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Active Absorption of Radiated Waves in a 3D Boundary Element Model

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INTRODUCTION

The simulation of open boundaries in three dimensional (3D) Rankine type numerical models is a subject which has been given particular attention during recent years. For simulations of the interaction between waves and a structure (fixed or floating) in the time domain, a number of open boundary conditions may be found in the literature. They include the Orlanski type radiation conditions, sponge layers, and active wave absorption. In general the Orlanski type conditions are not an attractive choice since they require the instantaneous phase velocity vector at the open boundary which is difficult to determine. A sponge layer has the disadvantage that the size of the computational domain must be increased along with the length of the sponge layer considered, and thus can become prohibitively expensive when applied to general problems. Furthermore, a sponge layer may allow some reflection of oblique incident waves. Active absorption of waves by a wavemaker has been applied in physical wave flume experiments for many years. In numerical models active wave absorption has also been considered as a method to allow waves to propagate out of the computational domain (see e.g. Clément and Domgin, 1995, and Skourup and Schäffer, 1995). Techniques for active wave absorption have so far only been developed for wave flumes, i.e. for 2D waves. For 3D waves an exact active wave absorption technique has not yet been developed. However, one step towards active wave absorption in 3D is to consider a quasi-3D wave absorber simulated by an array of independently controlled 2D active wave absorbers. This idea is investigated in the present abstract.

MATHEMATICAL FORMULATION

A potential flow is assumed, with boundary conditions expanded up to second order and applied on the mean positions of the free surface and body boundaries (see Isaacson and Cheung, 1992, for example). The numerical implementation is, however, not yet complete, so results will be presented at first order only. The total velocity potential is separated into a known incident potential ϕ_I, and a scattering potential ϕ_S representing the effects of the body and its motions,

$$\phi(x,t) = \epsilon\,[\,\phi_I^{(1)}(x,t) + \phi_S^{(1)}(x,t)\,] + \epsilon^2\,[\,\phi_I^{(2)}(x,t) + \phi_S^{(2)}(x,t)\,] + \ldots \tag{1}$$

where ϵ is the perturbation parameter and the superscripts denote the order of the expansion. By formulating the boundary value problem for the scattered field alone all waves in the domain are outgoing waves, and all lateral boundaries should thus be formulated as absorbing boundaries.

The active wave absorption method used here is similar to the one used at the Danish Hydraulic Institute for wave absorption in physical flumes. The motion of a wave absorber is a function of the time history of the wave absorber position and of the free surface elevation at the wave absorber. These are transformed to an updated wave absorber position by use of a digital filter designed to match a theoretically determined transfer function (see Schäffer et al., 1994, for details). The same technique may also be used in a 3D model by considering a finite number of 2D wave absorbers placed next to each other and working independently. Each absorber is then governed by the same digital recursive filter and by the local time history of the position of and the elevation at the absorber. In the present simulations the absorbers all work in the piston mode, but digital filters are also available for hinged flap wave absorbers. The wave absorber boundary condition is of the Neumann type.

To compute the potential, the boundary value problem is re-cast as a boundary integral equation via Green's 2nd identity

$$\alpha(\mathbf{x})\, \phi(\mathbf{x},t) = \int_\Gamma \phi(\xi,t)G_n(\mathbf{x},\xi) - G(\mathbf{x},\xi)\phi_n(\xi,t)\ d\Gamma \tag{2}$$

where $\xi = (\xi_1,\xi_2,\xi_3)$ is the position vector of an integration point situated at the boundary Γ of the domain, and the factor $\alpha(\mathbf{x})$ depends on the position of the observation point $\mathbf{x}$ ($\alpha(\mathbf{x})=2\pi$ for $\mathbf{x}$ situated at a smooth part of the boundary). Equation (2) is discretized using a panel method where the kernel function $G(\mathbf{x},\xi) = 1/r = 1/|\xi-\mathbf{x}|$, and the variation over a panel of both the potential and the geometry is taken to be linear. Collocation is performed at the corners of each panel, and the resulting linear system of equations is solved by LU factorization at the first time level (i.e. at $t = 0$) and then by back-substitution at each time step. The free surface boundary conditions are integrated using 4th order Adams-Bashforth and Adams-Moulton schemes. Further details concerning the numerical solution can be found in Skourup et al. (1992).

Once the potential has been computed the forces and moments on the structure are determined by integrating the pressure over the wetted surface of the body.

NUMERICAL EXAMPLES

The following results were obtained using a square wave tank of 12 metres length and with a water depth of 1 meter. A vertical circular cylinder with a radius of 1 metre is situated in the middle of the wave tank. The walls of the wave tank are equipped with a number of active piston wave absorbers. Two numerical tests are made: The first concerns diffraction of waves due to the presence of a fixed bottom mounted cylinder, while the other concerns a radiation problem due to forced motions of a floating cylinder of draft 0.5m.

Even though irregular waves can be simulated using the present model, we include only one regular wave test in this abstract. Numerical experiments involving irregular waves will be presented in a forthcoming paper. The wave considered has a period of 1.4s. This gives a (first order) wave length of 3.0m. Hence, there are less than two wave lengths between the cylinder and the walls of the tank, and any reflections from the wave absorbers will appear in the results after only a few wave periods.

A convergence study of the calculations using increasingly fine discretizations of the domain was undertaken, and the results presented here have converged to graphical accuracy using 3173 nodes over 1/2 of the computational domain. In Fig. 1 the time series of the in-line force on the cylinder is depicted for a time span of more than 40 wave periods. For comparison, the analytical (linear) in-line force amplitude determined by the MacCamy and Fuchs (1954) theory is also shown.

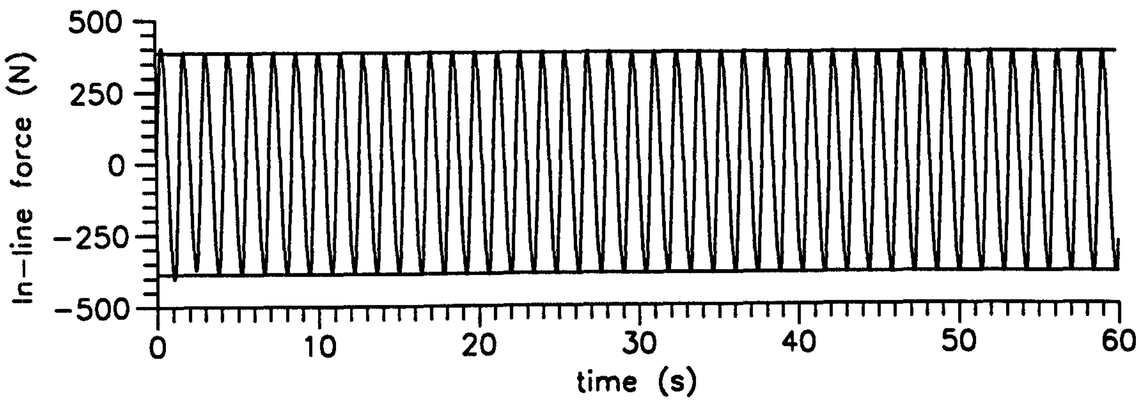

Figure 1. BEM solution time series of the in-line force on a vertical circular cylinder. The force amplitude determined by the MacCamy and Fuchs theory is also depicted.

The agreement is excellent, and the effects of reflection from the active wave absorbers are not graphically visible. The difference in force amplitudes between the simulations and the MacCamy and Fuchs result is less than 1 per cent. The computed overturning moment on the cylinder is within 2 per cent of the MacCamy and Fuchs result.

The wave height enhancement factor (i.e. the wave run-up) on the cylinder is determined as the mean wave amplitude taken over a few wave periods during the simulation. It is compared with the theoretical result by the MacCamy and Fuchs theory. The comparison is depicted in Fig. 2.

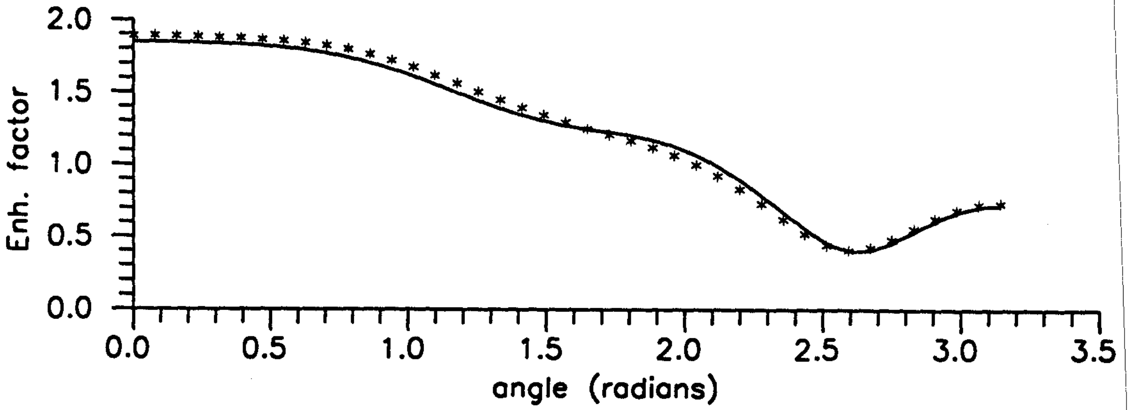

Figure 2. Wave height enhancement factors on the cylinder. The solid line is the MacCamy and Fuchs solution while the asterisks are the BEM solution. The angle 0 radians correspond to the up-wave side on the cylinder while the angle π correspond to the down-wave side on the cylinder.

The agreement between the computed and theoretical wave height enhancement factors on the cylinder is good. The largest local deviation is about 6 per cent.

The second example concerns a floating cylinder with draft 0.5m, which is forced in unit amplitude surge motion at a of period of 1.4s in the same domain as described above. There are no incoming waves in this simulation. In Fig. 3 the force on the cylinder is depicted for more than 20 periods of oscillation of the cylinder. The time series is compared to converged results from the frequency domain code WAMIT (1995).

The agreement both in phase and amplitude is seen to be excellent. The deviation between the force amplitudes using the two methods is about 3 per cent. The reason for the slightly higher deviation here than in the diffraction problem is that the force is determined by the scattered potential alone, while the force in the diffraction problem was given by the sum of the incident and scattered potentials.

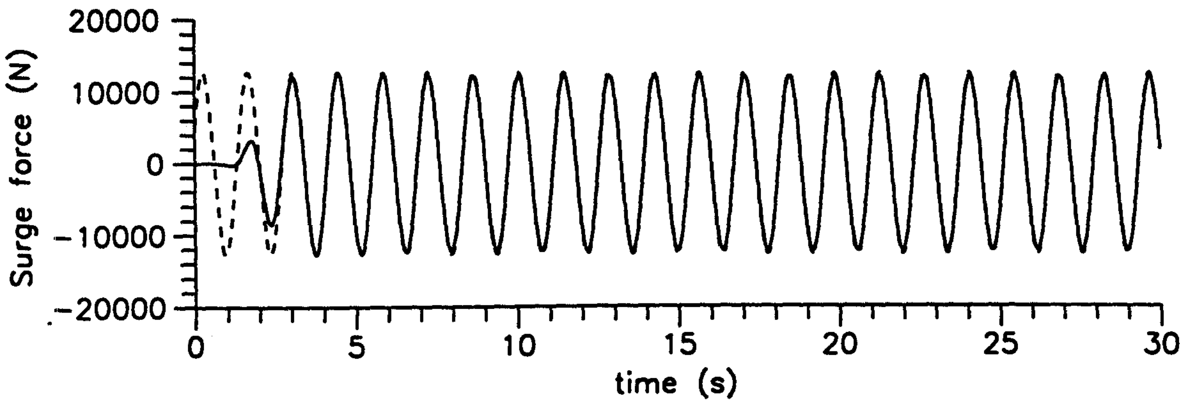

Figure 3. Force on the cylinder due to surge motion in calm water. The solid line depicts the 3D BEM result and the dotted line is the force determined by use of the WAMIT code.

CONCLUSIONS

In the present abstract some preliminary results have been presented to show the efficiency of using an active wave absorption technique as the open boundary condition in a 3D boundary element model. Both a diffraction and a radiation problem have been considered, and excellent agreement with analytical and established numerical results was found. The present results are correct to first order, but the model is still under development, and the second order extension of it is straightforward, and it will be implemented in the near future. The active absorption technique used here works well over a broad range of frequencies and may be used to absorb irregular waves as well. This technique allows the tank walls to be situated close to the structure and thus decreases the necessary computational domain, so that the computing time may be reduced compared to other methods.

ACKNOWLEDGEMENT

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DISCUSSION

Clement: Comment: It is easy to show (Milgram (1970), Naito (1985), Clement and Maisondieu (1993)) that the optimal filter for the control of the absorbing paddle in the time domain is not causal. Then going from frequency domain to time domain is not evident and requires a long and careful study; it took three years for Maisondieu's Ph.D.! Thus we should appreciate more details about the "secret" filter coefficients.

Question: It is well known that the piston is able to be an ideal (i.e. $C_a = 100\%$) wave absorber in the low frequency range. Could you comment about the poor results of your absorption strategy in this frequency range?

Skourop & Bingham: I agree with your comment that an optimal filter for the control of the absorbing paddle in the time domain is not causal. However, a stable causal filter can be constructed for practical applications (see e.g. Schäffer et al. (1994), Skourup and Schäffer (1995)). The filter coefficients are part of the commercial software AWACS (Active Wave Absorption Control System) developed at the Danish Hydraulic Institute and can therefore not be published.

As you mention in your question, the piston should be an ideal wave absorber in the low-frequency range, which clearly is not the case for the AWACS absorber. The reason for that is that there is a high-pass filter in the AWACS software in order to ensure a stable system (i.e. no drift) for physical wave tank experiments. In numerical tests, this may not be necessary (as long as the drift of the wave paddle is small). Numerical simulations, in which this filter is omitted, will be made in the future.