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On energy arguments applied to the slamming force

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For the sake of geometric simplicity, we consider the case of a symmetric body entering calm water vertically, so that the impact force is vertical.

When it is derived through conservation of momentum, the impact force is usually given as

$$F = -U \frac{d}{dt} (M_a)$$

U being the impact velocity, and M_a the 'added-mass' (e.g. see Faltinsen, 1990, p. 286).

Apparently conservation of the kinetic energy (potential energy being negligible at the initial time of impact) leads to

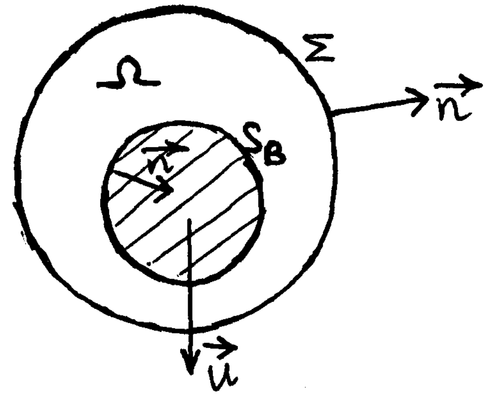
$$F = -\frac{1}{2} U \frac{d}{dt} (M_a)$$

(e.g. see Miloh & Shukron, 1991), in disagreement with the previous expression.

A first idea to resolve this contradiction could be that the added masses these two expressions refer to are not the same; it is known that so-called wetting correction introduces a factor 2 that has been omitted by some investigators. However this is not the explanation here, as we shall see.

1. Rigid boundaries

In this first section we consider a body moving inside a fluid domain limited by solid boundaries. A simple case, illustrated below, is that of a cylinder moving inside another cylinder, along a diameter.



1.a Momentum conservation

Integration of the pressure p on S_B gives the hydrodynamic force

$$\vec{F} = -\rho \iint_{S_B} \left(\Phi_t + \frac{1}{2} (\nabla\Phi)^2 \right) \vec{n} \, ds$$

Following Newman (1977, pp 132-134), we can rewrite $\vec{F}$ as

$$\vec{F} = -\rho \frac{d}{dt} \left(\iint_{S_B} \Phi \, \vec{n} \, ds \right) - \rho \iint_{\Sigma} \left(\frac{\partial \Phi}{\partial n} \nabla\Phi - \frac{1}{2} (\nabla\Phi)^2 \, \vec{n} \right) ds$$

Σ being fixed, and by definition of the added-mass M_a:

$$\vec{F} = -\frac{d}{dt} \left(M_a \, \vec{U} \right) + \rho \iint_{\Sigma} \frac{1}{2} (\nabla\Phi)^2 \, \vec{n} \, ds$$

1.b Energy arguments

The kinetic energy in the fluid is

$$\begin{aligned} E_C &= \frac{1}{2} \rho \iiint_{\Omega} (\nabla\Phi)^2 \, dv = \frac{1}{2} \rho \iint_{S_B \cup \Sigma} \Phi \, \frac{\partial \Phi}{\partial n} \, ds \\ &= \frac{1}{2} \rho \iint_{S_B} \Phi \, \frac{\partial \Phi}{\partial n} \, ds = \frac{1}{2} M_a \, U^2 \end{aligned}$$

Its time derivative is equal to

$$\begin{aligned} \frac{dE_C}{dt} &= \frac{d}{dt} \left(\frac{1}{2} \rho \iiint_{\Omega} (\nabla\Phi)^2 \, dv \right) \\ &= \frac{1}{2} \rho \iiint_{\Omega} \frac{\partial}{\partial t} (\nabla\Phi)^2 \, dv + \frac{1}{2} \rho \iint_{S_B \cup \Sigma} (\nabla\Phi)^2 \, \Phi_n \, ds \\ &= \rho \iiint_{\Omega} \nabla\Phi_t \, \nabla\Phi \, dv + \frac{1}{2} \rho \iint_{S_B} (\nabla\Phi)^2 \, \vec{U} \cdot \vec{n} \, ds \\ &= \rho \iint_{S_B \cup \Sigma} \Phi_t \, \nabla\Phi \cdot \vec{n} \, ds + \frac{1}{2} \rho \iint_{S_B} (\nabla\Phi)^2 \, \vec{U} \cdot \vec{n} \, ds \\ &= \rho \iint_{S_B} \left(\Phi_t + \frac{1}{2} (\nabla\Phi)^2 \right) \vec{U} \cdot \vec{n} \, ds \\ &= -\vec{F} \cdot \vec{U} \end{aligned}$$

Hence, in the simple case considered here:

$$F = -\frac{1}{2U} \frac{d}{dt} \left(M_a \, U^2 \right)$$

To summarize, momentum conservation gives

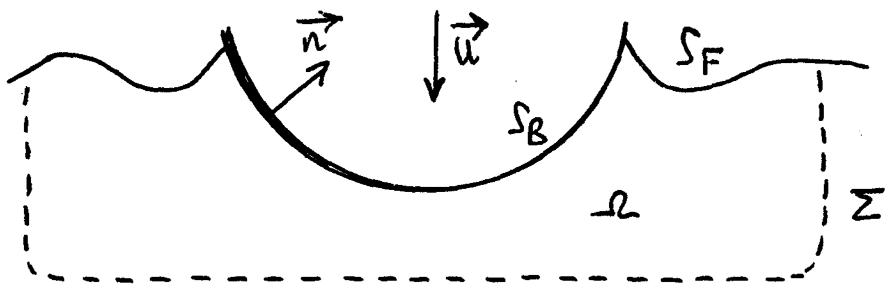
$$F = -M_a \frac{\partial U}{\partial t} - U \frac{\partial M_a}{\partial t} + \frac{1}{2}\rho \iint_\Sigma (\nabla\Phi)^2 \, n_z \, ds \tag{1}$$

while energy conservation gives

$$F = -M_a \frac{\partial U}{\partial t} - \frac{1}{2} U \frac{\partial M_a}{\partial t} \tag{2}$$

the second expression being apparently much simpler to use. It has been checked numerically that, in the simple example considered (a circular cylinder moving inside a larger one), they do yield identical results.

2. The impact problem



Now we consider the case of a symmetric body entering initially calm water at high speed.

The previous analysis based on momentum conservation remains valid if we replace S_B with $S_B \cup S_F$, since the free surface S_F is a material surface at zero pressure. Taking Σ to infinity gives the slamming force

$$F = -M_a \frac{\partial U}{\partial t} - U \frac{\partial M_a}{\partial t} \tag{3}$$

Usually, in water impact problems, the free surface conditions are simplified into $\Phi = \Phi_t = 0$ at $z = \eta$. As a result the kinetic energy

$$E_C = \frac{1}{2}\rho \iiint_\Omega (\nabla\Phi)^2 \, dv = \frac{1}{2}\rho \iint_{S_B \cup S_F} \Phi \frac{\partial \Phi}{\partial n} \, ds$$

reduces apparently to

$$E_C = \frac{1}{2}\rho \iint_{S_B} \Phi \frac{\partial \Phi}{\partial n} \, ds = \frac{1}{2} M_a U^2$$

so that we obtain, as before:

$$F = -M_a \frac{\partial U}{\partial t} - \frac{1}{2} U \frac{\partial M_a}{\partial t} \tag{4}$$

different from (3), with, apparently, no hope, this time, to resolve the contradiction.

The problem actually results from the fact that the correct free surface condition $\Phi_t + 1/2 \, (\nabla\Phi)^2 = 0$ (gravity playing no role at the initial instant) has been replaced with $\Phi = \Phi_t = 0$, which is in fact valid only in an outer domain (away from the body and free surface intersection). Armand & Cointe

(1986), retaining both terms, have solved the inner problem (for a circular cylinder of radius R), and obtained two jets with velocity:

$$V_j = 2\left(\frac{U\,R}{t}\right)^{1/2} = 2\,U\,\epsilon^{-1} = 2\,V_{sr}$$

and thickness:

$$\delta_j = \frac{\pi\,R}{4}\left(\frac{U\,t}{R}\right)^{3/2} = \frac{\pi\,R}{4}\,\epsilon^3$$

based on a small parameter $\epsilon = (U\,t/R)^{1/2}$. V_{sr} is the spray root velocity, equal to half the velocity in the jet.

From these expressions it can be checked that the mass and momentum fluxes through the jets are of orders ϵ^2 and ϵ, hence negligible. However the energy flux, be G, is not negligible:

$$\begin{aligned} G &= 2\,\delta_j\,\frac{1}{2}\,\rho\,V_j^2\,(V_j - V_{sr}) \\ &= \rho\,\pi\,R\,U^3 \\ &= \frac{1}{2}\,U^2\,\frac{\partial M_a}{\partial t} \end{aligned}$$

These expressions are based on the solution with wetting correction, the added mass being then equal to

$$M_a = 2\,\pi\,\rho\,U\,t\,R$$

(e.g. Faltinsen, 1990, pp. 286-288).

Hence we obtain that as much kinetic energy has been imparted to the jets as to the remainder of the fluid. This solves the contradiction and the correct force expression, for the impact problem, is

$$F = -U\,\frac{\partial M_a}{\partial t}$$

Since equations (3) and (4) are general, it may be concluded that, in all impact situations, at the initial instant, the kinetic energy is equally transfered to the jets and to the bulk of the fluid.

References

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4. J.N. Newman, 1977. *Marine hydrodynamics*, MIT Press.

DISCUSSION

Korobkin: The problem analysed in the report is connected with the effect of "energy loss" under the impact on the free surface of an incompressible liquid. This effect was analysed in my paper published in the 20th Symposium of Naval Hydrodynamics (Santa Barbara, 1994) for an arbitrary body (plane case). The kinetic energy of the spray jets was calculated and the relation presented in the report was derived as well. The axisymmetrical case was also considered. The relation between the kinetic energy of the spray sheet and the energy of the liquid bulk is the same as for the plane case. The distribution of the energy under the impact by a floating body (initial wetted area is not zero) was studied in my report at the 10th WWWFB (Oxford, 1995). The same problem but for the impact by a vertical wall (impulsively started wavemaker) was analysed in my paper published 2 months ago (JFF, Vol.307, 1996). In all three cases the explanations of the "lost energy" effect are different.