NONLINEAR WAVE-BODY INTERACTIONS IN A NUMERICAL WAVE TANK

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1 Introduction

The time-domain nonlinear free-surface waves and wave-body interactions are investigated in a 3-dimensional numerical wave tank using an Indirect Boundary Integral Method (IBIM). Simple Rankine sources are used outside the solution domain to desingularize boundary integrals (Cao et al., 1991). To update the position of the fluid particles on the free surface, fully-nonlinear free-surface boundary conditions are integrated with respect to time using the Eulerian-Lagrangian time marching technique. A regridding algorithm is used to eliminate the possible instabilities in the region of high gradients without using artificial smoothing. The input waves entering from the up-stream boundary are generated by either a piston-type wave maker or by prescribing actual wave data or analytic solutions. The energy of outgoing waves are gradually removed in the artificial damping zone by viscous dissipation rather than by being transmitted out of the solution domain. When simulating open-sea conditions instead of a numerical wave tank, the artificial damping zone (absorbing beach) is employed at all side walls to prevent possible contamination due to wall reflection. Unlike Cao *et al.* (1991), side walls and ϕ_n-type damping zone are used in the present numerical wavetank. The developed computer program was verified through mass and energy conservations and comparisons with experiments as well as analytic first- and second-order diffraction solutions.

2 Mixed boundary value problem

The ideal fluid is assumed so that a velocity potential exists and the fluid velocity is given by its gradient. The value of the potential, at each time step, is given on the free surface (Dirichlet boundary condition) and the value of the normal derivative of the potential (Neumann boundary condition) is known on the body surface and the bottom surface. The free-surface potentials and elevations are determined by integrating the following nonlinear free-surface boundary conditions with respect to time.

$$\frac{\delta\eta}{\delta t} = \frac{\partial\phi}{\partial z} - (\nabla\phi - \mathrm{v}).\nabla\eta - U_o(t)\frac{\partial\eta}{\partial x} \qquad \text{on } S_F \qquad (1)$$

$$\frac{\delta\phi}{\delta t} = -g\eta - 1/2\nabla\phi.\nabla\phi + \mathrm{v}.\nabla\phi - \frac{P_a}{\rho} - U_o(t)\frac{\partial\phi}{\partial x} \qquad \text{on } S_F \quad (2)$$

where

$$\frac{\delta}{\delta t} \equiv \frac{\partial}{\partial t} + \mathrm{v}.\nabla$$

is the time derivative following the moving node, U_o is forward velocity, and $\mathrm{v} = -U_o i + \nabla\phi$ dictates the material node approach. $\nabla\phi$ on the right-hand side can be determined after solving the boundary value problem for ϕ. Using the material node approach, $\nabla\eta$ term drops in eq.(1). A Lagrangian-Eulerian method, in which a mixed BVP is solved at each time step, is used on the free surface to time step the unknown potentials and wave elevations. A Runge-Kutta-Fehlberg method is employed for this purpose. Indirect boundary integral methods utilize the source density $\sigma(\vec{r}_s)$ which is used to determine the unknown velocity potential. Then, a weighted residual method (collocation method) is used to solve the integral equations for the unknown $\sigma(\vec{r}_s)$. In order to determine the unknown source strengths, an efficient iterative method called Generalized Minimal Residual (GMRES) Technique (Saad and Schultz, 1986) is used.

For an accurate free-surface flow computation, mass/volume, momentum, and energy conservations should be satisfied in the computational domain. For instance, the total energy conservation in a wave tank, following Contento and Casole (1995), can be expressed as

$$\varepsilon(t) = \dot{W}_W(t) + \dot{E}_o(t) - \dot{W}_B(t) - \dot{E}_\Omega(t) \qquad (3)$$

where $\dot{W}_W(t)$ is the power delivered by the wavemaker and given by

$$\dot{W}_W(t) = \int_{\partial W} p\frac{\partial\phi}{\partial n}dS$$

$\dot{E}_o(t)$ is the rate of energy flux through the open boundary and given by

$$\dot{E}_o(t) = \int\limits_{\partial W_O} \frac{\partial \phi}{\partial t}\frac{\partial \phi}{\partial n} dS \tag{4}$$

$\dot{W}_B(t)$ is the rate of work done by fluid on the body and given by

$$\dot{W}_B(t) = -\int\limits_{\partial B} p\frac{\partial \phi}{\partial n} dS \tag{5}$$

$\dot{E}_\Omega(t)$ is the rate of energy in the fluid and defined by a potential and kinetic contribution

$$\begin{aligned} \dot{E}_\Omega(t) &= \dot{E}_{\Omega_{POT}}(t) + \dot{E}_{\Omega_{KIN}}(t) \\ &= \frac{d}{dt}[\frac{1}{2}\rho g \int\limits_{\partial W \cup \partial F \cup \partial B} \eta^2 \, \mathrm{d}S - \frac{1}{2}\rho \int\limits_{\partial\Omega} \phi\frac{\partial \phi}{\partial n} \, \mathrm{d}S] \end{aligned} \tag{6}$$

where Ω is the boundary of 3-D solution domain. Then, $\varepsilon(t)$ can be compared with the amplitude of the power delivered by the wavemaker to obtain the absolute error in the solution domain.

3 Numerical results

First, the developed computer program was verified through mass, momentum, and energy conservation. The performance of artificial damping zone was tested for various wave conditions. As can be seen in Figure 1, the ϕ_n-type beach is more effective for shorter waves. Second, we conducted two fully-nonlinear diffraction computations with bottom-mounted and truncated uniform vertical cylinders. The simulation results are compared with Mercier & Niedzwecki*'s* (1994) experiments and Kim & Yue's (1989) second-order diffraction computation. The comparison with Mercier & Niedzwecki (1994) showed that the present fully-nonlinear computation agreed better with experiments than the second-order diffraction computation, as can be seen in Figure 2.

4 References

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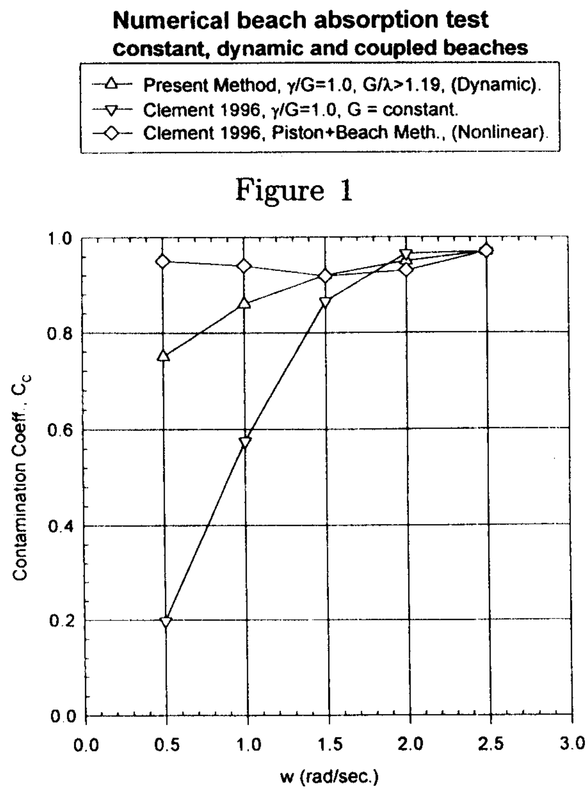

Figure 1

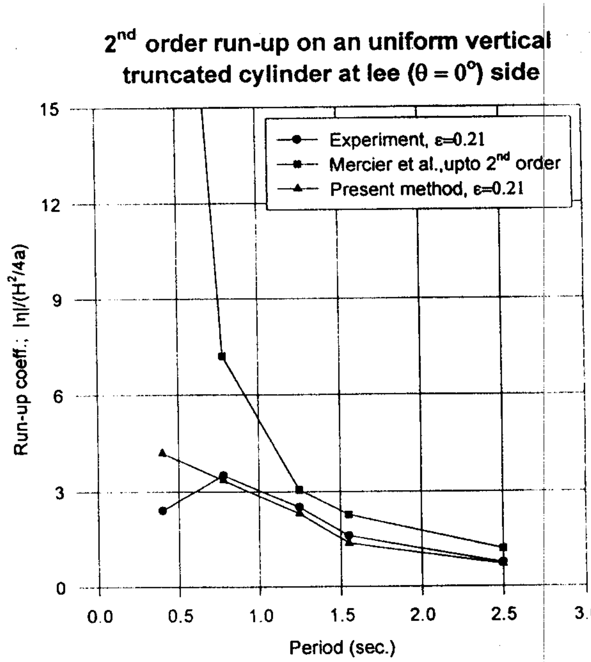

Figure 2

DISCUSSION

Berkvens P.J.F.: In your results on waves diffracting around a cylinder, some very short waves are visible along the waterline. Do you think there is a relation between these short waves and the problems that P. Ferrant encounters when he has waves diffracting around a cylinder in the presence of current?

Celebi S., Kim M-H.: The very small kinks along the waterline is just a graphical noise and short waves around the cylinder are diffracted waves. The desingularized BIEM method is relatively robust at the body-free surface intersection line and we did not experience any numerical problems there.

Grilli S.: Which phase velocity did you use in your Orlanski condition for the bi-chromatic problem?
And what did you do in case of singularity of the celerity?

Celebi S., Kim M-H.: We numerically calculated the phase velocity on the free surface directly from Orlanski condition. In doing this, we selected several points close to the open boundary and the phase velocities are averaged.
In this procedure, the point where singularity occurs is excluded.

Laget O.: Can you tell some precisions on the outside boundary condition you have used?
Do you use both the absorbtion beach and the Orlanski condition? How do you compute the phase velocity and on which variable do you apply the Orlanski condition (free surface deformation, velocity, pressure?)

Celebi S., Kim M-H.: We applied Orlanski condition for the velocity potential and the phase velocity was numerically obtained. For the time derivative of the velocity potential, dynamic free-surface condition was used. We did not combine Orlanski and numerical beach yet.