Volume of Fluid Method (VOF)
Applied to Non-Linear Wave Problems on Body-Fitted Grids

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Abstract

A computational code for solving three-dimensional non-linear wave problems is presented in this paper. The flowfield and the unknown free surface location are calculated by coupling the Reynolds-averaged Navier-Stokes/Euler equations with a Volume Of Fluid (VOF) method for tracking the free surface. The method is based on a finite-volume technique. The Euler or Navier-Stokes equations are solved using a pseudo-compressibility approach for unsteady (dual time-stepping) and steady flows on an eulerian curvilinear grid. Numerical results illustrate the solution of the Euler equations, on one hand, for unsteady confined problem such as sloshing in tanks and, on the other hand, for steady waves on a submerged airfoil in infinite domain. The test cases are compared with experimental and numerical results. The model is valid for viscous flows and its implementation for the Navier-Stokes equations is being conducted.

1 Introduction

A CFD code, EOLE, has been developed since these last five years. Two different methods for tracking free surface are implemented:
• an adaptative grid method [3],
• the Volume Of Fluid method using a fixed but curvilinear mesh which is well adapted to tight walls (ship hulls, tank sides).
In the first method, the free surface is treated by means of a moving boundary fitted to the mesh. Its advantages are a good conservation of the bounded fluid volume, a precise representation of the free surface, but it is limited to track complicated free surfaces such as plunging breaking. That is the reason why a method is developed, in which the Navier-Stokes/Euler equations are coupled with the VOF method, more efficient for treating complicated free boundaries configurations.
All these developments preserve the capability to conduct multi-domains calculations. The advantage of such a software architecture is to solve the Navier-Stokes equations in a sub-domain, for example, around the hull of a boat, matched to a second one in which the fluid viscosity can be neglected and the Euler equations solved. These equations have as intrinsic solutions some discontinuities, such as vortex sheets, which are generated in the Navier-Stokes region and convected into the Euler region. In this way, the rotationality of the flow is well represented. The interest of the numerical technique is that the matching of the solutions between several domains is straight-forward, thanks to the resolution method, valid for viscous as well as inviscid flows.

2 Pseudo-compressibility method

A method of pseudo-compressibility for solving the steady incompressible Euler or Navier-Stokes equations was initially developed [2]. This is an iterative method introducing into the continuity and the momentum equations some derivatives with respect to a fictitious time called pseudo-time τ, which imposes an hyperbolic behaviour in τ to the differential system. The basis of this technique for computing steady flows is to use the evolutionary equations governing unsteady flows and to obtain the steady solution as the asymptotic limit of the pseudo-transient solution, when time goes to infinity.
This method of pseudo-compressibility was then extended [4] for the computation of unsteady flows by means of a second order fully implicit time scheme.
The numerical method, for pseudo-time integration, is based on the method previously developed by Jameson et al. [1] and Vatsa [8]. The main features are a finite-volume method based on a space-centered scheme, second and fourth order artificial viscosity terms, five stages Runge-Kutta time stepping and implicit residual smoothing.

3 VOF method

The VOF method is based on a concept of a fractional volume of fluid, as previously proposed by Hirt and Nichols [5]. The fraction of fluid in each cell is representated by a function F, whose value is unity in a cell full of fluid, while a zero value indicates that the cell contains no fluid. Cells with F values between zero and one must then contain a free surface.
The evolution of the F field is governed by the following transport equation, written in a conservative form :

$$\frac{\partial F}{\partial t} + \frac{\partial uF}{\partial x} + \frac{\partial vF}{\partial y} + \frac{\partial wF}{\partial z} = 0 \qquad (1)$$

where t is the physical time, (x, y, z) the cartesian system, (u, v, w) cartesian components of the velocity.
The evolution of the function F in each cell is made from fluxes calculations of F through all the faces of a cell. The technique to calculate the fluxes is based on the donor-acceptor method [5].
This technique is extended for the 3D case, using the basic assumptions of the original method. A further development is implemented to deal with arbitrary block-structured meshes using curvilinear coordinates transformation in order to obtain a good description of the velocity profile in the boundary layer and to calculate the hydrodynamic loads with a great accuracy.
The transport equation (1), when expressed in the local basis attached to the grid, still keeps an identical form, with the cartesian velocity components replaced by the contravariant ones. This identity permits to transpose the cartesian algorithm with slight modifications in the F fluxes calculations.

4 Coupling algorithm between the fluid equations and VOF method

The coupling of the unsteady Euler/Navier-Stokes equations with the VOF method was performed first. At the end of each time step, when the convergence of the velocity and the pressure fields are reached, the transport equation of the F field (1) is solved.
Nevertheless, as the purpose of this research is to adress the steady problem of the wave resistance of an advancing hull in calm water, the steady Navier-Stokes/Euler equations are now considered. The derivative with respect to the physical time t in the formula (1) is replaced by a derivative with respect to the pseudo-time τ. The numerical integration scheme is identical.

The reason of this choice is to calculate the steady solution, avoiding the resolution of the unsteady algorithm in order to reduce computational cpu time. Thus, we solve the mass and the momentum equations and the transport equation of F at each iteration of pseudo-time.

5 Results

Some numerical results are presented to show that the VOF method may be used to track the free surface for both steady or unsteady problems.

1- Unsteady simulation of sloshing in tanks : These numerical simulations have been realized by solving the unsteady Euler equations coupled with the VOF method. For a tank in a surge motion in 2D, we compare numerical solutions given by the method of the adaptative mesh and the VOF method. The method of adaptative mesh has been validated by analytical solutions given by the potential theory [7].
Three-dimensional Euler computations have then been realized to represent the sloshing in a tank in an oscillated roll motion around one of the diagonales of the bottom of the tank. This test case allows to generalize the algorithm to 3D without taking into account far-field boundaries conditions, since a zero normal velocity condition is applied on solid surfaces. The convergence speed of the numerical simulation is fast at each time step: a four order reduction of the residuals is achieved in 30 iterations. Figure 1 illustrates the non-linear free surface elevation at a fixed instant.

2- Steady waves on a submerged NACA0012 profile in finite depth : Steady Euler equations are solved to calculate flow around the NACA0012 wing section with 5 degrees attack angle under the free surface. The computations results are compared [6] with experimental results and other numerical solutions obtained by Hino and Liu by solving the Navier-Stokes equations.
A non-reflexion condition is fixed at the upstream boundary, using the hyperbolic property of the pseudo-unsteady system.
The velocity, the pressure and the F field are extrapolated at the downstream boundary.
The flowfield becomes stationary after about 1000 iterations. Calculations show good agreements with the measuments for the first wave, while the second crest is slightly moved downstream. The wave elevation compared with the measurements and the other CFD results is given in figure 2.

6 Code development

Three-dimensional Euler calculations are underway around the submerged Farell ellipsoïde and a fixed Wigley hull. The on-going developments concern the evaluation of the pitch and heave degrees of freedom, through the coupling of the flow equations with the equations of the ship rigid body dynamics. The overall stability of the coupled algorithm is obtained by a pseudo-time integration of the rigid body dynamics.

7 Acknowledgments

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Sloshing in tanks

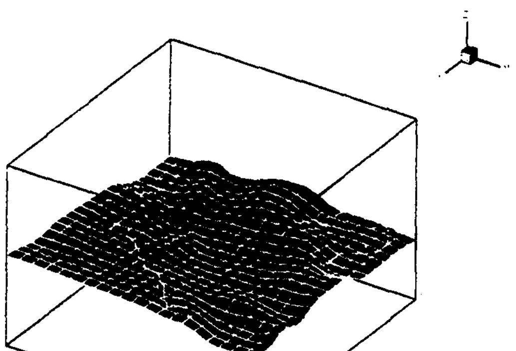

Figure 1 : Roll motion of a 3 D tank

Steady waves over a wing section NACA0012

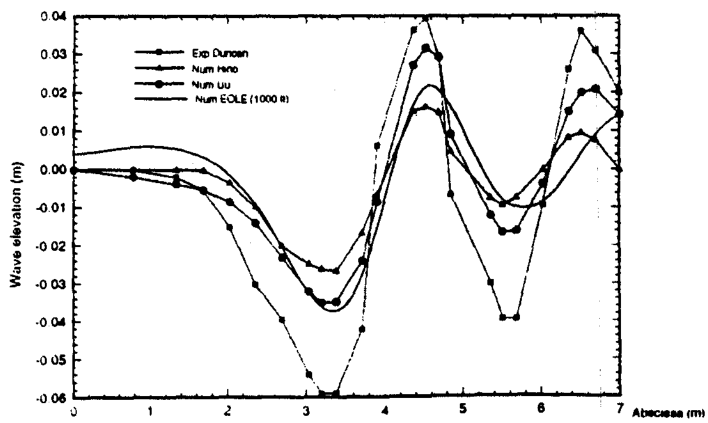

Figure 2 : Free surface elevations obtained by experimental and numerical methods

DISCUSSION

Armenio: Why do you prefer to use a VOF technique instead of a Marker-and-Cell algorithm for treating the free surface?

de Jouette et al.: We choose a VOF technique to solve the problem of wave resistance because we have already got a first experience of this method for tracking complicated free surfaces, such as the phenomenon of the bubble stretching or the jets due to the bubble collapse behind torpedos during the launching stage from a submarine.

Clement: Did you make accuracy and convergence tests with regard to the discretization? Are you sure that your results have converged when you compare to other algorithms (Reva, Eliza, experiments...)?

de Jouette et al.: We have only started the implementation of the VOF method for solving three-dimensional non-linear wave problems for 6 months. We have extended this method relative to solve unsteady problems to steady flows calculated by means of an Eulerian curvilinear grid. Of course, many numerical tests are to be made, such as the influence of the discretization on results in the case of the immersed profile and the Wigley hull. Concerning the comparisons of our results of flows above the submerged ellipsoid and around a Wigley hull with experiments and other computational codes, we are sure that the iterations we made are enough to obtain a wave profile along the hull with a good accuracy, but are not sufficient to describe the waves in the wake. Moreover, we are going to monitor in detail the convergence of the VOF field.