The 11st International Workshop on Water Waves and Floating Bodies, Hamburg, 1996

Time-domain calculations on a sailing vessel in waves, study on increasing speed.

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1. Introduction

In recent years many studies have been carried out solving the unsteady ship motion problem. This problem is very important in predicting the behaviour of a ship in real sea-keeping, which includes the interaction between waves and the velocity of a ship. Computing this interaction can be done by using the frequency domain or the physical time domain. The disadvantage of the studies in the frequency domain is their restriction to harmonic waves. Real waves are not harmonic. In the time domain we can also handle non-harmonic waves.
Prins [2] developed a two- and three-dimensional time-domain algorithm to compute the behaviour of a cylinder, a sphere and a commercial tanker in current and waves. The results are satisfactory. We have extended this method with, among other things, a frequency independent absorbing boundary condition [3].
Our next goal is to apply the method to a LNG carrier at service speed. The forward speed of the commercial tanker considered up to now is very low, the maximum Froude number is 0.018, i.e. 2 knots, while the usual speed of a $125,000m^3$ LNG carrier is about 20 knots (i.e. Froude number is 0.2). This fact causes some problems in our algorithm. We study increasing speed and the the effect on our absorbing boundary condition. To remove the instabilities on the free-surface due to increasing forward speed, we introduce upwind discretization. Both cases were done in the two-dimensional algorithm. Some results of the tree-dimensional algorithm will be shown at the presentation.

1. The absorbing boundary condition

The physical fluid domain is infinite (or large). The computational domain cannot be infinite, so we have to introduce artificial boundaries and proper boundary conditions. In the literature several methods have been proposed to absorb free surface waves. Keller and Givoli [1] introduce a method which use an artificial boundary, dividing the original domain into a computational and a residual domain (the interior and the exterior). In our method we use a boundary condition independent of the wave frequency, using the idea of the Givoli's method with Prins' algorithm. Like the interior, we also discretize the free surface of the exterior by dividing this into panels. We developed a special Green's function in the exterior. The condition absorbs the outgoing waves. The method also reduces computer time, when computing the behaviour of an object in harmonic waves. Firstly, the boundary will be

closer to the object and, secondly, it is not necessary to implement the conditions dependent on every frequency. We are also able to use a step-response function to calculate the hydrodynamic coefficients and are able to calculate the drift forces and wave drift damping.
The advantage of using our algorithm compared to using the conventional time-domain Green's function in the exterior, is that it will be easier to implement the effects of higher speed, because the boundaries of the exterior are already divided into panels.

In our first set up [3], we assumed the interior to be moving together with the object, while the exterior was fixed to the earth. To solve the problem in the interior as an overall matrix equation, we applied Green's theorem on the domain between the boundaries, see figure 1.

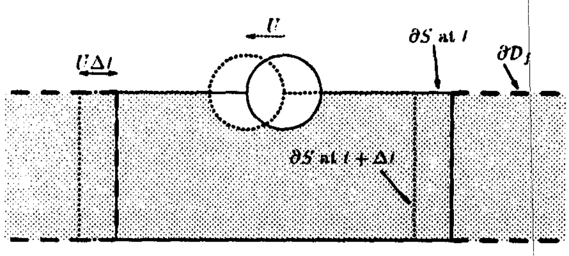

FIGURE 1: *The 2-D geometry*

In our new set up we assume the exterior also to be moving together with the object. In the exterior we get the following linearized free surface condition

$$\phi_{tt} + g\phi_z + 2U\phi_{xt} + U^2\phi_{xx} = 0 \qquad \text{at } z = 0 \ . \tag{1}$$

Where $\phi(\boldsymbol{x}, t)$ is the unsteady potential and the steady potential is approximate by the undisturbed flow potential Ux. We use the following discretizations for the time derivatives

$$\begin{aligned} \phi_{tt}\big|^{n+1} &= \frac{1}{(\Delta t)^2}\left(2\phi^{n+1} - 5\phi^n + 4\phi^{n-1} - \phi^{n-2}\right) + \mathcal{O}\left((\Delta t)^2\right) \\ \phi_t\big|^{n+1} &= \frac{1}{2\Delta t}\left(5\phi^n - 8\phi^{n-1} + 3\phi^{n-2}\right) + \mathcal{O}\left((\Delta t)^2\right) \ , \end{aligned}$$

with superscripts denoting the time level. Now we may write equation (1)

$$\begin{aligned} \phi_z^{n+1} + \frac{2}{g(\Delta t)^2}\phi^{n+1} &= \frac{1}{g(\Delta t)^2}\left(5\phi^n - 4\phi^{n-1} + \phi^{n-2}\right) - \\ &- \frac{U}{g\Delta t}\left(5\phi_x^n - 8\phi_x^{n-1} + 3\phi_x^{n-2}\right) - \frac{U^2}{g}\left(3\phi_{xx}^n - 3\phi_{xx}^{n-1} + \phi_{xx}^{n-2}\right) + \mathcal{O}\left((\Delta t)^2\right) \ . \end{aligned} \tag{2}$$

We use an upwind discretization scheme calculate the numerical differentiation of the potential (see next section), and solve our problem by using Green's theorem, the same way we did in our first set up,

$$\boldsymbol{D}_1\boldsymbol{\psi}^{n+1} = \boldsymbol{D}_2\boldsymbol{\psi}^n + \boldsymbol{D}_3\boldsymbol{\psi}^{n-1} + \boldsymbol{f}^{n+1} + \boldsymbol{E}_{\mathcal{D}}\tilde{\boldsymbol{\phi}} \ , \tag{3}$$

with $\boldsymbol{\psi}$ a vector containing $(\phi_f|\phi_{\mathcal{H}}|\phi_{\mathcal{B}}|\phi_{\mathcal{B}_n})$ and $\boldsymbol{E}_{\mathcal{D}}\tilde{\boldsymbol{\phi}}$ is the right-hand side of (2).

The reflections due to the artificial boundary are with both set ups are both very small, less then $\frac{1}{2}$% of the total surface elevation, when the boundary is one wave-length away from the object. The advantage of the second algorithm is that is will be easy to implement the effects of the double body potential. The right-hand side of (2) will then also contain terms of the double body potential.

3. Upwind discretization

Increasing the speed, the potential is showing point-to-point oscillations. In figure 2 and 3 is shown that the potential shows local extremes, after one period of forced oscillation in the heave direction, if the Froude number $Fn > 0.50$. These numerical oscillations arise, when we use central discretization.

$$\phi_x|_i = \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x} + \mathcal{O}(\Delta x) ,$$

with subscripts denoting the element number of the clockwise numbered uniform mesh (like figure 1), with mesh size Δx. These oscillations, called wiggles, occur in the stationair convection-diffusion equation, when the Péclet number $Pe = UL/k$ is large and in the instationair convection-diffusion equation, when the Courant-Friedrichs-Lewy number $CFL = U\Delta t/\Delta x$ is large. To remove these numerical instabilities upwind discretization is used or artificial viscosity is added.

We noticed that in our case also the CFL-number plays an important role in the determination of stability of the scheme. For increasing values of the CFL-number the scheme becomes unstable. Therefore it seems to be that our problem will also be solvable by using upwind discretization and we replace the central discretization in our algorithm by

$$\phi_x|_i = \frac{\phi_i - \phi_{i-1}}{\Delta x} + \mathcal{O}(\Delta x) .$$

This is called upwind discretization, because it is biased in upstream direction. Now the wiggles disappeared, see figure 4.

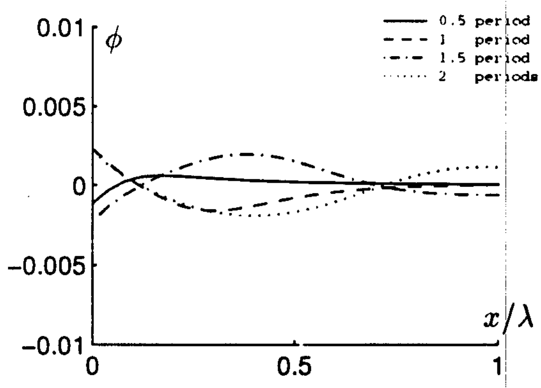

FIGURE 2: *The potential, using central discretization, $Fn = 0.2$*

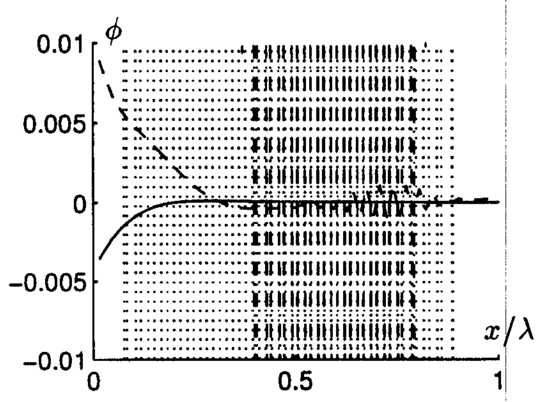

FIGURE 3: *The potential, using central discretization, $Fn = 0.5$*

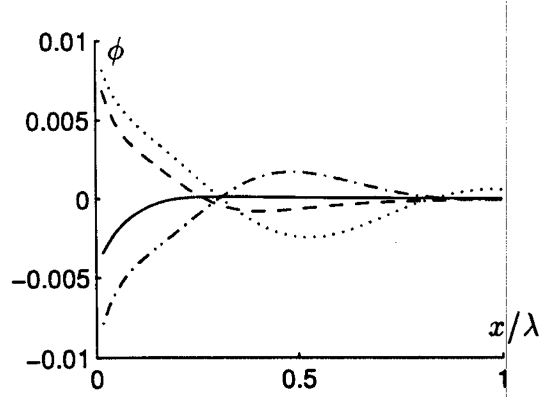

FIGURE 4: *The potential, using upwind discretization, $Fn = 0.5$*

To get more accuracy we can also use second-order upwind discretization. And less necessary, but also possible is to use upwind discretization for the second derivative.

$$\phi_x|_i = \frac{3\phi_i - 4\phi_{i-1} + \phi_{i-2}}{2\Delta x} + \mathcal{O}((\Delta x)^2) \qquad \phi_{xx}|_i = \frac{\phi_i - 2\phi_{i-1} + \phi_{i-2}}{(\Delta x)^2} + \mathcal{O}((\Delta x)^2) \,.$$

4. Conclusions and further research

In our two-dimensional algorithm, we get good results increasing the speed using our boundary condition with an explicit time derivative. More increasing the speed gives numerical instabilities and using upwind discretization makes them disappear, In the future we will carry out the stability analysis.
Our goal is to calculate the behaviour of a LNG carrier, so we also have to adjust our three-dimensional algorithm. Applying the speed dependent absorbing boundary condition in that algorithm is the same as in the 2D algorithm. In the two dimensional mesh it is easy to see which element is in the upstream direction of another element. In our three-dimensional mesh of the carrier it will be more complicated, see figure 5.

FIGURE 5: *the mesh of the LNG carrier and the free surface*

Acknowledgements

Financial support for this work and the mesh of the ship-hull are given by the Maritime Research Institute Netherlands.

References

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[2] H.J. Prins. *Time-domain calculations of the drift forces and moments.* PhD thesis, Delft University of Technoloy, The Netherlands, 1995.

[3] L.M. Sierevogel and A.J. Hermans. Absorbing boundary condition for floating two-dimensional objects in current and waves. *Journal of engineering mathematics (to appear)*, 1996.

DISCUSSION

Clement: In 2D applications, your computational domain is moved in the same direction as the waves to be absorbed at the boundary, but in 3D, it is not the case and waves exit the box with an incidence angle. Is it a problem with regard to wave absorption? Could you comment this point?

Sierevogel and Hermans: In our 2D algorithm the computational domain is not moving in the same direction as the waves, but in the same direction as the current. Thus the computational domain is fixed to the body. Also in the 3D case the domain is fixed to the body. We absorb the outgoing waves by using an exterior domain. In our first 2D setup, the exterior domain did not move, thus was fixed to the earth. This gives some problems in the 3D case, so in the new setup and the 3D-case the exterior is also moving with the body.