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A Study on a Higher-Order Rankine Panel Method for Nonlinear Steady Wave Making Problems

by Xin LIN and Mikio TAKAKI
Hiroshima University, JAPAN

1 Introduction

Nonlinear steady ship wave problems have been studied more than ten years. In most early works, the problem is solved by the mixed Eulerian-Lagrangian method or direct N-S solvers in time domain. Accuracy of these methods is dominated by the scheme of time integration that is quite difficult to improve. In this paper, a velocity domain method is proposed for solving the nonlinear steady wave making problem. Velocity stepping is used instead of time stepping and bi-cubic B-spline panels with Rankine source Green's function are applied in the direct boundary element method in present study.

2 Theory

We consider a ship advancing at constant forward speed U in an incompressible, inviscid and irrotational fluid domain. The x-axis of Cartesian frame fixed with the ship points to the direction of the ship forward velocity, and z-axis is positive upward. S_h denotes the wetted hull surface and S_f denotes the free surface. The normal vector of the boundary surfaces $\boldsymbol{n}$ is taken to point inward the fluid domain Ω. The velocity field $\boldsymbol{V}(\boldsymbol{r})$ is determined by its potential $\Phi = -x + \phi$ like $\boldsymbol{V} = U\nabla\Phi$. Free surface condition and the wave field are given by

$$\left.\begin{array}{r} \dfrac{1}{2K_0(F_n)}\nabla\Phi\cdot\nabla(\nabla\Phi\cdot\nabla\Phi) + \nabla z\cdot\nabla\Phi = 0 \\ \zeta = -\frac{1}{2}\left(\nabla\Phi\cdot\nabla\Phi - 1\right) \end{array}\right\} \quad \text{on } z=\zeta \tag{1}$$

where $K_0(F_n) = 1/F_n^2 L$. This nonlinear problem is solved progressively from the homogeneous solution of zero speed case to the high velocity region. The velocity potential of each new velocity step is solved iteratively based on the potential of previous velocity step. On condition that the velocity increment ΔF_n is small, the potential increment of each iteration φ can be assumed to satisfy the following linearized boundary value problem

$$\nabla^2\varphi = 0 \qquad \text{in } \Omega \tag{2}$$

$$\frac{\partial\varphi}{\partial n} = 0 \qquad \text{on } S_h \tag{3}$$

$$\begin{array}{r} \nabla\Phi\cdot\nabla\left(\nabla\Phi\cdot\nabla\varphi\right) + \frac{1}{2}\nabla\varphi\cdot\nabla\left(\nabla\Phi\cdot\nabla\Phi\right) \\ -C(\Phi)\nabla\Phi\cdot\nabla\varphi + K_0(F_n+\Delta F_n)\nabla z\cdot\nabla\varphi \\ +\left[K_0(F_n+\Delta F_n) - K_0(F_n)\right]\left[\nabla z\cdot\nabla\Phi - C(\Phi)\zeta(F_n)\right] = 0 \end{array} \qquad \text{on } z=\zeta \tag{4}$$

where Φ and ζ are the potential and wave amplitude of former iterative step. Coefficient $C(\Phi)$ is a function of known Φ as

$$C(\Phi) = -\frac{[K_0(F_n+\Delta F_n)-K_0(F_n)]\,\nabla z\cdot\nabla(\nabla z\cdot\nabla\Phi)}{K_0(F_n+\Delta F_n)+\nabla\Phi\cdot\nabla(\nabla z\cdot\nabla\Phi)} \tag{5}$$

The increment of wave amplitude $\Delta\zeta$ is given by

$$\Delta\zeta = -\frac{\nabla\Phi\cdot\nabla\varphi+[K_0(F_n+\Delta F_n)-K_0(F_n)]\,\zeta}{K_0(F_n+\Delta F_n)+\nabla\Phi\cdot\nabla(\nabla z\cdot\nabla\Phi)} \qquad \text{on } z=\zeta \tag{6}$$

$\varphi(\boldsymbol{r})$ is determined by the following integral equation

$$\alpha(\boldsymbol{r})\varphi(\boldsymbol{r}) - \iint_{S_h\cup S_f}\varphi(\boldsymbol{r}')\frac{\partial G(\boldsymbol{r};\boldsymbol{r}')}{\partial n'}ds + \iint_{S_f}G(\boldsymbol{r};\boldsymbol{r}')\frac{\partial\varphi(\boldsymbol{r}')}{\partial n'}ds = 0 \tag{7}$$

in which $\alpha(\boldsymbol{r})$ is the interior solid angle at field point $\boldsymbol{r}$, $G(\boldsymbol{r},\boldsymbol{r}') = 1/|\boldsymbol{r}-\boldsymbol{r}'|$ and S_f is defined by $z=\zeta$.

The iteration is repeated until the nonlinear free surface condition (1) is satisfied.

3 Numerical Method and Implementations

Notice that continues second derivatives in condition (4) is required, fourth order B-spline panels are adopted in the boundary element method for both the geometry of ship hull, the wavy free surface and the potential discretizations. The boundary surfaces and potential on them can be expressed as

$$\boldsymbol{Q}(u,v) = \sum_{i=1}^{N_u}\sum_{j=1}^{N_v}\boldsymbol{q}_{i,j}B_{i,4}(u)B_{j,4}(v) \tag{8}$$

where $B_{i,4}(u)$ is the one-dimensional B-spline basis function of fourth order along u-direction, function $\boldsymbol{Q}$ and its polygon vertices $\boldsymbol{q}_{i,j}$ are of

$$\boldsymbol{Q} = \left\{\begin{array}{c} x \\ y \\ z \\ \Phi \end{array}\right\}, \qquad \text{and} \qquad \boldsymbol{q}_{i,j} = \left\{\begin{array}{c} \alpha_{i,j} \\ \beta_{i,j} \\ \gamma_{i,j} \\ \lambda_{i,j} \end{array}\right\} \tag{9}$$

The derivatives with respective to u and v can be simply obtained as

$$\frac{\partial^n\boldsymbol{Q}}{\partial u^n} = \sum_{i=1}^{N_u}\sum_{j=1}^{N_v}\boldsymbol{q}_{i,j}\frac{\partial^n B_{i,4}(u)}{\partial u^n}B_{j,4}(v) \tag{10}$$

$$\frac{\partial^n\boldsymbol{Q}}{\partial v^n} = \sum_{i=1}^{N_u}\sum_{j=1}^{N_v}\boldsymbol{q}_{i,j}B_{i,4}(u)\frac{\partial^n B_{j,4}(v)}{\partial v^n} \tag{11}$$

Substituting relation (8) as well as (10) and (11) into formula (7) and enforce the boundary condition at collocation points, we can get a system of equations for the unknown potential polygon vectices $\lambda_{i,j}$.

The numerical scheme explained above is of third order and free of numerical damping as been proved by Sclavounos[4].

To realize the proposed method, we must pay attention to the evaluation of second derivative terms. The tangential derivatives can be evaluated from formulas (10) and (11), but the normal one Φ_{nn} that appears in $\nabla z \cdot \nabla (\nabla z \cdot \nabla \Phi)$ of coefficient $C(\Phi)$ of free surface condition (4) needs a special treatment. We have a few methods to evaluate Φ_{nn}. The simplest one in higher-order panel method is taking advantage of Laplace equation. After defining curvilinear frame $u^i = (u^1, u^2, u^3)$ and getting the metric tensor g^{ij} as well as determinant $D = \det \|g_{ij}\|$, we have Laplace equation in this frame like

$$\frac{1}{\sqrt{D}}\frac{\partial}{\partial u^i}\left(\sqrt{D}g^{ij}\frac{\partial \Phi}{\partial u^j}\right) = 0 \tag{12}$$

Unfortunely accuracy of Φ_{nn} evaluated by using this equation is strongly affected by the second tangential derivatives, therefore is not satisfied in some case. In present study an integral equation method is proposed for evaluating Φ_{nn}. For any smooth vector field $\boldsymbol{A}$, we have

$$\alpha(\boldsymbol{r})\boldsymbol{A}(\boldsymbol{r}) = -\iiint_{\Omega} G(\boldsymbol{r},\boldsymbol{r}')\Delta \boldsymbol{A}(\boldsymbol{r}')ds + \iint_{S}\left[\boldsymbol{A}(\boldsymbol{r}')\frac{\partial G(\boldsymbol{r};\boldsymbol{r}')}{\partial n'} - G(\boldsymbol{r};\boldsymbol{r}')\frac{\partial \boldsymbol{A}(\boldsymbol{r}')}{\partial n'}\right] ds \tag{13}$$

$\boldsymbol{r}$ is inside the domain Ω which is bounded by the closed surface S. For the uncommpressible irrotational velocity field

$$\Delta \boldsymbol{V} = \nabla(\nabla \cdot \boldsymbol{V}) - \nabla \times (\nabla \times \boldsymbol{V}) = 0 \qquad \text{in } \Omega \tag{14}$$

Therefore Φ_{nn} can be determined by the integral equation

$$\iint_{S_h \cup S_f} G(\boldsymbol{r};\boldsymbol{r}')\frac{\partial \boldsymbol{V}(\boldsymbol{r}')}{\partial n'}ds = \iint_{S_h \cup S_f} \boldsymbol{V}(\boldsymbol{r}')\frac{\partial G(\boldsymbol{r};\boldsymbol{r}')}{\partial n'}ds - \alpha(\boldsymbol{r})\boldsymbol{V}(\boldsymbol{r}) \tag{15}$$

Evaluation of Φ_{nn} in this integral equation is independent from the second tangential derivatives, therefore the accuracy of Φ_{nn} is much higher than that by using equation (12) in the case of inaccurate evaluation of second tangential derivatives. We confirm this through a calculation of a double-body flow problem for an ellipsoid with $L/B = 5$ and $B/D = 1.2$. 50×9 panels are used on 1/4 body surface. Compare to the analytic results we fine that the maximum relative error of disturbance potential ϕ is smaller than 0.005%, while the maximum relative error of $(m_1, m_2, m_3) = -(\boldsymbol{n} \cdot \nabla)\nabla\phi$ is smaller than 1.0%.

Status This study is still in progress.

Reference

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