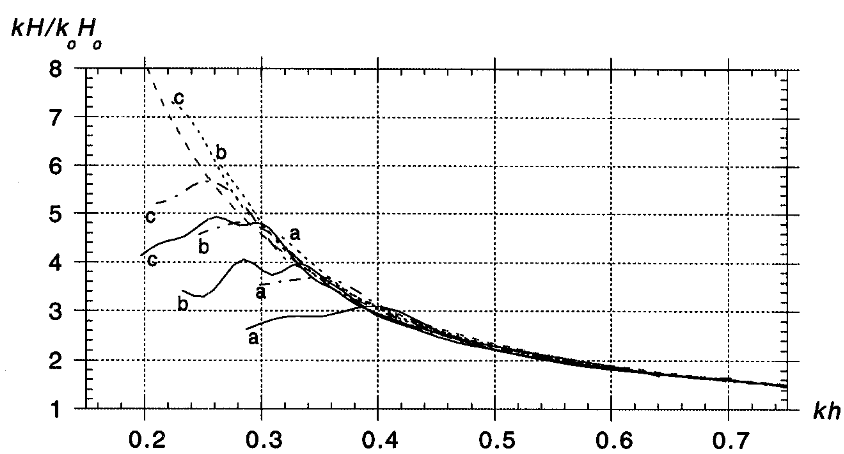

Figure 3: Normalized wave steepness kH/k_oH_o for periodic waves shoaling over a 1:50 slope. H'_o = (- - - -) 0.04, (— - —) 0.06, and (——) 0.08, and T' = : 5.5 (curves a); 6.5 (curves b); 7.5 (curves c). (– – –) LWT results.

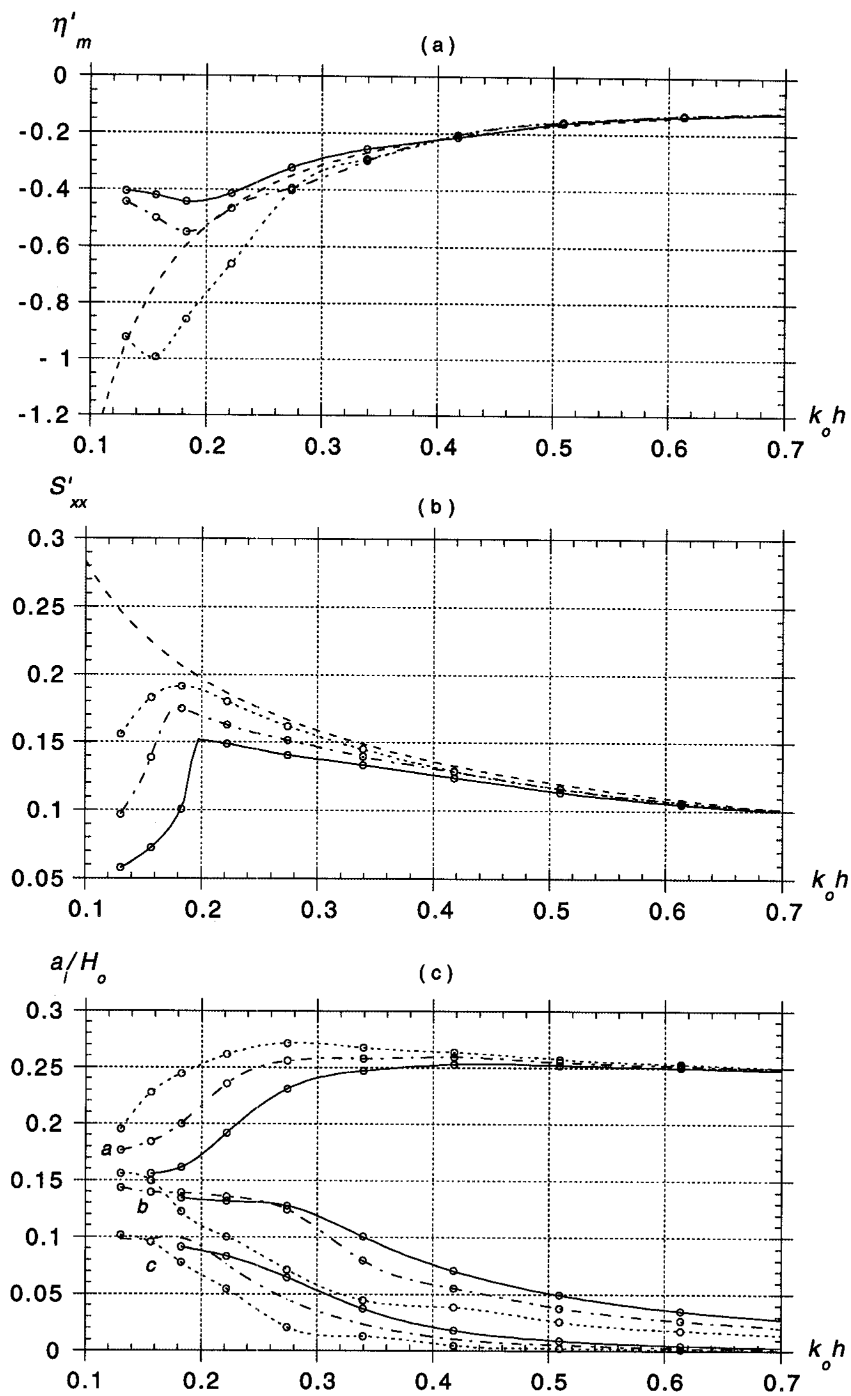

Figure 4: Normalized (a) mean water level $\eta'_m = \eta_m / h_o H_o'^2$; (b) radiation stress $S'_{xx} = S_{xx}/\rho g H_o^2$ (with ρ the fluid density); and (c) first three harmonics amplitudes (a,b,c $\equiv a_i$, i = 1,2,3), for three periodic waves shoaling over a 1:50 slope. Symbols and definitions are as in Fig. 2. Results have been averaged over $3T$ in the quasi-steady regime. Symbols (o) denote locations of "numerical gages". Corrections, $\Delta\eta'_{mo} = -0.0274$ and $\Delta S'_{xxo} = h'_o(\Delta\eta'_{mo}) + (\Delta\eta'_{mo})^2/2$, have been applied to the linear results for η'_m and S'_{xx}, respectively, to account for the actual mean water level in depth h_o in the FNPF results.