Analysis of Oscillating-Water-Column Device using a Panel Method

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1. INTRODUCTION

An oscillating-water-column (OWC) is a device designed to extract energy from the ocean waves. Typically it is a (partially) submerged hollow structure trapping water and air. The water, driven by the waves, moves through the submerged aperture and the moving internal free surface causes the pressure fluctuation of the air in the chamber. The air, in turn, drives turbine while it moves back and forth through contraction. Theories accounting for the wave interactions with OWC were set forth in the early 80's, based on linear theory of water waves and an assumption of simple harmonic oscillation of the air pressure with its mean value equal to atmospheric pressure. Commonly further simplification was made considering only constant vertical displacement of the internal free surface; namely a piston mode. For a review of the theory and application see [1] and [2].

The unique difference of wave interactions with OWC, compared to other structures, is the oscillating pressure acting on the interior free surface. However, in principle, the problem can be analysed using the three dimensional panel method which has been applied extensively to the analysis of wave interactions with a solid boundary. Here we describe two approaches to solve the problem based on a panel method. In the first approach the velocity potentials are evaluated explicitly. The wave source potentials are distributed on the interior free surface as well as on the body surface. A special form of integral equation is suggested which requires only minor modifications on the existing panel codes developed for wave interactions with offshore structures or ships. In the second approach, the physical parameters of interest are evaluated from the conventional diffraction and the radiation solutions of the rigid body motion using Haskind relation and other symmetry relations, without solving for any additional potentials.

The computational results demonstrate that the panel method is applicable to this kind of problem. The results from the two approaches converge to each other. These results are compared with other results obtained from the mode decomposition developed in [3] where the internal free surface is described by a superposition of the piston mode and other oscillatory higher-order modes.

2. ANALYSIS

In accordance with the usual assumptions of the first-order radiation-diffraction theory, the velocity potential Φ exists and the flow field is governed by the Laplace equation

$$\nabla^2 \Phi = 0 \tag{2.1}$$

Assuming regular incident waves, Φ can be expressed in the complex form $\Phi = \mathrm{Re}(\phi e^{i\omega t})$, where Re denotes the real part, ω is the frequency of the incident wave and t is time. A Cartesian coordinate system (x, y, z) is fixed in space with $z = 0$ the undisturbed position of the free surface and the z-axis positive upwards. On $z = 0$, ϕ is subject to

$$\phi_z - \frac{\omega^2}{g}\phi = -\frac{i\omega}{\rho g}p, \tag{2.2}$$

where ρ is the fluid density, g is gravity and p is the complex amplitude of the oscillatory pressure, $P = \mathrm{Re}(pe^{i\omega t})$, acting on the free surface. The constant atmospheric pressure does not contribute to the power transfer. $p = 0$ except on the interior free surface.

To consider a general situation, we suppose that there are M separate interior free surfaces and the pressure may be different on each surface. The velocity potential ϕ then can be expressed as

$$\phi = \phi_D + i\omega \sum_{j=1}^{6} \xi_j \phi_j + i\omega \sum_{j=7}^{6+M} \frac{-p_m}{\rho g}\phi_j. \tag{2.3}$$

Here ϕ_D is the sum of the incident wave potential and the scattered potential due to the presence of the fixed body. ϕ_j denote radiation potentials and they correspond to the mode of rigid body motion when $j = 1, ..., 6$. When $j \geq 7$, ϕ_j are defined to be the potentials due to the pressure on one of the interior free surfaces while no pressure is applied on the others. An index m is used as a pointer for the interior free surfaces and $m = j - 6$. Thus ϕ_j is due to the pressure p_m applied on the m-th interior free surface S_i^m.

The incident wave system is defined by the potential

$$\phi_I = \frac{igA}{\omega} \frac{\cosh[k(z+h)]}{\cosh kh} \exp(-ikx\cos\beta - iky\sin\beta), \tag{2.4}$$

where A is the amplitude, k is the wavenumber defined by the dispersion relation $K \equiv \omega^2/g = k\tanh kh$, h is the fluid depth, and β is the angle between the direction of propagation of the incident wave and the positive x-axis. On the undisturbed position of the body boundary (S_b), the diffraction and radiation potentials are subject to the conditions

$$\phi_{Dn} = 0 \quad \text{and} \quad \phi_{jn} = n_j, \tag{2.5}$$

where $(n_1, n_2, n_3) = \mathbf{n}$, $(n_4, n_5, n_6) = \mathbf{x} \times \mathbf{n}$, $n_j = 0$ for $j \geq 7$ and $\mathbf{x} = (x, y, z)$. The unit normal vector $\mathbf{n}$ is defined to point out of the fluid domain. On $z = 0$, they are subject to

$$\phi_{Dz} - K\phi_D = 0 \quad \text{and} \quad \phi_{jz} - K\phi_j = n_j, \tag{2.6}$$

where $n_j = 0$ except $n_{m+6} = 1$ on S_i^m. Note that n_j is defined on $S_b + S_i \equiv S_{bi}$.

Other than the point-wise value of the velocity potentials, the physical parameters of interest are the forces on the body and the power transfer from the water to air. These are essentially expressed in terms of the added-mass and damping coefficients defined by

$$A_{ij} - \frac{i}{\omega}B_{ij} = \rho \iint_{S_{bi}} n_i \phi_j dS \quad (i, j = 1, 2, ..., 6 + M) \tag{2.7}$$

and the exciting-force components

$$X_i = -i\omega\rho \iint_{S_{bi}} n_i \varphi_D dS \quad (i = 1, 2, ..., 6 + M). \tag{2.8}$$

As discussed in Section 4, $A_{ij} = A_{ji}, B_{ij} = B_{ji}$ and$B_{ii} \geq 0$ for all i and j. They are the same as those of the rigid body motion when $i, j \leq 6$.

With the definitions (2.7-8), the hydrodynamic force component F_i on the body is expressed in the form

$$F_i = X_i + \sum_{j=1}^{6} (\omega^2 A_{ij} - i\omega B_{ij})\,\xi_j + \sum_{j=7}^{M+6} (\omega^2 A_{ij} - i\omega B_{ij})\,(\frac{-p_m}{\rho g}) \quad (i = 1, ..., 6). \tag{2.9}$$

The power transferred across the interior free surface is equal to the time-average of the rate of energy flux

$$\overline{\frac{dE}{dt}} = \sum_{m=1}^{M} \iint_{S_i^m} \overline{P_m \Phi_z} dS = \tfrac{1}{2} \sum_{m=1}^{M} \mathrm{Re}(p_m^* \iint_{S_i^m} \phi_z dS) = \frac{\omega^2}{2g} \sum_{m=1}^{M} \mathrm{Re}(p_m^* \iint_{S_i^m} \phi dS). \tag{2.10}$$

where (2.2) has been invoked. When the body is not fixed, the net rate of energy absorption is obtained by subtracting the rate of work done by the air pressure to the body, denoted by $\partial w/\partial t$, from (2.10) as shown in [3].

The 'capture width', W, is the net energy absorption rate normalized in respect to the corresponding rate of input in the incident-wave system, per unit width of the wave crests, equal to $\frac{1}{2}\rho g A^2 v_g$ where $v_g = d\omega/dk$ is the group velocity. Upon substituting (2.3) into (2.10) and subtracting $\partial w/\partial t$, we have

$$\begin{aligned} W = &-\frac{\omega}{\rho^2 g^2 A^2 v_g} \operatorname{Im} \sum_{m=1}^{M} p_m^* \left[X_m + \sum_{j=1}^{6} (\omega^2 A_{m+6,j} - i\omega B_{m+6,j}) \xi_j \right] \\ &-\frac{\omega^2}{\rho^3 g^3 A^2 v_g} \operatorname{Re} \sum_{m=1}^{M} p_m^* \sum_{n=1}^{M} p_n B_{m+6,n+6} + \frac{\omega}{\rho g A^2 v_g} \operatorname{Im} \sum_{m=1}^{M} p_m^* S_m (\xi_3 + y_m \xi_4 - x_m \xi_5) \, . \end{aligned} \tag{2.11}$$

where S_m, x_m and y_m are the area and x and y coordinates of the centroid of S_i^m, respectively. The optimum values of p_m may be obtained from a system of algebraic equations which consists of the equations of motion and M equations obtained from $\partial W/\partial p_m = 0$.

3. COMPUTATION OF THE VELOCITY POTENTIALS

All components of the exciting forces and the hydrodynamic coefficients may be evaluated directly from (2.7-8). The required velocity potentials are obtained from the following integral equations. With an operator L defined by

$$\mathrm{L}(\phi(\mathbf{x})) = 2\pi\phi(\mathbf{x}) + \iint_{S_b} d\xi \phi(\xi) \frac{\partial G(\xi;\mathbf{x})}{\partial n_\xi}, \qquad \text{for } \mathbf{x} \in S_b, \tag{3.1}$$

and

$$\mathrm{L}(\phi(\mathbf{x})) = 4\pi\phi(\mathbf{x}) + \iint_{S_b} d\xi \phi(\xi) \frac{\partial G(\xi;\mathbf{x})}{\partial n_\xi}, \qquad \text{for } \mathbf{x} \in S_i. \tag{3.2}$$

where G is a wave source potential, the diffraction potential is obtained from

$$\mathrm{L}(\phi_D(\mathbf{x})) = 4\pi\phi_I(\mathbf{x}) \quad \text{for } x \in S_{bi}, \tag{3.3}$$

and the radiation potentials from

$$\mathrm{L}(\phi_j(\mathbf{x})) = \iint_{S_{bi}} n_j G(\mathbf{x};\xi) \quad \text{for } x \in S_{bi}. \tag{3.4}$$

It is customary to solve (3.3-4) only for $x \in S_b$ for the unknown potentials on S_b and then to evaluate potentials on S_i from (3.3-4) for $x \in S_i$. However, we find it is easier to extend the existing panel codes by assuming that the potentials are unknown on S_{bi}. Computationally, however, the latter entails more CPU because of the increase of the dimensions of the unknowns.

4. SYMMETRY AND HASKIND RELATIONS

As noted in [1], B_{ij} in the second term of (2.11) can be evaluated from the Haskind relations. Thus we only need to solve for the diffraction potential to evaluate the exciting forces (2.8). All other hydrodynamic coefficients in (2.9) and (2.11) can be evaluated from the rigid body radiation potentials because of the symmetry property of A_{ij} and B_{ij} as shown below.

First we consider the hydrodynamic force on the body in mode i due to the pressure on S_i^m. This is the case when $i \leq 6$ and $j = m + 6 \geq 7$. Using boundary conditions (2.5) and (2.6), we have

$$\begin{aligned} X_{ij} &= -i\omega\rho \iint_{S_b} \phi_j n_i dS = -i\omega\rho \iint_{S_b} (\phi_j \phi_{in} - \phi_{jn} \phi_i) dS \\ &= -i\omega\rho \iint_{S_i} (\phi_{jn} \phi_i - \phi_j \phi_{in}) dS = -i\omega\rho \iint_{S_i} (\phi_{jz} - K\phi_j) \phi_i dS = -i\omega\rho \iint_{S_i^m} n_j \phi_i dS. \end{aligned} \tag{4.1}$$

This implies the symmetry of the hydrodynamic coefficients defined in (2.7). Similarly, it can be shown that A_{ij} and B_{ij} are symmetric for $i, j \geq 7$. Since the wave elevation $\zeta = -i\omega\phi/g$ due to the balance between the hydrodynamic pressure and the gravitational force, the last term in (4.1) is merely the weight of the net oscillatory volume of the water on S_i^m caused by ϕ_i.

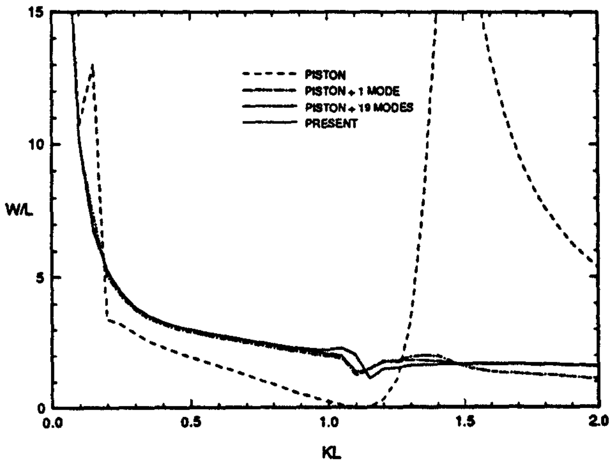
Finally the Haskind relations can be derived in the similar manner as in [1], with the result

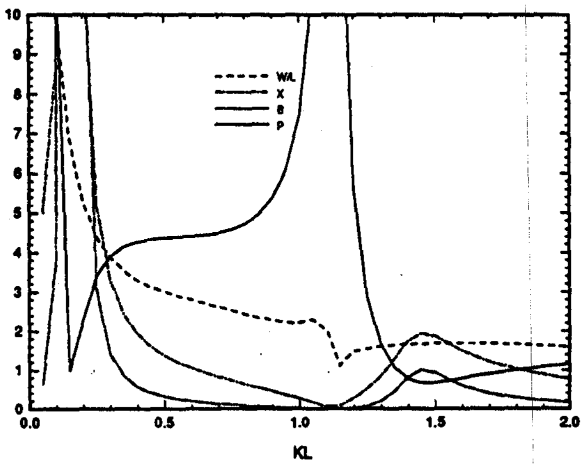
$$B_{ij} = \frac{k}{8\pi\rho g A^2 v_g} \mathrm{Re} \int_0^{2\pi} X_i X_j^* d\theta. \tag{4.2}$$

Thus B_{ii} is non-negative.

5. NUMERICAL RESULT

An example of the computational results is shown below. The OWC is a box shape with one internal free surface and assumed fixed on the free surface in the infinite water depth. The external horizontal dimension is 20m, the draft is 5m and the wall thickness is 0.5m. The aperture is on the bottom half of the weather side wall ($-2m > z > -4.5m$) with the breadth the same as that of the OWC's internal dimension: 19m. The outer and inner surfaces of the OWC are discretized with 1520 and the internal free surface with 256 panels. The figure on the left shows the nondimensional optimum capture width calculated using the approach in Section 3. Also shown are the piston-mode approximation and the additions of higher-order modes. The first higher-order mode is the lowest antisymmetric Fourier mode in the wave direction. The figure on the right shows the modulus of the nondimensional exciting force ($X_7/\rho g A^2 L$), the damping coefficient ($B_{77}/\rho L^3 \omega$) and the optimum pressure ($P_1/\rho g A$), where $L = 10m$. It shows two kinds of resonances: a low-frequency piston-mode resonance with large flux of water and a high-frequency sloshing mode.





6. ACKNOWLEDGEMENT

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