3-d Rankine panel computations for a ship in head waves

Michael Hughes
Analytical Methods, Inc.

This paper presents an extension of the work presented on the last workshop, Hughes (1995). Unspecified notation follows general custom, resp. ITTC standard. I consider a ship moving with mean speed U in harmonic head waves of small amplitude. For $\tau = U\omega_e/g > 0.25$, diffracted and radiated waves cannot propagate upstream. For $\tau < 0.25$, the method might require an extension which could either use absorbing boundary conditions ("beach") or match the near-field Rankine solution to a far-field Green function solution. However, MIT experience with similar Rankine singularity methods suggests that the errors in not fulfilling the radiation condition for the upstream waves is not so important for $\tau < 0.2$.

The linear ship motion problem is still the seakeeping problem of principal practical interest as it is the keystone for added resistance predictions. The most common tools in practice to solve the ship motion problem are based on strip theory, because this approach is cheap, fast, and for most cases also quite accurate. However, strip methods do not perform so well for high speeds, full hull-forms (tankers), ships with strong flare, and generally for low encounter frequencies which typically occur in following seas. Approaches to improve prediction of added resistance should capture

- 3-D effects of the flow

  3-D effects are important for low encounter frequencies and full hull forms. 3-D diffraction at the bow region of tankers contributes considerably to added resistance.

- Forward-speed effects

  Strip methods include forward speed by the change in encounter frequency. But forward speed enters the ship motion problem in additional ways: the local steady flow field, the steady wave pattern of the ship, and the change of the hull form and wetted surface due to squat (dynamic sinkage and trim).

I will present a Rankine source method to capture both 3-D and all forward-speed effects for linearized seakeeping problems as proposed by Bertram (1990a) and McCreight (1991). However, McCreight never realized his proposal and Bertram 'faked' a solution by using a crude approximation for the second-order derivatives on the hull.

The problem is linearized around the fully nonlinear solution of the steady flow (derivatives of velocity potential $\phi^{(0)}$, wave elevation $\zeta^{(0)}$, wetted surface S including trim and sinkage and wave profile on hull). In a first step, the steady flow is solved iteratively using higher-order panels (parabolic in geometry, linear in source strength), Hughes (1995), Hughes and Bertram (1995). The relevant boundary conditions are given here without further comment:

The particle acceleration in the stationary flow is:	$\vec{a}^{(0)} = (\nabla\phi^{(0)}\nabla)\nabla\phi^{(0)}$
For convenience we introduce abbreviations:	$\vec{a}^{g} = \vec{a}^{(0)} - \{0,0,g\}^T$
	$B = -\frac{1}{a_3-g}\frac{\partial}{\partial z}(\nabla\phi^{(0)}\vec{a}^{g})$
In the whole fluid domain:	$\Delta\phi^{(0)} = 0$
On the steady free surface:	$\nabla\phi^{(0)}\vec{a}^{g} = 0$
	$\frac{1}{2}(\nabla\phi^{(0)})^2 - g\zeta^{(0)} = \frac{1}{2}U^2$
On the hull:	$\vec{n}\nabla\phi^{(0)} = 0$

$\vec{n}$ is the unit normal on the hull. Radiation and open-boundary condition are enforced by shifting sources vs. collocation points.

In a second step, the diffraction/radiation problem is solved using almost the same techniques as for the steady flow computation except that now flat panels with constant source strength distributions are used as second derivatives on the hull are no longer required. For head waves, the problem is symmetric and sway, yaw, and roll motions are zero.

The instationary potential $\phi^{(1)}$ is divided into the potential of the incident wave ϕ^w, the diffraction potential ϕ^d, and the 3 symmetric radiation potentials:

$$\hat{\phi}^{(1)} = \hat{\phi}^d + \hat{\phi}^w + \hat{\phi}^1\hat{u}_1 + \hat{\phi}^3\hat{u}_3 + \hat{\phi}^5\hat{u}_5 \tag{1}$$

The symbol $\hat{}$ denotes a complex amplitude. $\hat{u}_i$ are the motion amplitudes in the 3 symmetric degrees of freedom. The boundary conditions were derived by Newman (1978) and Bertram (1990a).

On the free surface at $z = \zeta^{(0)}$, the boundary condition is:

$$(-\omega_e^2 + Bi\omega_e)\hat{\phi}^{(1)} + ((2i\omega_e + B)\nabla\phi^{(0)} + \vec{a}^{(0)} + \vec{a}^g)\nabla\hat{\phi}^{(1)} + \nabla\phi^{(0)}(\nabla\phi^{(0)}\nabla)\nabla\hat{\phi}^{(1)} = 0 \tag{2}$$

Setting the steady flow to uniform flow simplifies (2) to the usual Kelvin condition at $z = 0$. However, this crude linearization loses many physical properties.

The boundary condition on the average position of the hull is:

$$\vec{n}\nabla\hat{\phi}^{(1)} + \hat{\vec{u}}\vec{m} + \hat{\vec{\alpha}}(\vec{n} \times \nabla\phi^{(0)} + \vec{x} \times \vec{m}) - i\omega_e(\hat{\vec{u}}\vec{n} + \hat{\vec{\alpha}}(\vec{x} \times \vec{n})) = 0 \tag{3}$$

with $\vec{m} = (\vec{n}\nabla)\nabla\phi^{(0)}$, $\vec{u} = \{u_1, u_2, u_3\}$, $\vec{\alpha} = \{\alpha_1, \alpha_2, \alpha_3\} = \{u_4, u_5, u_6\}$. The 'average position' of the hull is the solution of the steady flow problem and includes trim and sinkage. This is deemed to be important for ships with strong flare and considerable squat, e.g. yachts or slender container ships.

Diffraction and radiation problems for unit amplitude motions can be solved independently using Rankine sources (panels). Each source fulfills automatically Laplace's equation and the decay condition that the disturbance of the flow vanishes at infinity. For the diffraction problem, all u_i are set to zero. For a radiation problem, the relevant motion amplitude is set to 1. All other motion amplitudes, $\hat{\phi}^w$, and $\hat{\phi}^d$ are set to zero. Then the boundary conditions at the free surface (2) and the hull (3) are fulfilled in a collocation scheme. Radiation and open-boundary condition are enforced by shifting the Rankine sources vs. collocation points as proposed by Bertram (1990b). The collocation scheme forms 4 systems of linear equations in the unknown complex source strengths sharing the same matrix of coefficients. Only the right-hand sides differ. The 4 systems are solved simultaneously using a complex-variable version of Söding's (1994) semi-iterative solver.

The first-order forces $\vec{F}$ and moments $\vec{M}$ acting on the body result from the body's weight and from integrating the pressure over the average wetted surface S, Bertram (1990a). The relation between forces, moments and motion acceleration is:

$$\begin{aligned} F_1^{(1)} &= -m\omega_e^2(\hat{u}_1 + \hat{\alpha}_2 z_g) \\ F_3^{(1)} &= -m\omega_e^2(\hat{u}_3 - \hat{\alpha}_2 x_g) \\ M_2^{(1)} &= -m\omega_e^2(z_g\hat{u}_1 - x_g\hat{u}_3 + k_y^2\hat{\alpha}_2) \end{aligned} \tag{4}$$

k_y is the radius of the moment of inertia $\Theta_y = \int(x^2 + z^2)\,dm$, m the mass, x_g and z_g the center of gravity. Inserting the pressure integrals (for unit motion) for the forces and the moment forms a system of linear equations for u_1, u_3, α_2, quickly solved by Gauss elimination.

Preliminary computations have been performed for a Series-60 ($C_B = 0.7$) in head waves, which was discretized using 827 panels on the hull. For a ship of 121.92m length, the mass was set to 9924t, $\Theta_y = 9.6 \cdot 10^9$tm^2, $x_g = 0.6$m before amidships, $z_g = 0.9$m below CWL.

Exciting forces in head waves for $F_n = 0.2$ agree well with experiments of Gerritsma and Beukelman (1960,1967). The phase for pitch exciting force is given as about $+90°$ in Gerritsma and Beukelman (1960) for all Froude numbers, but as about $-90°$ in Gerritsma and Beukelman (1967) for the two Froude numbers considered there. I therefore determined the phase by taking the value from Gerritsma and Beukelman (1960) and subtracting 180°. Results for motions, Fig.2, indicate some errors which could possibly be due to (in order of likeliness):

1. programming errors
2. input errors (steady solution or additional data)
3. discretisation errors
4. theoretical errors in the formula work

The method offers the most extensive physical model for frequency-domain computations intended for added resistance computations. Subsequently good results are expected for the ultimate prediction of added resistance. However, at present

- the method is limited to $\tau > 0.25$,
- added resistance has not been included yet even in the formula work,
- and testing and debugging of the seakeeping code is not yet finished.

The expectations for better results have yet to be proven.

Acknowledgement

This research was carried out with the support of a research fellowship from the Alexander von Humboldt Foundation.

References

BERTRAM, V. (1990a), *Ship motions by a Rankine source method*, Ship Techn. Res. 37/4, pp.143-152

BERTRAM, V. (1990b), *Fulfilling open-boundary and radiation condition in free-surface problems using Rankine sources*, Ship Techn. Res. 37/2, pp.47-52

GERRITSMA, J.; BEUKELMAN, W. (1960), Shipmotions in longitudinal waves, ISP 7/66, pp.49-71

GERRITSMA, J.; BEUKELMAN, W. (1967), Analysis of th modified strip theory for the calculation of ship motions and wave bending moments, ISP 14/156, pp.319-337

HUGHES, M. (1995), *Application of a higher-order panel method for computing higher derivatives of the steady potential in a ship motions program*, 10th WWWFB, Oxford

HUGHES, M.; BERTRAM, V. (1995) *A higher-order panel method for steady 3-d free-surface flows*, IfS-Report 558, Univ. Hamburg

McCREIGHT, W. (1991), *Ship motions with nonlinear high speed effects*, 6th Workshop Water Waves and Floating Bodies, Woods Hole, pp.169-172

NEWMAN, J. (1978), *The theory of ship motions*, Advances in Applied Mechanics 18

SÖDING, H. (1994), *Incomplete Gauss Elimination*, Ship Technology Research 41/2, pp.111-112

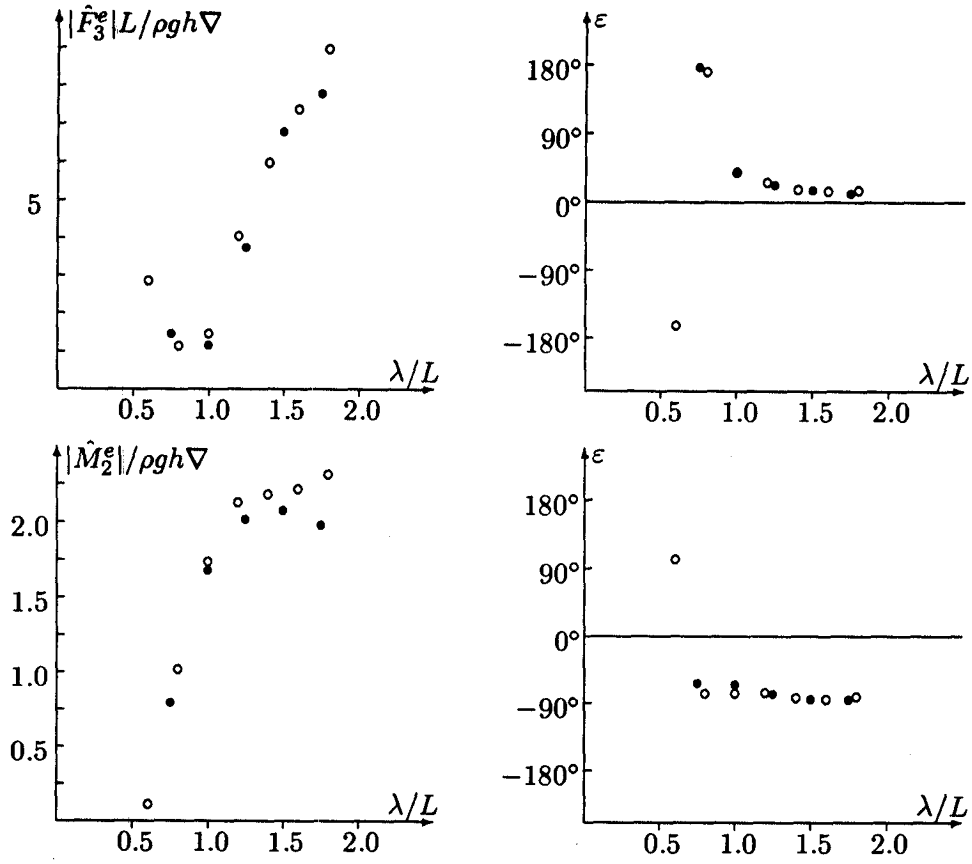

Fig.1: Exciting forces for $F_n = 0.2$, ∘ computation, • experiment

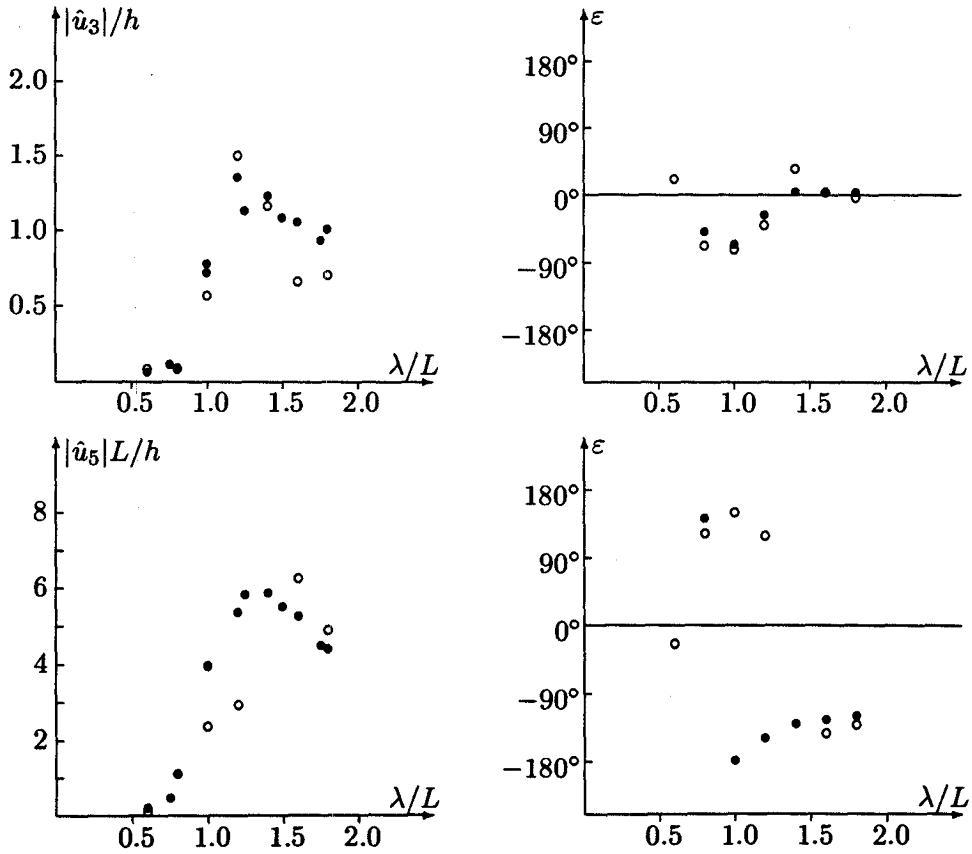

Fig.2: Response amplitude operators for $F_n = 0.2$, ∘ computation, • experiment

DISCUSSION

Korsmeyer: What is the convergence criterion for the steady problem?

Hughes: At the end of each iteration step the nondimensional error in the boundary condition (defined as $\varepsilon = \max(|\frac{1}{2}\nabla((\nabla\phi)^2)\nabla\phi + g\phi_z|/gU)$ and the nondimensional error in the dynamic boundary condition (defined as $\varepsilon_{dyn} = \max(|\frac{1}{2}((\nabla\phi)^2 - U^2) + g\bar{\zeta}|/U^2)$ are computed, where "max" refers to the maximum value at all of the collocation points on the free surface. This information is used to determine if the solution has converged or more iteration steps are needed.