An approximate technique for the hydrodynamic analyses of a huge floating structure

Tingyao Zhu, Hiroshi Kagemoto and Masataka Fujino
Department of Naval Architecture and Ocean Engineering
The University of Tokyo
7-3-1 Hongo, Bunkyo-ku, Tokyo 113, JAPAN

1. Introduction

In Japan a national project is now going on in which the feasibility of a huge floating structure as large as several kilometers long and wide are investigated. For the analysis of behaviours of such structures in waves, direct application of conventional numerical techniques is practically impossible, because the required computational burden is enormous. In order to overcome this difficulty, we proposed an approximate technique at the last workshop in Oxford. This time we present an extention of the technique and how the analysis of a floating structure of several kilometers long and wide can be carried out with reasonable computational time.

2. An analysis of a slender structure

Let us first consider an analysis of a slender structure which is very long compared to wave length while the width is small compared to wave length. As an example, we consider a linear diffraction problem of a regular wave train (wave length is 100m) due to a rectangular floating structure of 1000m long and 50m wide. (Since the draft of such rectangular structures should be very small, say less than 1m, we do not take the effect of the draft into account in the present analysis.) Supposing that we use a singularity distribution method for the analysis, since at least 10 panels are needed per one wave length (we assume the singularity strength on each panel is constant), the number of panels for the analysis amounts to 500(100×5). However, since the length of the structure is very large compared to the wave length, the number of panels need not be that large but, as we showed in the last workshop, can be reduced by representing the inner domain of the structure by a small number of long panels. For example, if we analyze a problem in beam waves, the singularity strength on the panels located at a certain distance from the both ends of the structure must be almost the same. Then the structure need not be divided into panels of equal size but, as shown in Fig.1, can be represented by a small number of long panels plus a number of small panels that take care of the end effects. In oblique waves, this approximation can still be used with a slight modification which assumes that the singularity strength on the panels located at a certain distance from the both ends of the strucutre varies according to the following law instead of assuming that they are the same.

$$\sigma_j = \sigma_i e^{ik(x_j - x_i)\cos\chi} \tag{1}$$

where $\sigma_j e^{-i\omega t}, \sigma_i e^{-i\omega t}$ stand for the singularity strength of j-th & i-th panel respectively, whose x coordinates are x_j, x_i and k represents the wave number of an incident wave train.

In head waves, we did not succeed in the last workshop to show how to simplify the computation, because the singularity strength does not vary in accordance with eq.(1) but decay gradually from upstream to downstream. This is because waves are reflected at each panel as they progress toward downstream, where the wave energy should be eventually diminished. In other words, the approximation given by eq.(1) should hold even in head waves but the effective area of the approximation is at an inifinite downstream, where $\sigma = 0$, which means the approximation is practically of no use. However, in a slender body theory, it is known that the singularity strength on a slender body in head waves decay in proportion to $s^{-1/2}e^{iks}$, where s stands for the distance from the weatherside edge of the corresponding structure [1]. Therefore, it is still possible to reduce the number of panels in the analysis of head waves by assuming the lengthwise distribution of singularity strength varies as $s^{-1/2}e^{iks}$ at the panels located at more than certain distance from the weatherside edge of a structure. The singularity distribution obtained by a reduced number panels based on this assumption(see Fig.2(55 panels)) is compared with those obtained by 500 panels of equal size in Fig.3. The agreement is quite good.

3. An analysis of a very long and very wide structure

We showed how to simplify the ananlysis of a slender structure. However, the structure now proposed in Japan for an international airport is 5000m long and 1000m wide, which is not slender because the width is also very large compared to the usual wave length that will appear on the sea. In head waves, according to the computation obtained by a large number of small panels of equal size, lengthwise distribution of singularity strength of such huge structures still obey the same law as that of a slender structure along the rim of the structure. However, at the inner part of the structure, the singularity strength does not vary as $s^{-1/2}e^{iks}$ but simply decays exponentially and becomes zero quite fast at a small distance from the weatherside edge. The fluid field under the inner part of a huge structure, which is both long and wide, is bounded by the bottom of the structure at the upper-end while the lower-end is also bounded by a sea bottom. Therefore, the velocity potential $\phi e^{-i\omega t}$ that satisfies a Laplace equation and the boundary conditions should be written as:

$$\phi = (A_n^+ e^{\kappa_n x} + A_n^- e^{-\kappa_n x}) \cdot \cos \kappa_n (z + h) \qquad (2)$$
$$(\kappa_n = \frac{n\pi}{h-d})(h: \text{water depth}, d: \text{draft})$$

where z is positive upward and zero at an undisturbed free surface. Eq.(2) tells us that under the structure no wave of progressive mode can exist. This explains why the singularity strength decay exponentially at an inner part of a structure. However, since the decay ratio is explicitly known as $e^{-\kappa_n x}$, the analysis in head waves can still be simplified by assuming that the singularity strength decays in proportion to $s^{-1/2}e^{iks}$ along the panels near the rim of a structure while assuming that it decays in proportion to $e^{-\kappa_n s}$ along the panels at the inner part a structure. The panel division of a structure of 1500m long and 300m wide in head waves($\lambda = 100m$) based on this assumption is shown in Fig.4 (155 panels). The results are compared with those obtained by 1800 equally divided small panels in Fig.5. The agreement is quite good while the time for the computation is reduced to $(155/1800)^2 (= 0.0074)$ by this approximation.

If we proceed on to the analysis of a structure in oblique waves, according to the calculation obtained by a large number of small panels of equal size, the singularity strength along the rim of the structure does not decay as $s^{-1/2}e^{isx}$ but varies according to eq.(1). Here one question arises. So far we have shown that a singularity strength varies as $s^{-1/2}e^{iks}$ in head waves while it varies as $e^{iks\cos\chi}$(eq.(1)) in oblique waves. However, it is unrealistic to assume that the characteristics of the lengthwise singularity strength distribution changes discontinuously once χ deviates from 0 deg. There should be transient angle of incidence between $\chi = 0$ and $\chi =$(small angle) in which the singularity strength does decay as $s^{-\alpha}e^{iks}$, where $\alpha < 1/2$.

References

[1]Faltinsen, O.: Wave forces on a restrained ship in head-sea waves, 9th ONR Symp., 1972.

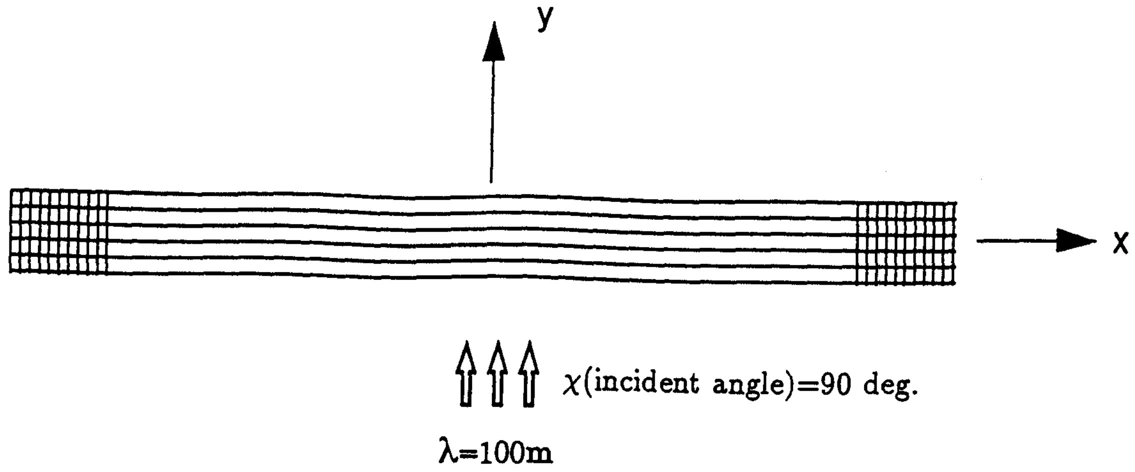

Fig.1 Panel subdivision in beam waves

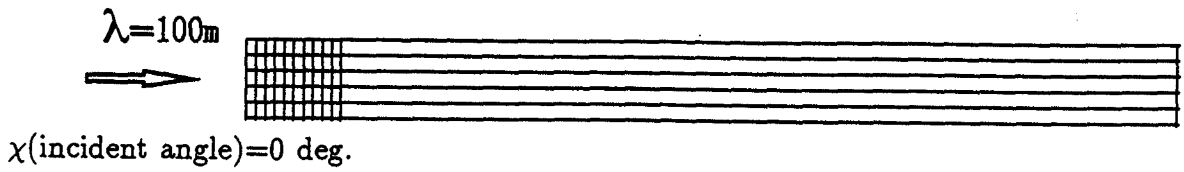

Fig.2 Panel subdivision in head waves

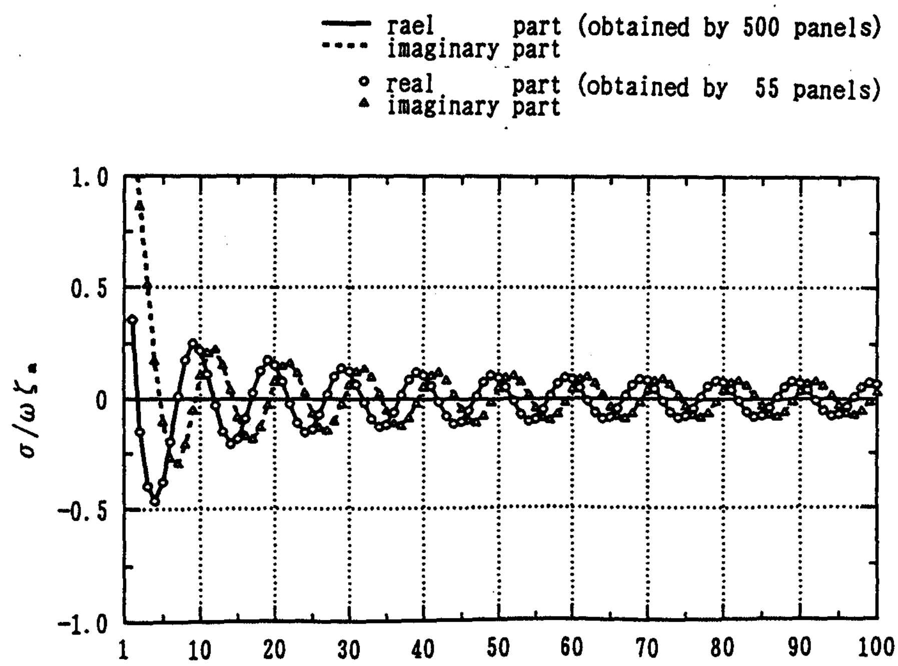

Fig.3 The lengthwise singularity strength distribution

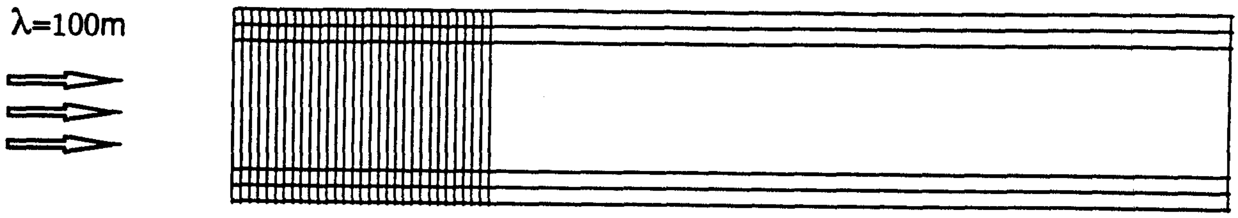

Fig.4 Panel subdivision of a structure which is both very long and very wide

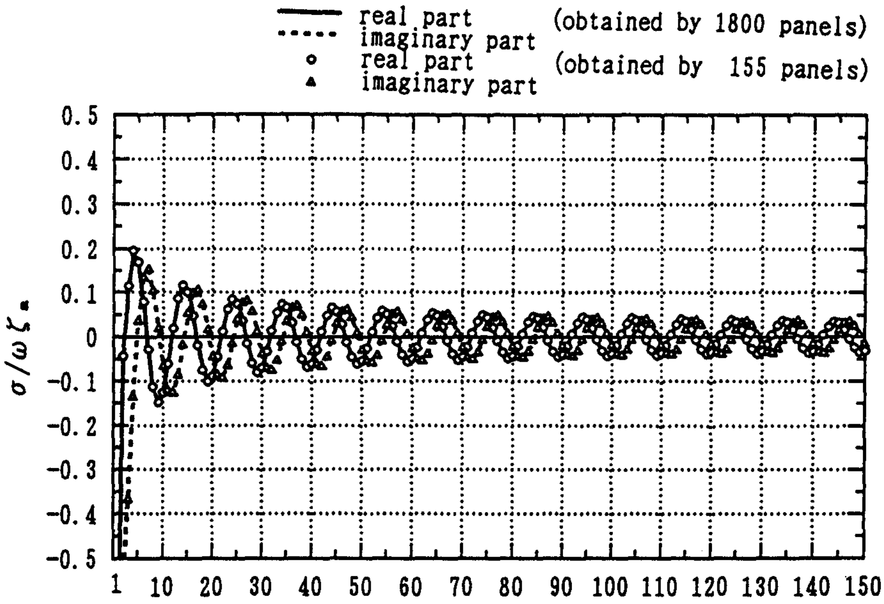

Fig.5 (a) The lengthwise singularity distribution at the rim of the structure

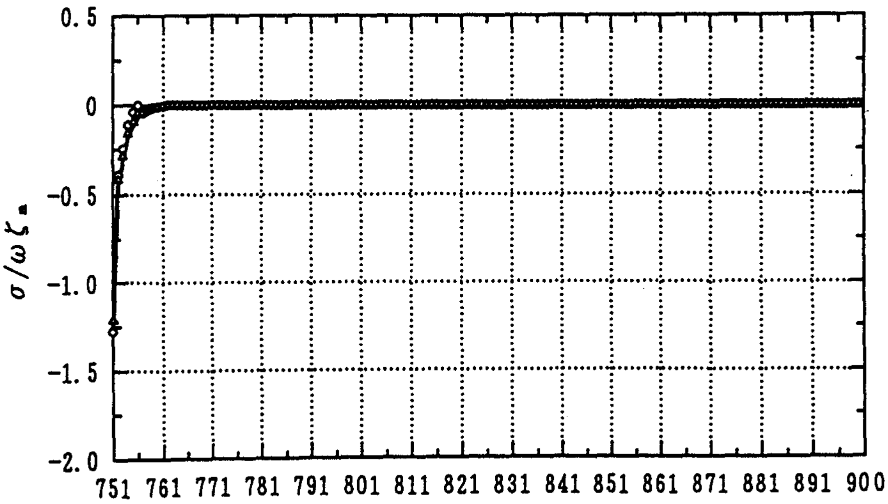

Fig.5 (b) The lengthwise singularity distribution at the inner part of the structure