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# "On the Wave Resistance of Twin-Hull Floating Bodies with Non-Symmetric Demi-Hulls"

by

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**SUMMARY**: In the past, the wave resistance problem of symmetric thin mono- or twin-hulls, moving at moderate to high Froude numbers on the free surface of an ideal liquid, could be successfully addressed by the Michell (1898) or Stretenskii (1936) thin ship theory, respectively. As a consequence, the wave resistance of a ship could be directly related to her hull form characteristics, thus enabling to set up efficient hull form optimization procedures (see, e.g., Papanikolaou and Androulakakis, Proc. FAST'91 Conf.). In the practice of catamaran design, however, many hull forms, developed mainly by intuition or semi-empirical procedures, are adopting non-symmetric demi-hull features, therefore an extension of the original Michell's and Stretenskii's theory seems, at least for the purpose of understanding the physics of the problem, long time overdue. The present paper addresses exactly this classical problem of wave resistance theory by extending Michell's approach to consider non-symmetric hulls through inclusion of a normal dipole distribution on the demi-hull's centerplane and by deriving an approximate solution for the resulting first-kind hyper-singular integral equation for the dipole strength. Since the simple structure of Michell's original wave resistance formula could be retained, the present theory for non-symmetric catamaran forms enables the further application of efficient hull form optimization procedures, as known from symmetric hull form design.

**THEORETICAL BACKGROUND**: Consider the potential flow caused by a twin-hull floating body $\mathcal{B}$ moving with constant forward speed $U$ on the, otherwise undisturbed, free surface of an ideal liquid of infinite depth and extent in a uniform gravitational field ($g$ will indicate the acceleration due to gravity). The body $\mathcal{B} = \mathcal{B}_1 \cup \mathcal{B}_2$ is assumed to be *thin*, i.e., the (common) length $L$ of the two demi-hulls $(\mathcal{B}_1, \mathcal{B}_2)$ is much greater than their (common) beam $B$. This geometrical assumption can be accurately formulated by attributing the role of *the perturbation parameter* $\epsilon$ to the geometrical ratio $B/L$, that is,

$$B/L = \epsilon, \quad 0 < \epsilon \ll 1, \tag{1}$$

and correlating it asymptotically with the second geometrical ratio $T/L$ – $T$ being the (common) draft of the two demi-hulls – and the Froude number $F_r = U/\sqrt{gL}$, which is the fundamental physical non-dimensional parameter of the flow under consideration. In the present work we assume

$$T/L = O(1) \quad \text{and} \quad F_r = O(1) \quad \text{with respect to } \epsilon, \tag{2}$$

where $O$ is the one of the two classical Landau order symbols $O$ (big oh) and $o$ (little oh).

We shall restrict our attention to that part of the flow field, which is time invariant with respect to a Cartesian system of co-odinates $O\tilde{x}_1\tilde{x}_2\tilde{x}_3$ fixed to the body $\mathcal{B}(\epsilon)$ with the $O\tilde{x}_3$ axis drawn vertically upwards, the $O\tilde{x}_1\tilde{x}_2$ plane lying on the quiescent liquid surface and the $O\tilde{x}_1$ pointing to the direction of the motion of the body. With respect to this system reference, the liquid motion is characterized by the velocity potential $\tilde{\Phi}(\tilde{x})$, $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) \in \tilde{\mathcal{D}}$, $\tilde{\mathcal{D}}$ being the region filled by the liquid

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and bounded by the free surface $\partial\tilde{\mathcal{D}}_F$, taken to be a non-parametric surface with respect to quiescent liquid surface $\tilde{x}_3 = 0$, namely $\tilde{x}_3 = \tilde{\eta}(\tilde{x}')$, $\tilde{x}' = (\tilde{x}_1, \tilde{x}_2)$. Since $\dim(\mathcal{B}(\epsilon)) = \dim(\mathcal{B}(0)$, the problem of determining $\tilde{\Phi}(\tilde{x})$ and $\tilde{\eta}(\tilde{x}')$ can be handled by appealing to the techniques of *regular perturbation theory*. Introducing the non-dimensional variables: $x_\ell = \tilde{x}_\ell/L$, $\ell = 1,2,3$, $\Phi(x) = \tilde{\Phi}(\tilde{x})/g^{1/2}L^{2/3}$ and $\eta(x') = \tilde{\eta}(\tilde{x}')/L$, and exploiting the fact that, for a thin hull with slowly varying shape lengthwise, the $x_1$-component of its outer (with respect to the liquid) normal vector $\mathbf{n} = (n_1, n_2, n_3)$ is asymptotically small, namely $n_1 = O(\epsilon)$, we arrive after simple asymptotic reasoning at the following result: **the velocity potential $\Phi(x;\epsilon)$ and the free-surface elevation $\eta(x';\epsilon)$ admit of the following asymptotic approximations:**

$$\Phi(x;\epsilon) = \Phi_0(x;\epsilon) + o(\Phi_0(x;\epsilon)), \quad \Phi_0(x;\epsilon) = O(\epsilon), \quad x \in \mathcal{D}(0), \tag{3}$$

$$\eta(x';\epsilon) = \eta_0(x';\epsilon) + o(\eta_0(x';\epsilon)), \quad \eta_0(x';\epsilon) = O(\epsilon), \quad x \in \partial\mathcal{D}_F(0), \tag{4}$$

where

$$\eta_0(x';\epsilon) = F_r\,\Phi_{0,1}(x;\epsilon), \quad x' \in \partial\mathcal{D}_F(0), \tag{5}$$

**and $\Phi_0(x;\epsilon)$ satisfies the following boundary value problem:**

$$\Phi_{0,11} + \Phi_{0,22} + \Phi_{0,33} = 0, \quad x \in \mathcal{D}(0), \tag{6a}$$

$$\left\{\begin{array}{lll} \Phi_{0,2} = -\epsilon F_r\, f_{(1 \text{ or } 2),1} & \text{as } x_2 \to (- \text{ or } +)s+ \\ \Phi_{0,2} = +\epsilon F_r\, f_{(2 \text{ or } 1),1} & \text{as } x_2 \to (- \text{ or } +)s- \end{array}\right\}, \quad x \in \partial\mathcal{D}^w_{B_{(1 \text{ or } 2)}}(0), \tag{6b}$$

$$\mathcal{K}^{-1}\Phi_{0,11} + \Phi_{0,3} = 0, \quad x \in \partial\mathcal{D}_F(0), \quad \mathcal{K} = F_r^2, \tag{6c}$$

$$\Phi_{0,\ell} \to 0, \qquad \ell = 1,2,3, \quad x_3 \to -\infty, \tag{6d}$$

*Radiation condition* $(R)$ : the energy flux associated with the disturbance of the moving body is directed away towards $x_1 \to -\infty$. (6e)

The subscript $\ell$ after a comma denotes partial differentiation with respect to the space variable $x_\ell$, while $\mathcal{D}(0)$ denotes the open domain bounded by the quiescent liquid surface $x_3 = 0$ and the flat surface pieces $\partial\mathcal{D}^w_{B_i}(0)$, $i = 1,2$, to which degenerates the wetted surface of the two demi-hulls at the limit $\epsilon = 0$. $\partial\mathcal{D}^w_{B_1}(\epsilon)$ is represented as

$$\left\{\begin{array}{ll} x_2 = -s + \epsilon\, f_1(x_1, x_3), & x_2 \geq -s \\ x_2 = -s - \epsilon\, f_2(x_1, x_3), & x_2 \leq -s \end{array}\right\}, \quad (x_1, x_3) \in \Omega, \tag{7}$$

with $\Omega$ being the projection of $\partial\mathcal{D}^w_{B_1}(0)$ onto the center-plane $x_2 = 0$, and analogously for $\partial\mathcal{D}^w_{B_2}(0)$. Finally, $\partial\mathcal{D}_F(0)$ denotes the open planar domain $\{x_3 = 0\}\backslash\cup^2_{i=1}\partial\mathcal{D}^w_{B_i}(0; x_3 = 0)$, where $\partial\mathcal{D}^w_{B_i}(0; x_3 = 0)$ is the slit representing the degenerate waterline of $\partial\mathcal{D}^w_{B_i}(0)$, $i = 1,2$.

Let $G(x;\xi)$, $x = (x_1, x_2, x_3)$, $\xi = (\xi_1, \xi_2, \xi_3)$, be the Green function, also referred to as *the Kelvin source*, associated with the Laplace field equation (6a), the linearized free-surface condition (6c), the "bottom" boundary condition (6d) and the radiation condition (6e). Furthermore, let the separation distance $2s$, between the axes of the two demi-hulls, be large enough so that local interference effects can be neglected. It can then be proved, with the aid of potential theory, that: **the leading-order potential $\Phi(x_0;\epsilon)$ can be approximately decomposed as below:**

$$\Phi_0(x;\epsilon) \approx \Phi_{0,1}(x;\epsilon) + \Phi_{0,2}(x;\epsilon), \quad x \in \mathcal{D}(0), \tag{8}$$

where

$$\Phi_{0i}(x;\epsilon) = \epsilon\frac{F_r}{4\pi}\int_{\partial\mathcal{D}^w_{B_i}(0)} G(x;\xi)(f_{1,1} + f_{2,1})\,d\xi - \frac{(-1)^i}{4\pi}\int_{\partial\mathcal{D}^w_{B_1}(0)} \nu_i(\xi;\epsilon)\frac{\partial G(x;\xi)}{\partial \xi_2}\,d\xi, \; i = 1,2. \tag{9a}$$

**Here $\nu_1(x;\epsilon)$ is the solution of the following hyper-singular integral equation of the first kind:**

$$\epsilon F_r(f_{2,1} - f_{1,1}) = \frac{1}{4\pi} \oint_{\partial\mathcal{D}^w_{B_1}(0)} \nu_1(\xi;\epsilon) \frac{\partial^2 G(x;\xi)}{\partial x_2 \partial \xi_2} d\xi, \quad x \in \partial\mathcal{D}^w_{B_1}(0), \tag{9b}$$

**with the double-dash signifying the finite part of the indicated integral, according to Hadamard, and**

$$\nu_2(x_1, s, x_3;\epsilon) = \nu_1(x_1, -s, x_3;\epsilon), \quad (x_1, x_3) \in \Omega. \tag{9c}$$

An asymptotic approximation of the dipole distribution $\nu_1(x;\epsilon)$ can be easily obtained by assuming that the demi-hulls $\mathcal{B}_i(\epsilon)$, $i = 1,2$ are not only thin but *a-bit-slender* too, i.e.,

$$T/L = \epsilon_1, \quad \epsilon_1 = \epsilon^a, \ 0 < a < 1. \tag{10}$$

Assume now that $\nu_1(\xi_1, \xi_3)$ is twice continuously differentiable in $\Omega$ and take its two-term Taylor expansion around the point $(x_1, x_3)$. Substituting this expansion into the right-hand side of (9b), and taking into account the following expression for the Kelvin source:

$$G(x;\xi) = \frac{1}{|x-\xi|} + H_{reg}(x;\xi), \quad x_3, \xi_3 \in (-\infty, 0], \tag{11}$$

where $H_{reg}(x;\xi)$ is the so-called *regular part* of the Kelvin source, we arrive at an integro-differential equation for $\nu_1(x_1, x_3)$. After careful asymptotic analysis, one can show that **the formula:**

$$\nu_1^{approx}(x_1, x_3) = 4\pi\epsilon F_r(f_{2,1} - f_{1,1}) \left[ \oint_{\partial\mathcal{D}^w_{B_1}(0)} \frac{\partial^2}{\partial x_2 \partial \xi_2} \left( \frac{1}{r(x;\xi)} \right) \right]^{-1}, \quad (x_1, x_3) \in \Omega, \tag{12}$$

**provides indeed an asymptotic approximation of the solution of this equation. Obviously, analogous results can be drawn for the dipole distribution $\nu_2(x_1, x_3)$; see equ. (9c).**

**<u>NUMERICAL RESULTS</u>**: Combining formulae (8), (9a) and (12) with the well known Kochin's formula (Kochin, 1936), we obtain an approximation for the wave resistance of twin-hull bodies, whose demi-hulls are thin and a-bit-slender in the sense specified by the asymptotic estimates (1) and (10). The numerical performance of this approximation is illustrated in Fig. 1 for a symmetric (type-B: symmetric Wigley demi-hulls) and two non-symmetric catamarans (type-A (-C): halved Wigley demihulls with outwards (inwards) flat face) with common geometric characteristics ($L = 1.0\ m$, $B = 0.1\ m$, $T = 0.2\ m$) and displacement ($\nabla = 8.889\ lt$), and two different tunnel widths ($w/L =$ 0.3 and 0.5). The depicted results suggest that, for higher Froude numbers ($Fr > 0.4$) type-A hulls give the lowest wave resistance coefficients, whereas for moderate Froude numbers ($0.30 < Fr < 0.40$) type-C seems superior to the others. In all cases the symmetric hull form arrangement (type-B) is between the others and exhibits a good overall performance.

Further numerical results can be found in Kaklis and Papanikolaou (1992) and Spanos (1995). The above results are qualitatively supported by existing experimental data for catamaran models with non-symmetric demi-hulls. It seems, however, necessary to validate the present theory by systematic model experiments with simplified standard hull forms (halved WIGLEY and strut-type hulls), before implementing the procedure into a systematic hull-form optimization scheme.

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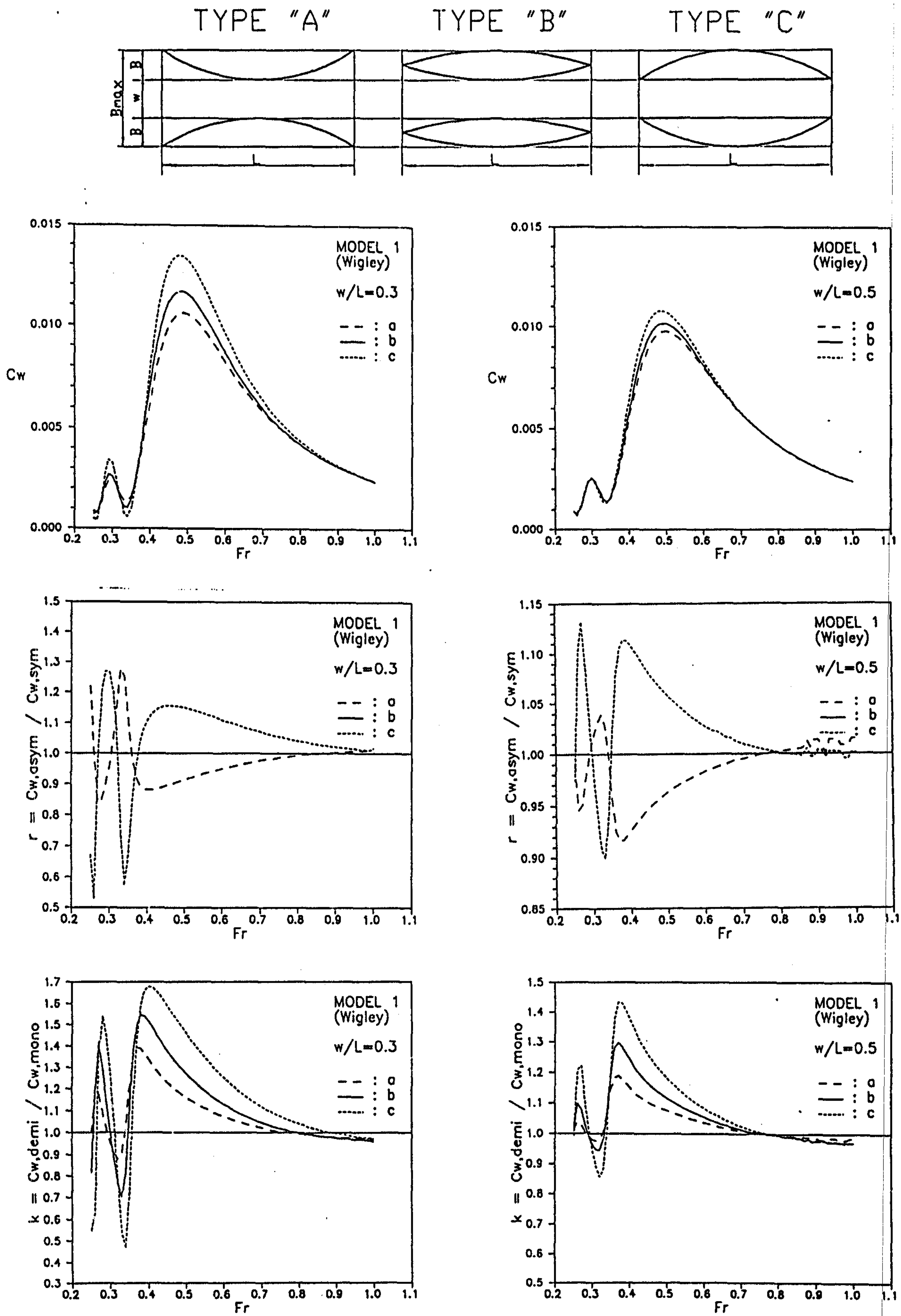


**Fig 1.** The influence of the demi-hull asymmetry on the wave resistance coefficient $c_w$ of thin catamarans ( $r$: asymmetry influence ratio, $k$: demi-hull interaction coefficient).