Time Domain Calculations in Finite Water Depth

A. K. Bratland* F. T. Korsmeyer† J. N. Newman‡

1 Introduction

Hydrodynamic computations in the time domain using a free-surface Green function have been presented in numerous papers [Bingham *et al.* 1994, Lin & Yue 1990, Beck & Magee 1990]. This abstract demonstrates the extension of this type of analysis to finite depth. This transient approach may be used to compute the first-order, frequency-domain hydrodynamic coefficients. We present these here to validate the method by comparison to computations made directly in the frequency domain for zero speed. With the addition of forward speed to the analysis the first-order steady force becomes important. It has been shown that this steady force (with components referred to as resistance, sinkage, and trim) is the limit as time becomes infinite of the force computed when the body is impulsively accelerated in surge. Since the prediction of squat (maximum draught; sinkage plus trim) is particularly important in finite depth, we present these results for various depth-based Froude numbers to demonstrate the forward-speed analysis.

The finite-depth analysis is carried out similarly to our previous infinite-depth approach. The potential problem is cast as a boundary integral equation. The only boundary appearing in this equation is the body boundary itself due to the choice of a Green function which satisfies the (transient) free-surface condition and, in the present work, the bottom (no-flux) boundary condition. One advantage of the time-domain approach is that the same Green function can be used both for moving-ship problems with nonzero forward velocity U and for fixed structures where $U = 0$.

2 The Green function

The appropriate Green function may be written as the sum of a Rankine and a wave part, which we define in the forms

$$G^{(0)} = \frac{1}{r} + \frac{1}{r'} - 2\int_0^\infty \frac{e^{-kh}}{\cosh kh} \cosh k(z+h) \cosh k(c+h) J_0(kR)dk \tag{1}$$

$$G_t^w = 2\int_0^\infty \frac{\sqrt{gk\tanh kh}}{\cosh kh \sinh kh} \sin(t\sqrt{gk\tanh kh}) \cosh k(z+h) \cosh k(c+h) J_0(kR)dk \tag{2}$$

[Newman 1992] describes effective algorithms for the integral in (1), and outlines the fundamental difficulties associated with the efficient evaluation of (2). The approach which we have implemented here is to express (2) as the sum of two terms involving the normalized function $F(X,V,T)$, as defined by [Newman 1992] equation 25, and then to consider the difference function $F - F_\infty$ where F_∞ can be evaluated from the corresponding infinite-depth Green function. Then we expand this difference function in triple Chebyshev expansions, in unit squares of the rectangular domain $0 \le X \le 16$ and $0 \le T \le 33$. (Physically, the variables X and T correspond to the horizontal distance from the source to the field point, and the time t, nondimensionalized in terms of the depth h and gravity g.) The coefficients of these Chebyshev expansions are pre-evaluated, and stored for routine use. At each time step of the convolution, T =constant and the triple expansions are reduced to double expansions in the normalized horizontal and vertical coordinates X and V, which are then evaluated for each combination of source and field points.

*Techpower, Norway
†Research Laboratory of Electronics, Massachusetts Institute of Technology
‡Department of Ocean Engineering, Massachusetts Institute of Technology

One point to note is the large-time asymptotic behavior of the Green function, since this affects the corresponding behavior of computed hydrodynamic forces. In the infinite-depth case (2) is exponentially small, but for $h < \infty$ $G_t^w \rightarrow \mathcal{O}(1/t)$. When $U \neq 0$, and coordinates are used which move with the ship, $G_t^w \rightarrow \mathcal{O}1/t + \mathcal{O}\sin(\omega_c t)/t$, where ω_c denotes the critical frequency. This is given in the infinite-depth limit by $\tau_c = \omega_c U/g = 1/4$, while the variation of τ_c with depth is shown in Figure 5.

3 Results

For zero speed we compare Fourier-transformed time-domain computations to computations made directly in the frequency domain with a similar boundary integral method, for the surge and heave motions of a hemisphere. From the asymptotic behavior of the Green function it is clear that the time record has to be longer for finite water depth than infinite water depth. Alternatively we could approximate the large time behavior based on the asymptotic analysis. For the added-mass and damping coefficients presented in Figure 1 we have calculated to a maximum time of T=25 and assumed the impulse-response function to be zero beyond that. The results from the frequency-domain code ("FD") and time-domain code ("TD") agree within graphical accuracy. At low frequency, the heave added-mass rises steeply, in a manner which appears consistent with the the result of [Yeung 1981] that $A_{33} \rightarrow \infty$ as $\omega \rightarrow 0$. Figure 2 shows the corresponding results for the exciting force, with similar confirmation from the frequency-domain computations.

The large time limit of the radiation potential forced by impulsive surge acceleration, ϕ^r, can be considered as the steady potential. That means the steady forces, with Neumann-Kelvin linearization, can be written as the large time limit of

$$F_j(t) = \rho U \iint_{\overline{S}_b} \frac{\partial}{\partial x}\phi^r \, dS. \tag{3}$$

Figure 3 shows computations of $F_3(t)$ and $F_5(t)$ for the Wigley hull. By applying the equations of hydrostatic equilibrium

$$\begin{aligned} C_{33}x_3 + C_{35}x_5 &= F_3 \\ C_{53}x_3 + C_{55}x_5 &= F_5 \end{aligned}$$

the "steady" sinkage and trim are found. The sinkage is shown in Figure 4. The sinkage increases with decreasing water depth, or with increasing values of the Froude number $F_{nh} = \frac{U}{\sqrt{gh}}$.

For large values of time, the transient results oscillate at the critical frequency, with slowly-decreasing amplitude, about the steady limiting values. While computations at higher depth-based Froude number are of interest, the finite computational domain for the evaluation of the Green function described in §2, poses a restriction. Work is presently underway to remove this limitation, permitting the estimation of squat closer to the critical Froude number.

References

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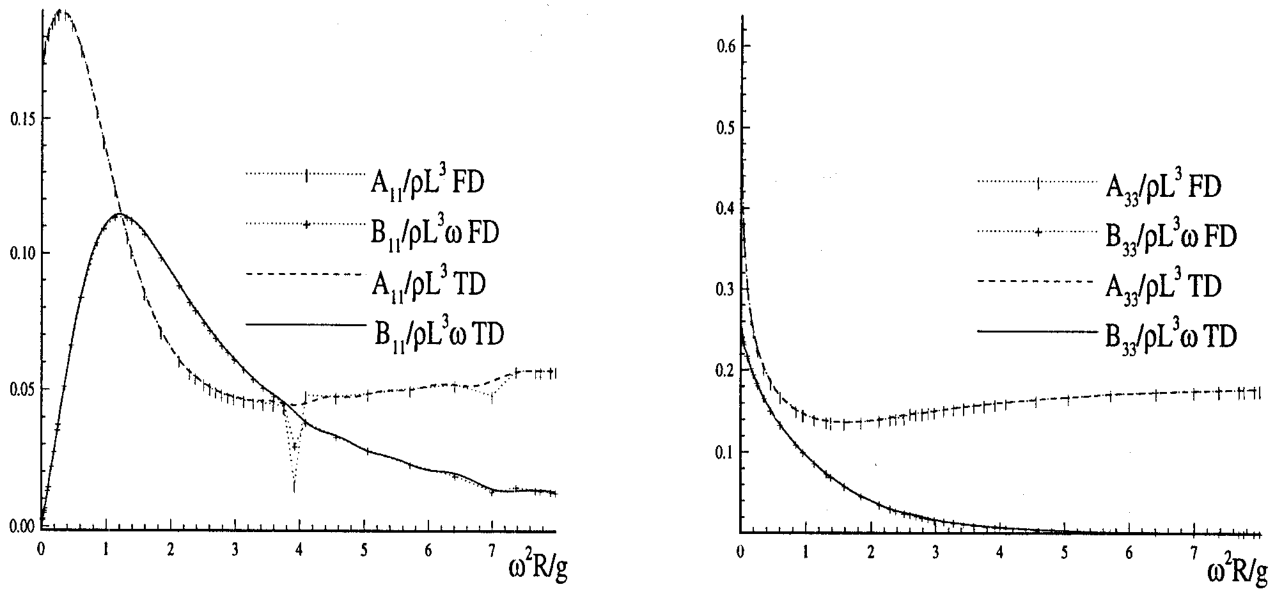

Figure 1: The added-mass and damping coefficients of a hemisphere for surge (left) and heave (right) for water depth $h = 1.2R$. "TD" denotes Fourier transformed time-domain computations, "FD" denotes frequency-domain computations.

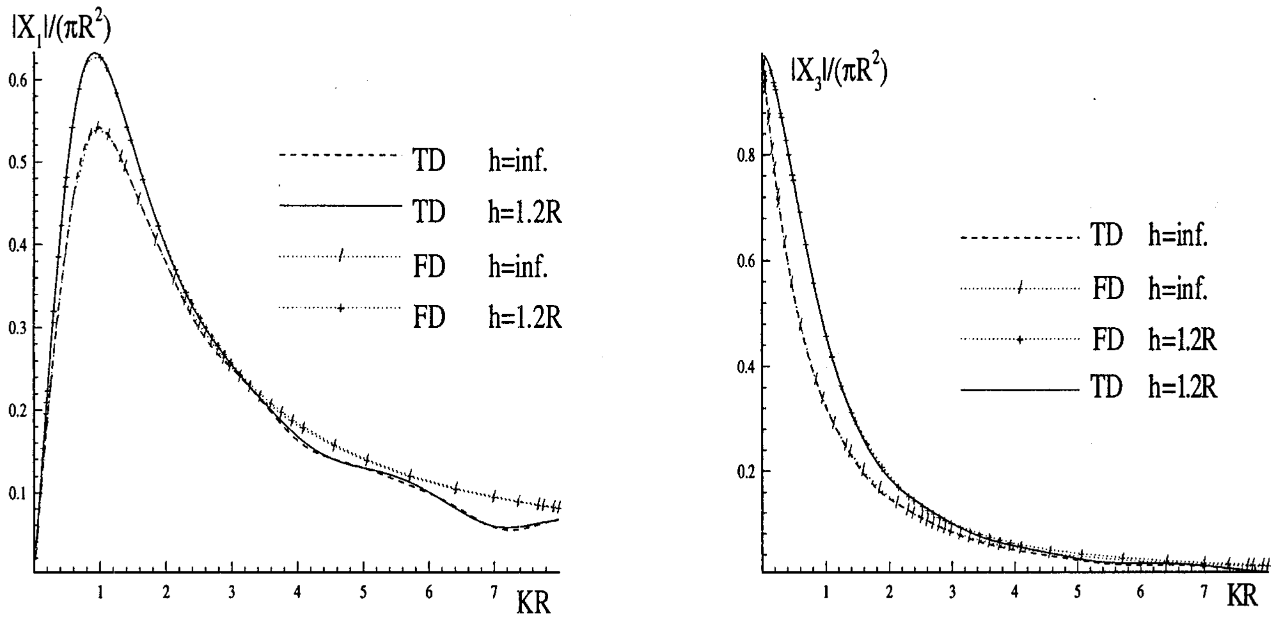

Figure 2: The exciting-force coefficients of a hemisphere for surge (left) and heave (right) for water depths $h = 1.2R$ and $h \to \infty$. "TD" denotes Fourier transformed time-domain computations, "FD" denotes frequency-domain computations.

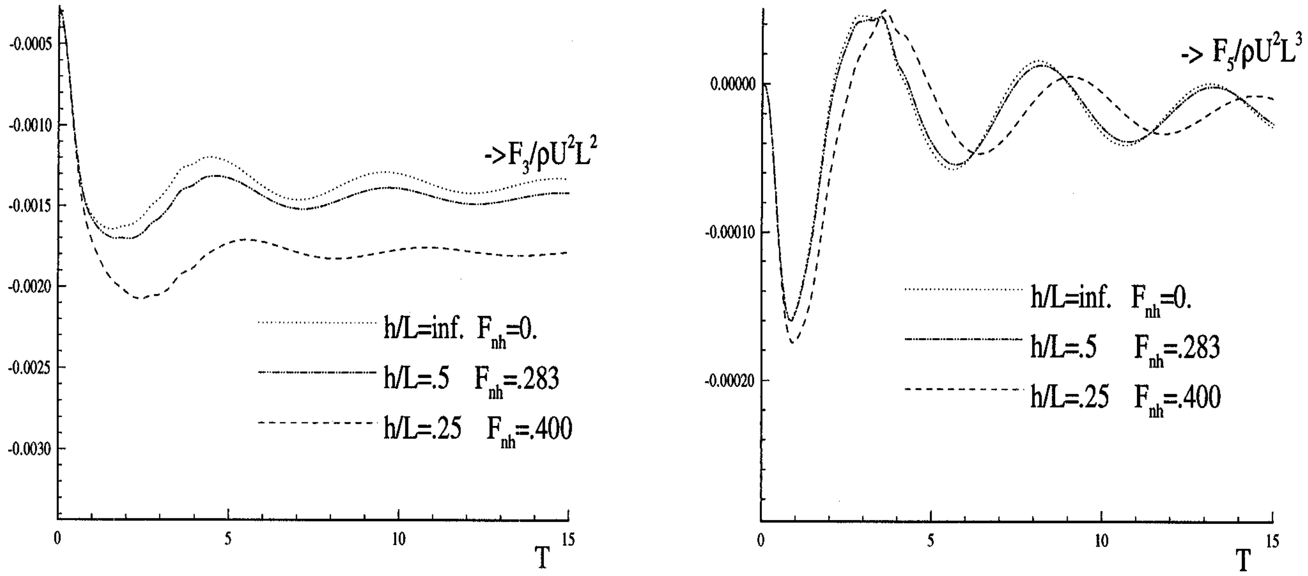

Figure 3: The approach to the steady vertical force and moment on a Wigley hull for $F_n = 0.2$, for the water depths and depth-based Froude numbers indicated.

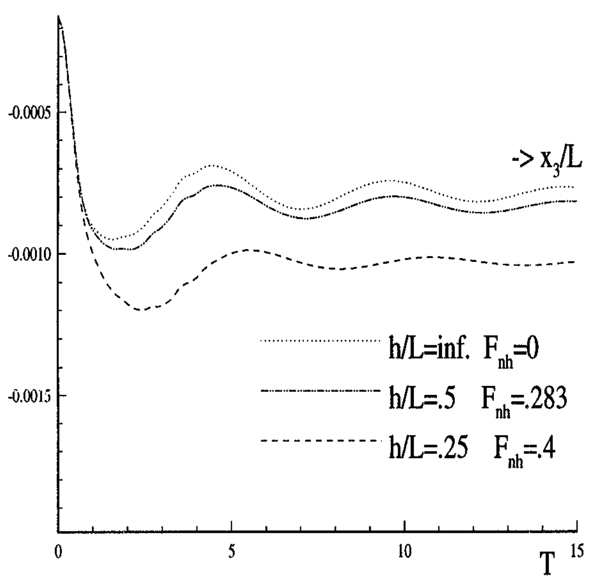

Figure 4. The approach to the "steady" sinkage for a Wigley hull for $F_n = 0.2$, for the water depths and depth-based Froude numbers indicated.

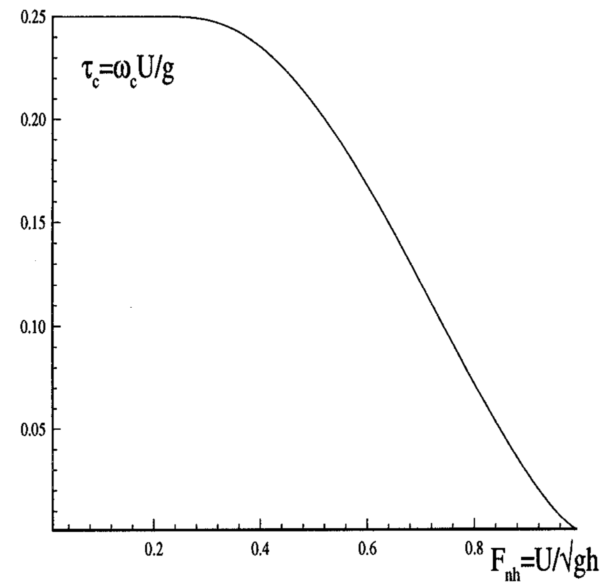

Figure 5. The depth dependence of τ_c.

DISCUSSION

Tuck E.O.: I am encouraged by the fact that the rate of decay of transients seems to be as rapid at the higher $F_{nh} = 0.400$ as it is at the lower $F_{nh} = 0.283$. As $F_{nh} \to 1$, one might expect that, since the flow is ultimately unsteady then, the transients might decay less rapidly, perhaps not at all.

Bratland A.K., Korsmeyer T., Newman J.N.: In our results $F_{nh} << 1$, and the Green function has the asymptotic behaviour as written in abstract. As $F_{nh} \to 1$ another asymptotic must be considered. Newman has shown that, as $F_{nh} \to 1$ and $t \to \infty$

$$G_t \approx T^{-2/3} A_i \left(-\left(\frac{1}{4} T \right)^{3/2} \left(1 - X^2 / T^2 \right) \right)$$

where $T = t\sqrt{g/h}$ $\quad X = \dfrac{Ut}{h}$.

This must be examined closer, but we think you are right in suggesting slower decay.

Clément A.: In preceeding Workshops (Kyushu, Oxford), I presented the results of S. Mas' work about numerical computations of the time-domain Green function in finite water depth, and we observed that the gradient of the function was much more difficult to obtain through series and asymptotic expansions. Do you need to compute the gradient of the Green function in your numerical method? If so, did you observe this difference in the divergence of the algorithms?

Bratland A.K., Korsmeyer T., Newman J.N.: The gradient is required in the kernel of the integral equation. The Green function is represented by Chebyshev expansions which can be differentiated term-by-term. This avoids the analytic difficulties which have been discussed at past Workshops, but we are limited to a finite computational domain in (X,T) as noted in Section 2 of our abstract.