SLENDER BODY APPROXIMATION FOR YAW VELOCITY TERMS IN THE WAVE DRIFT DAMPING MATRIX

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1. INTRODUCTION

Consider a *slender ship* with length L and beam B ($\varepsilon = B/L << 1$) and suppose a coordinated system with the x-axis being coincident with the longitudinal axis while the z-axis is vertical, pointing upwards, the origin of the system being at the intersection of the free surface with the mid section; the ship cross section is defined by the contour line $\partial C(x)$ and the ship water line by ∂W. Points in $\partial C(x)$ are defined by the radius vector

$$\mathbf{r}_c = y_c \mathbf{j} + z_c \mathbf{k} \in \partial C(x). \tag{1}$$

Let $p_{20}(x,\mathbf{r}_c)$ be the *steady second order* pressure field due to a harmonic wave with amplitude A, frequency ω and incident in the β direction; one assumes here that $p_{20}(x,\mathbf{r}_c)$ includes Dirac δ-functions at the points where $\partial C(x)$ intersects ∂W, these concentrated forces per unit of length being related with the change in the wetted surface of the ship. The sectional steady drift coefficients are then defined by the expressions

$$\begin{Bmatrix} d_x(x;\omega,\beta) \\ d_y(x;\omega,\beta) \\ n_z(x;\omega,\beta) \end{Bmatrix} = \int_{\partial C(x)} p_{20}(x,\mathbf{r}_c) \cdot \begin{Bmatrix} n_x(x,\mathbf{r}_c) \\ n_y(x,\mathbf{r}_c) \\ -y_c n_y(x,\mathbf{r}_c) \end{Bmatrix} d\partial C(x), \tag{2}$$

where $(n_x;n_y)$ are components of the normal $\mathbf{n}$; defining the *moments* of (2) by the expressions

$$\begin{Bmatrix} M_{x,j} \\ M_{y,j} \\ M_{z,j} \end{Bmatrix} = \int_{-L/2}^{L/2} x^j \cdot \begin{Bmatrix} d_x(x;\omega,\beta) \\ d_y(x;\omega,\beta) \\ n_z(x;\omega,\beta) \end{Bmatrix} dx, \; j = 0,1,2, \tag{3}$$

the generalized steady drift force vector in the horizontal plane has obviously the components

$$
\begin{aligned}
D_x(\omega,\beta) &= M_{x,0}(\omega,\beta); \\
D_y(\omega,\beta) &= M_{y,0}(\omega,\beta); \\
N_z(\omega,\beta) &= M_{z,0}(\omega,\beta) + M_{y,1}(\omega,\beta).
\end{aligned}
\tag{4}
$$

The intention now is to express *all* elements of the Wave Drift Damping matrix, including the ones related to the *yaw motion*, in terms of the *moments* $\{M_{x,j}; M_{y,j}; M_{z,j}\}$. In the present study the influence of the coupling between the second order steady potential and the slow ship motion will not be considered, although it can be obtained by integration of quadratic functions of the first order (linear) solution.

2. THE YAW TERMS IN WDD MATRIX

The first column of the WDD matrix, the one related with the *surge* velocity U_x, is exactly given by the expression

$$
\begin{Bmatrix} B_{11}(\omega,\beta) \\ B_{21}(\omega,\beta) \\ B_{61}(\omega,\beta) \end{Bmatrix} = \frac{\omega}{g}\left[\omega\cos\beta\frac{\partial}{\partial\omega} - 2\sin\beta\frac{\partial}{\partial\beta} + 4\cos\beta\right]\cdot\begin{Bmatrix} D_x(\omega,\beta) \\ D_y(\omega,\beta) \\ N_z(\omega,\beta) \end{Bmatrix};
\tag{5a}
$$

the second column, related to the *sway* velocity U_y, is given by

$$
\begin{Bmatrix} B_{12}(\omega,\beta) \\ B_{22}(\omega,\beta) \\ B_{62}(\omega,\beta) \end{Bmatrix} = \frac{\omega}{g}\left[\omega\sin\beta\frac{\partial}{\partial\omega} + 2\cos\beta\frac{\partial}{\partial\beta} + 4\sin\beta\right]\cdot\begin{Bmatrix} D_x(\omega,\beta) \\ D_y(\omega,\beta) \\ N_z(\omega,\beta) \end{Bmatrix},
\tag{5b}
$$

both results being proven in Aranha (1996).

Observing the essentially two-dimensional feature of the wave diffraction by a slender body, one can introduce here, by inspection, the *sectional* WDD coefficients influenced by the *sway* velocity, given by (see (5b))

$$
\begin{Bmatrix} b_{12}(x;\omega,\beta) \\ b_{22}(x;\omega,\beta) \\ b_{62}(x;\omega,\beta) \end{Bmatrix} = \frac{\omega}{g}\left[\omega\sin\beta\frac{\partial}{\partial\omega} + 2\cos\beta\frac{\partial}{\partial\beta} + 4\sin\beta\right]\cdot\begin{Bmatrix} d_x(x;\omega,\beta) \\ d_y(x;\omega,\beta) \\ n_z(x;\omega,\beta) \end{Bmatrix}.
\tag{5c}
$$

Expression (5c) can be also proven exactly, as a blend of a two-dimensional result derived in Aranha (1994) and the three dimensional one given in Aranha (1996), and it can be used to obtain a *slender body approximation* for the elements of the WDD matrix related with the *yaw motion*. In fact, for a *slender body* the *yaw* motion is seen, at the cross section x, as being a *sway* motion with amplitude x.Ω, Ω being the yaw angular velocity; it turns out that the related *seccional* WDD coefficients are then given by

$$\begin{aligned} b_{16}(x;\omega,\beta) &= x \cdot b_{12}(x;\omega,\beta); \\ b_{26}(x;\omega,\beta) &= x \cdot b_{22}(x;\omega,\beta); \\ b_{66}(x;\omega,\beta) &= x \cdot b_{62}(x;\omega,\beta). \end{aligned} \tag{5d}$$

Integrating (5d) along the ship length, observing the contribution of the sway term b_{26} to the yaw moment and ignoring terms of relative order ε^2, one obtains finally, with the help of (3), that:

$$\begin{Bmatrix} B_{16}(\omega,\beta) \\ B_{26}(\omega,\beta) \\ B_{66}(\omega,\beta) \end{Bmatrix} = \frac{\omega}{g}\left[\omega \sin\beta \frac{\partial}{\partial\omega} + 2\cos\beta \frac{\partial}{\partial\beta} + 4\sin\beta\right] \cdot \begin{Bmatrix} M_{x,1}(\omega,\beta) \\ M_{y,1}(\omega,\beta) \\ M_{y,2}(\omega,\beta) \end{Bmatrix}. \tag{6}$$

3. GEOMETRIC OPTICS APPROXIMATION

The slender body approximation (6) can be checked directly against numerical results, as the ones derived by Grue & Palm (1996), for instance. While waiting the slender body code that allows one to determine the *moments* $\{M_{x,j}; M_{y,j}; M_{z,j}\}$, one presents here analytical expressions for the high frequency limit, where geometric optics approximation can be used. These limits have an importance in themselves, since they are analytic and hold in a range of frequencies where numerical results are most questionable.

Consider a wave incident on a vertical wall with α being the angle between the wave direction and the normal **n**. It is trivial to show in this case that the elementary drift force on an element d*s* of the wall is given by

$$dF = \frac{1}{2}\rho g A^2 \cos^2\alpha \, \mathbf{n} \, ds \, .$$

In high frequency one can consider the body as if it were a vertical cylinder infinitely long with cross section coincident with the water line ∂W. Assuming symmetry with respect to y-axis and that r(x) is the half beam of the body, the transition between the "illuminated" and "shadow" zones in the geometric optics limit is defined by a single variable $x_0(\beta)$, given by the expression

$$\text{Max}\,|r'(x)| \le |\tan\beta| \Rightarrow x_0(\beta) = \frac{L}{2};$$
$$\text{Max}\,|r'(x)| > |\tan\beta| \Rightarrow |r'(x_0(\beta))| = |\tan\beta|. \tag{7a}$$

Introducing the variables $\{\xi = 2x/L;\ \xi_0(\beta) = 2x_0(\beta)/L\}$ and the integrals

$$I_1 = \int_0^1 \frac{\xi . r'(\xi)}{1+(r'(\xi))^2} d\xi;$$
$$S_0(\beta) = \int_0^{\xi_0(\beta)} \frac{1}{1+(r'(\xi))^2} d\xi\ ;\ C_0(\beta) = \int_{\xi_0(\beta)}^1 \frac{r'(\xi)}{1+(r'(\xi))^2} d\xi; \tag{7b}$$
$$S_2(\beta) = \int_0^{\xi_0(\beta)} \frac{\xi^2}{1+(r'(\xi))^2} d\xi\ ;\ C_2(\beta) = \int_{\xi_0(\beta)}^1 \frac{\xi^2 . r'(\xi)}{1+(r'(\xi))^2} d\xi.$$

The following expressions are then obtained in the high frequency limit:

$$\hat{B}_{16} = \frac{B_{16}}{\rho B^2 \omega A^2} = \left(\frac{L}{B}\right)^2 I_1 \sin\beta;$$
$$\hat{B}_{26} = \frac{B_{26}}{\rho B^2 \omega A^2} = \left(\frac{L}{B}\right)^2 I_1 \cos\beta;$$
$$\hat{B}_{66} = \frac{B_{66}}{\rho B^3 \omega A^2} = \frac{1}{2}\left(\frac{L}{B}\right)^3 [S_2(\beta)\ \sin\beta + C_2(\beta)\ \cos\beta]; \tag{7c}$$
$$\hat{B}_{22} = \frac{B_{22}}{\rho B \omega A^2} = 2\left(\frac{L}{B}\right) [S_0(\beta)\ \sin\beta + C_0(\beta)\ \cos\beta].$$

Table (1) compares (7c) with high frequency numerical results obtained by Grue & Palm (1996) for different values of β.

	$\hat{B}_{16}$		$\hat{B}_{26}$		$\hat{B}_{66}$	
β	NUM	(7c)	NUM	(7c)	NUM	(7c)
π/2	–1.2	–3.4	0.	0.	21.4	24.6
3π/4	–0.8	–2.4	1.1	2.4	18.7	17.4
π	0.	0.	1.7	3.4	5.0	7.9

TABLE (1):Comparision between numerical results for KL=16 and (7c)

* * *

DISCUSSION

Eatock Taylor R.: I do not think one should try to draw conclusions about the reliability of your drift damping formula for moving bodies by refering to the slow convergence of results for a truncated cylinder. It is well known that the sharp corners in this case lead to slow convergence, particularly in the surge-heave coupling coefficient which is proportional to the slow forward speed (see the 1993 OTC paper by Teng and myself). The problem vanishes when a small corner radius is used, but the hemisphere analysed by John Grue is a much better test case than the truncated cylinder in the context of this controversy.

Aranha J.: I agree, in some aspect at least, with you, since the cylinder problem seems to be plagued with small numerical imprecisions to which the WDD formula is very sensitive. If I recall well, Kinoshita & al. results, shown in the presentation, were obtained by a quasi-analytic method and the convergence does not seem to be very good when the cylinder is free to oscillate.
However, the cylinder problem is one of the most obvious in our field and it seems natural, in this context, to look at it to confirm the validity of the formula.
With respect to your suggestion, that it would be better to look to the sphere, Grue's results, shown in the conference, together with a similar result, shown in my JFM (1996) paper, point both to a perfect agreement between the WDD formula and numerical results.

Grue J.: Your abstract and presentation are based on the work Aranha (1996) JFM, which you claim provides a formula for wave drift damping based on strict proof. First you find an expression for the far-field amplitude of the diffracted-radiated waves (with current), next conservation of momentum is applied to find the force.
Denote the far-field amplitude of the diffracted-radiated waves by H_u, which may be expanded by $H_u = H_o + \tau H_1$, where $\tau = U w/g$.
Consider the difference between your and our formulae for H_1. I have tried to show that this difference is zero, however, it is generally not. The following figure illustrates this. The body is a freely floating hemisphere moving with small forward speed. Dashed line: Aranha (1996), solid line: Nossen et al. (1991).
In this example your and our wave drift damping coefficients (B_{11}) are in close agreement, see figure 1a. For other geometries, e.g. ships, there is a general disagreement, of the order 100 %, except possibly for long waves.

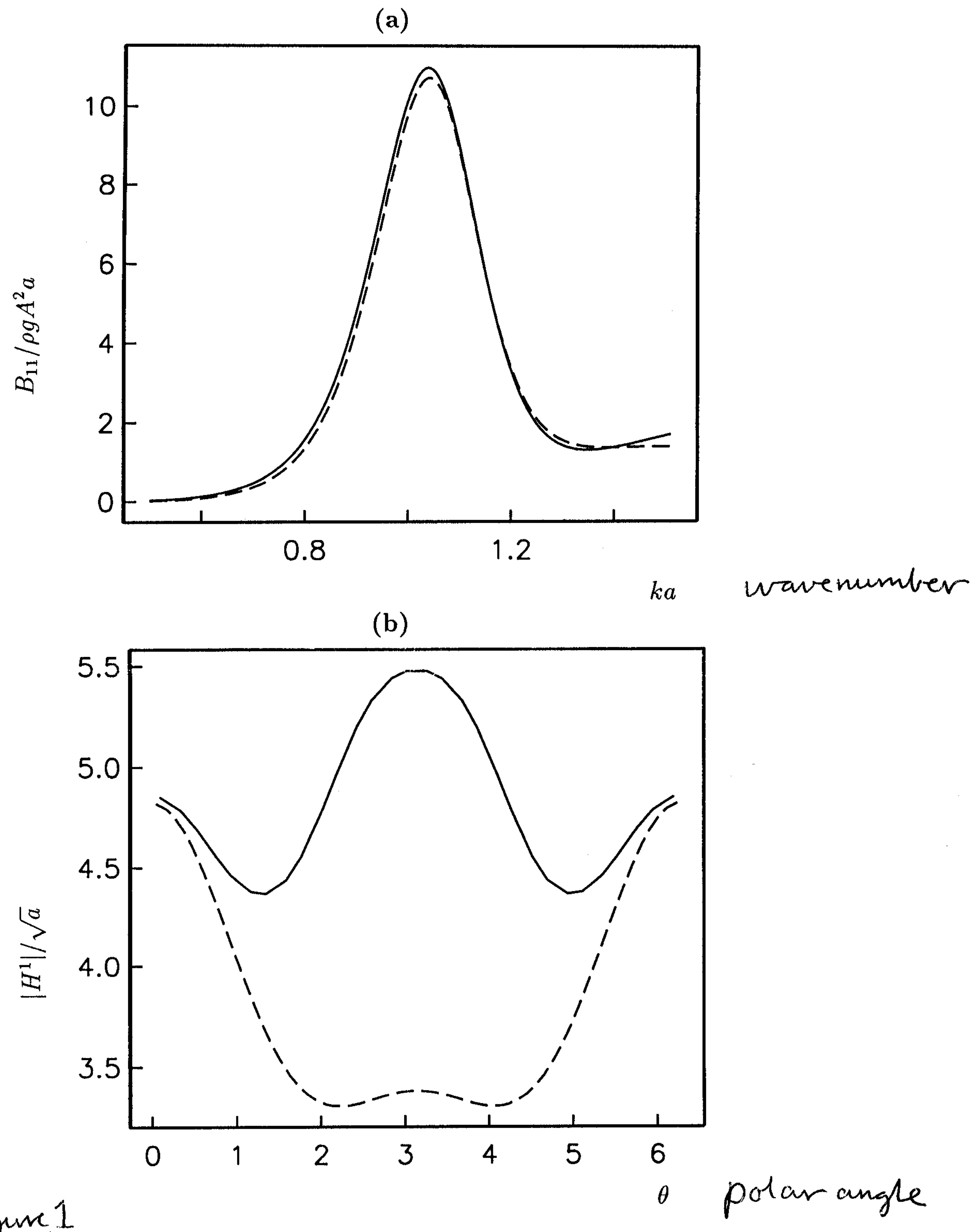

Figure 1

~~Figure 1~~: For a half immersed sphere, 400 panels on S_B and 880 panels on S_F, in head waves, the figures shows: (a): B_{11} computed by complete theory (solid line), and Aranha's formulae (dashed line). (b): The far-field amplitude function H^1 for translatory motion at $ka = 0.9$, computed by complete theory (solid line), and Aranha's theory (dashed line).

Aranha J.: Two things must be said about this:

1) I should thank J. Grue since he provided just another example of a 3D-body, free to oscillate, where the agreement between the numerical results and WDD formula is perfect.

2) With respect to the behavior of the $H_1(\theta)$ function one has obviously a misunderstanding since, otherwise, how could one obtain a complete agreement in the force computation with a complete disagreement in the far-field behavior? The point is that in my work the far-field is well behaved, it does not have the secular term that Grue's approach has (recall Malenica in the 10th IWWWFB, Oxford, 1995).

If Grue intends to make a comparison, it is not enough to differentiate the $A_u(\theta)$ coefficient with respect to U ; it is necessary to differentiate also the wavenumber, that depends also on U. In this way he would obtain secular terms in my expression that should be matched to his secular terms.

The way he has done compares two distinct things and it has no relevance for the discussion.