A time-domain algorithm for motions of high speed vessels using a new free surface condition.

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1 Introduction

When a ship is designed, it is important to know its behaviour in real sea-keeping at forward speed. This behaviour can be predicted performing model tests in towing tanks, but this is quit expensive. The introduction of fast and large computers has given the possibility to write simulation programs that can partly replace physical tests. A lot of research has therefore been carried out recent years. Prins [2] developed a time-domain algorithm to compute the behaviour of several floating bodies in current and waves based on potential flow. Sierevogel [1] contributed an absorbing boundary condition independent of frequency. Both used the double body potential to approximate the steady potential. This approximation is valid for low speeds, but when we increase speed, non-linear effects in the steady potential become more important. At MARIN a program has been developed (RAPID) by Raven [3] that calculates the steady potential satisfying the exact non-linear free surface condition. We use this potential to linearize the time dependent free surface condition. We solve the potential flow problem with this boundary condition in the time domain using a Rankine source distribution. We put the source panels at some distance above the free surface. This promising 'raised panel approach' was also used by Raven [3]. It has the advantage of resulting in a much smoother potential. Besides it is easier to include non-linear effects in the future. Because we assume the speed to be high, upwind differences must be used to obtain a stable iteration procedure. An absorbing boundary condition seems not to be necessary in the frequency range we're interested in (Strouhal number $\tau \gg \frac{1}{4}$). The calculations are carried out for a fictive analytical hull shape. For this hull hydrodynamic coefficients like added mass and damping are calculated. In our presentation we will compare these coefficients with results from other methods, investigate the influence of some of our most important parameters and look at the influence of reflected waves.

2 Mathematical model

We consider a ship moving at constant speed U. A coordinate system $Oxyz$ is introduced in the frame of reference following the forward speed of the body, with the x- and y-axes in the mean free surface and the z-axis vertical upward. The forward speed is in the direction of the negative x-axis. The fluid is assumed to be incompressible and inviscid, and the flow irrotational. We can therefore introduce a velocity potential Φ whose gradient equals the fluid velocity and that satisfies the Laplace equation. On the free surface $z = \zeta(x, y)$ this potential must satisfy:

$$\frac{\partial^2 \Phi}{\partial t^2} + 2\vec{\nabla}\Phi \cdot \vec{\nabla}\left(\frac{\partial \Phi}{\partial t}\right) + \frac{1}{2}\vec{\nabla}\Phi \cdot \vec{\nabla}\left(\vec{\nabla}\Phi \cdot \vec{\nabla}\Phi\right) + g\frac{\partial \Phi}{\partial z} = 0 \quad \text{at } z = \zeta \tag{1}$$

We linearize this condition by splitting the potential in a steady and unsteady part:

$$\Phi(\vec{x}, t) = \phi(\vec{x}, t) + \Phi_R(\vec{x}) \tag{2}$$

The steady potential satisfies the exact time-independent free surface condition and a zero normal velocity condition on the hull and is calculated by RAPID (RAised Panel Iterative Dawson). If we assume the unsteady potential to be small (small amplitudes of motion and waves), we can neglect higher order terms in ϕ and find:

$$\frac{\partial^2 \phi}{\partial t^2} + 2\vec{\nabla}\Phi_R \cdot \vec{\nabla}\frac{\partial \phi}{\partial t} + \vec{\nabla}\Phi_R \cdot \vec{\nabla}\left(\vec{\nabla}\phi \cdot \vec{\nabla}\Phi_R\right) + \frac{1}{2}\vec{\nabla}\phi \cdot \vec{\nabla}\left(\vec{\nabla}\Phi_R \cdot \vec{\nabla}\Phi_R\right) + g\frac{\partial \phi}{\partial z}$$

$$-\frac{1}{g}\left(\frac{\partial\phi}{\partial t}+\vec{\nabla}\Phi_R\cdot\vec{\nabla}\phi\right)\frac{\partial}{\partial z}\left(\frac{1}{2}\vec{\nabla}\Phi_R\cdot\vec{\nabla}\left(\vec{\nabla}\Phi_R\cdot\vec{\nabla}\Phi_R\right)+g\frac{\partial\Phi_R}{\partial z}\right)=0 \quad \text{at } z=\zeta_R \tag{3}$$

The last term is a transfer term that occurs because we linearize around the steady free surface in stead of the actual free surface. Most of the terms contain derivatives of steady velocities. First order derivatives can be calculated accurate. The transfer term contains second order derivatives. We are still busy finding a numerical scheme to obtain these derivatives. Until then the transfer term is omitted. On the hull of the ship we have the same linearized boundary condition Prins and Sierevogel used:

$$\frac{\partial\phi}{\partial n}=\frac{\partial\vec{\alpha}}{\partial t}\cdot\vec{n}+\left(\left(\vec{\nabla}\Phi_R\cdot\vec{\nabla}\right)\vec{\alpha}-\left(\vec{\alpha}\cdot\vec{\nabla}\right)\vec{\nabla}\Phi_R\right)\cdot\vec{n} \tag{4}$$

with $\vec{\alpha}$ the displacement vector. Again, we need stationary speed derivatives. We solve the Laplace equation with these linear boundary condition using a source distribution on the hull and above the free surface. The boundaries are divided in N panels, and on each panel the source strength σ is assumed to be constant. The potential is now given by:

$$\phi(\vec{x},t)=\sum_{j=1}^{N}\sigma_j(t)\iint_{\partial\Omega_j}G\left(\vec{x},\vec{\zeta}\right)dS_\zeta \qquad G=\frac{-1}{4\pi r} \tag{5}$$

If we choose N collocation points on the hull and free surface and apply the corresponding boundary conditions, we obtain N equations for the N unknown source strengths.

3 Test case

Before developing a numerical algorithm we have to choose some kind of hull. The calculations will be made for a mathematical hull shape given by the formula:

$$\left(\frac{z}{d/L}\right)^2+\left(\frac{y}{b(x)}\right)^2=1$$

$$b(x)=\frac{B}{2L}\left(1-8x^2+16x^4\right)$$

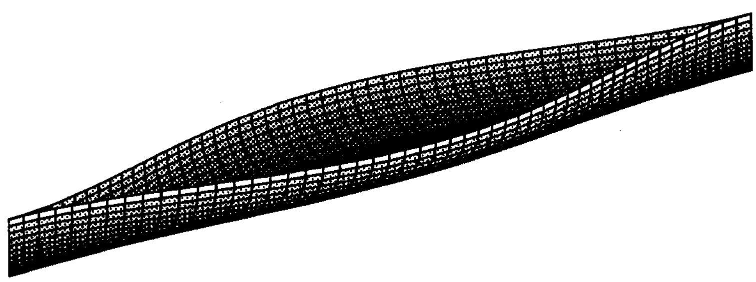

Figure 1: Mathematical hull shape used in calculations

We use $\frac{d}{L}=0.1$ and $\frac{B}{L}=0.2$. The advantage of a mathematical hull is the easy refinement of the grid. In our test study, that's very useful.

4 Numerical method

In our new free surface condition, time and spatial derivatives of the unsteady potential occur. The time derivatives are discretized by second order explicit schemes, therefore using only the current potential and potentials on previous time levels. The spatial derivatives are more complicated. They are decomposed in a derivative in a direction perpendicular and parallel to the free surface (see figure 2). The derivative parallel to the free surface is obtained using upwind differences because of it's well known stabilising properties. We found out that in our case it's absolutely necessary to use upwind differences to avoid wiggles in the solution. The difference becomes this way:

$$\frac{\partial\phi}{\partial l_{||}}(\vec{x}_i)=\sum_{j=1}^{m}\gamma_{ij}\phi(\vec{x}_j) \tag{6}$$

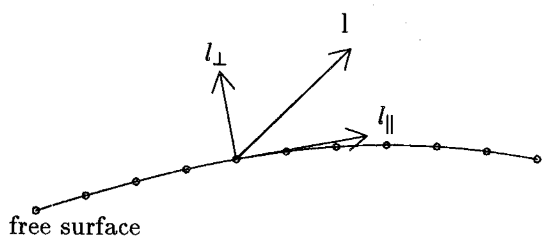

Figure 2: Decomposition of differentiation direction

with m the number of collocation points on the free surface. The coefficients γ_{ij} are non-zero only for some nearby collocation points upstream of the point $\vec{x}_i$. In case of a rectangular grid and uniform flow, this means for example:

$$\vec{\nabla}\Phi_R \cdot \vec{\nabla}\phi(\vec{x}_i) = U \frac{\frac{3}{2}\phi(\vec{x}_i) - 2\phi(\vec{x}_i - \Delta x) + \frac{1}{2}\phi(\vec{x}_i - 2\Delta x)}{\Delta x} + \mathcal{O}\left((\Delta x)^2\right) \tag{7}$$

The derivative in a direction perpendicular to the stationary free surface is obtained by changing the order of integration and differentiation in (5):

$$\frac{\partial \phi}{\partial l_\perp} = \sum_{j=1}^{N} \sigma_j(t) \iint_{\partial \Omega_j} \frac{\partial G}{\partial l_\perp}\left(\vec{x}, \vec{\zeta}\right) dS_\zeta \tag{8}$$

Second order derivatives of the unsteady potential are treated the same way. After discretizing the boundary conditions, the boundaries are divided in panels and collocation points are chosen. Applying the discretized boundary conditions in each collocation point gives us a matrix equation for the unknown source strengths:

$$A\vec{\sigma} = \vec{f} \tag{9}$$

Initially all source strengths are put zero. The time iteration starts by giving the ship one of possible 6 sinusoidal motions (translational or rotational). After a few periods a periodical wave pattern arises around the ship. When there is no reflection, the wave pattern is the same after each period of movement. When that state has been reached, the forces on the ship can be calculated and hydrodynamic coefficients like added mass and damping can be calculated.

5 Results

We mentioned we don't need an absorbing boundary condition because we consider high speeds and frequencies. In that case, the waves propagate downstream. The upwind difference scheme causes the waves not to reflect against the edge of the computational domain behind the ship. We only get reflections from the edge beside the ship. If we choose that edge far away enough, the reflected wave will end up behind the ship and not affect the wave pattern around the ship. Because our main goal is to calculate forces on the hull we can accept these reflections. If we want to predict the total wave pattern, or decrease the speed of the ship, we must add an absorbing boundary condition.

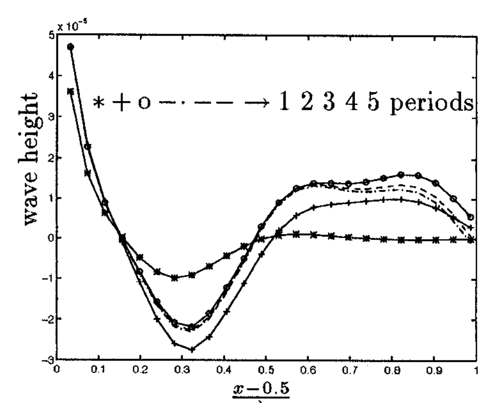

Figure 3: Wave pattern behind ship. Distances : 0.75λ and λ

In figure (3), (4) and (5) is shown what happens to the wave pattern after 1,2,3,4 and 5 periods behind the ship if we change the size of the computational domain. In the absence of reflections the wave pattern should not change anymore after a few periods. The pictures show a change in wave pattern at some distance behind the ship, an indication of the presence of reflected waves there. Choosing the edge of the free surface behind the ship further away doesn't influence the reflections. If we choose the edge of the free surface beside the ship further away, the reflections end up further downstream. In figure (3) the the free surface edge beside the ship was 0.75 wavelength away from the ship and the edge behind the ship one wavelength away, with the wavelength $\lambda = \frac{8\pi F_n^2}{(1-\sqrt{1+4\tau})^2}$. All calculations were done for $F_n = \frac{U}{\sqrt{gL}} = 0.4$ and $\tau = \frac{\omega U}{g} = 2.55$. In figure (4) we see the same reflections if we choose the edge behind the ship further away and don't change the distance to the free surface edge beside the ship. In figure (5) is shown that the reflections occur closer to the ship if the distance to the free surface edge beside the ship is 0.25λ.

In our presentation we will also look at hydrodynamic coefficients like added mass and damping and compare these with results from other methods.

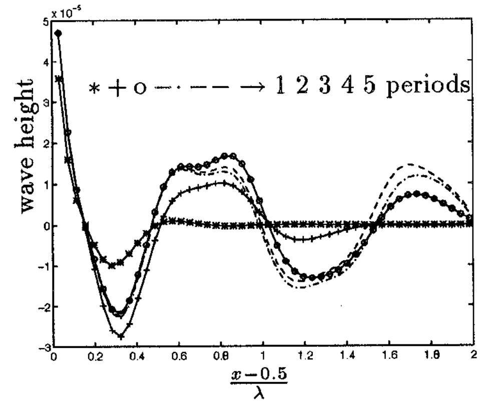

Figure 4: Wave pattern behind ship. Distances : 0.75λ and 2λ

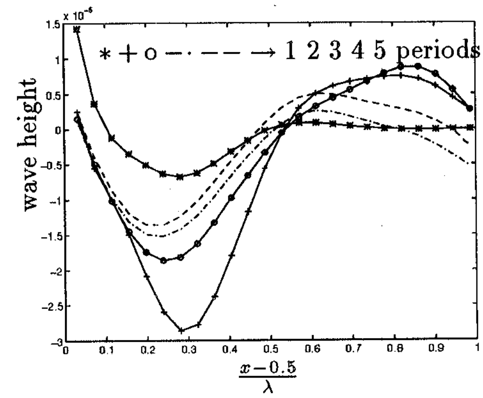

Figure 5: Wave pattern behind ship. Distances : 0.25λ and λ

6 Conclusions and further research

We have developed an algorithm to determine added mass and damping of a ship at high speed using a new boundary condition. Because of the high speed the reflected waves don't spoil the results. Next step is to add an absorbing boundary condition so we can decrease speed. After that we want to determine drift forces by introducing an incoming wave field. Also some attention still has to be paid to the transfer term in our free surface condition.

Acknowledgements

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References

[1] L.M. Sierevogel and A.J. Hermans. Absorbing boundary condition for floating two-dimensional objects in current and waves. *Journal of engineering mathematics, 1996*

[2] H.J. Prins. *Time-Domain Calculations of Drift Forces and Moments.* PhD Thesis, Delft University of Technology, 1995.

[3] H.C. Raven. *A Solution Method for the Nonlinear Ship Wave Resistance Problem.* PhD Thesis, Delft University of Technology, 1996.

DISCUSSION

Ferrant P.: I would like to know how you plan to account for incoming waves in your model. It is not clear to me how you can combine a linearised incident wave with the non-linear steady solution given by RAPID.

Bunnik T.H.J., Hermans A.J.: There are two ways of doing this. The first and easiest way is to split up the time-dependent potential ϕ into an incoming potential and an extra potential: $\phi_{tot} = \phi_{inc} + \tilde{\phi}$.

The separate potentials no longer represent physical waves close to the body, but the sum does.

If this approach doesn't work a wavemaker can be introduced, upstream of the body. The interaction between incoming wave and steady wavefield is then automatically included when the wave travels downstream.

Bertram V.:

1) The 'new' condition has been derived already by Newman (1978), but it is satisfying to see it now implemented in codes.

2) I would recommend modifying RAPID for a frequency approach for $\tau > 0.25$; the problem is then very similar to a shallow-water steady wave resistance problem. So all techniques including radiation and open-boundary conditions work just as well. Since the computations are easily parallelized in the frequency domain, this approach is in my experience very efficient.

Congratulations on a most interesting paper!

Bunnik T.H.J., Hermans A.J.:

1) Thank you for noticing.

2) It is possible in a time-domain approach to obtain information about a range of frequencies by using some kind of impulse response functions.

If we want to extend the method to non-linear, we can't use the frequency-domain approach because of its limitation to linear problems.

Magee A.: Since you have developed a time-domain method, why not calculate the impulse response functions to obtain the added mass and damping at <u>all</u> frequencies? This may help as well with wave reflection difficulties as you can finish the calculations before the wave reflections reach the ship.

Bunnik T.H.J., Hermans A.J.: We are certainly going to try this. The main problem is, that when waves of all frequencies are generated, high frequencies cannot be resolved on the frequency-independent grid (short waves) and low-frequency waves travel very fast and their reflections will reach the ship before stopping the calculation.