OCTREE-BASED FINITE ELEMENT ANALYSIS FOR THREE-DIMENSIONAL STEEP WAVES

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1. INTRODUCTION

Fully non-linear theory is commonly used for the simulation of steep wave interaction with large volume offshore structures. This problem is particularly relevant to the oil industry where concern has arisen due to the ringing response of some offshore structures. The present authors have described a fully non-linear finite element method for tackling this problem in two-dimensions at the last Workshop (1,2).

This paper presents details of an octree-based finite element mesh generator whereby tetrahedral meshes are produced (i) by generating an octree grid about planes of seeding points which describe the free surface and body geometry, and then (ii) by dividing the octree grid into a mesh of tetrahedral finite elements.

In addition, the paper describes a three-dimensional finite element numerical model for inviscid free surface flows. A time marching method is used which assumes that the wave profile and position of the structure are known at a particular instant, and the problem is solved numerically at successive time steps using potential theory. The potential at the free surface provides the boundary condition for the next time step calculation, and the new free surface profile is calculated from velocities at the surface. Preliminary results are presented for standing waves in a rectangular tank calculated using regular meshes.

2. OCTREE-BASED MESH GENERATION

2.1 Octree Grid

Octree grids are the three-dimensional extension of quadtree grids (1,2). The basic element is the octree cube, within which the grid is generated by recursive subdivision about planes of seeding points. Each division splits the divided cell into eight equal sized smaller cubes. The resulting grids are refined at the boundaries. The ratio of adjacent cell edges is restricted to be at most 2:1 by further subdivisions in order to limit the possible arrangements of neighbouring cubes. Each cell thus has a maximum of four possible face neighbours on each face, as shown in Figure 1 for the top face of cell A; two possible neighbours on each edge, as shown for the North East edge of cell A in Figure 2; and one corner neighbour on each vertex. In total, there exist 56 possible neighbours for each octree grid cell.

Octree grids have a hierarchical tree structure which enables efficient data storage and fast neighbour finding using tree traversal techniques. The numbering system adopted herein is based on that proposed by Samet (4). The reference number for each panel contains information about its location within the octree stored as a set of x, y and z translations, and combined into a single integer.

The reference numbering system can be summarised as

$$N = \sum_{i=0}^{m-1} N_i 9^i \quad (1)$$

where m is the division level of the cell and N_i takes the integer value 1, 2, 3, 4, 5, 6, 7 or 8 depending on the position in which the cell is located within its parent cell. In this scheme 1 refers to front North West, 2 to front South West, 3 to front North East and 4 to front South East, and 5 refers to back North West, 6 to back South West, 7 to back North East and 8 to back South East. The numbering of octree children is summarised in Figure 3. The reference number of a given cell can be used to obtain directly the reference number of its parent cell, its centre co-ordinates, and by manipulation the reference numbers of each neighbouring cell can be derived.

In Figure 4 planes of boundary seeding points are shown describing a rectangular tank containing water of sinusoidal free surface elevation of amplitude $0.08d$, where d is the water depth. Figure 5 shows the octree grid generated from the seeding points which has a maximum division level equal to 5.

2.2 Finite Element Mesh

Tetrahedral finite elements are obtained from the octree grid by joining the centres of four neighbouring cubic cells. This eliminates hanging midside nodes which occur in octree grids. Six tetrahedra are generated from a basic unit of eight cells of the same size, as shown in Figure 6. Tetrahedra are obtained by this standard configuration where possible throughout the grid, and additional neighbour combinations are also used where necessary at boundaries and within the grid at transitions between cells of different sizes. Finite element nodes are initially located at the centres of octree cells. Figure 7 shows the tetrahedra obtained from the octree grid in Figure 5. The presence of any holes in the mesh is prevented by checking that each internal face has exactly two elements connected to it.

Boundary nodes are then moved by interpolation using the co-ordinates of boundary seeding points, in order to produce a close approximation to the free surface, as shown in Figure 8.

3. SOME PRELIMINARY RESULTS

The finite element formulation and solution method have been developed in previous papers (2,3). In this work, preliminary results have been obtained using a regular mesh generator. The calculation domain is divided into cubes by planes parallel to the bottom and walls of the tank, and six tetrahedral elements are generated within each cube. The meshes are adapted to follow the free surface by adjusting nodes in the vertical direction only.

Two-dimensional standing waves have been simulated in a three-dimensional rectangular tank of depth d, width b=$0.3d$ and length l=$2d$. The initial free surface elevation is given by:

$$\eta(x,y,t=0)=a\cos\frac{2\pi}{l}x \qquad (2)$$

where a is the amplitude. Figure 9 shows the wave elevation history, η/a, measured at the centre of the tank for amplitudes a=0.01d and a=0.05d respectively, plotted with the linear analytical solution. In both cases the numerical results agree well with the analytical solution, however the discrepancy is more visible in the latter case as the non-linear effect becomes more significant.

In the next stage of this work the locally refined octree-based finite element mesh generator outlined in Section 2 will be combined with the solution algorithm discussed in Section 3 in order to tackle more complex geometries. This combined approach will be applied to realistic problems related to the offshore industry, and results presented at the Workshop.

5. REFERENCES

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6. ACKNOWLEDGEMENTS

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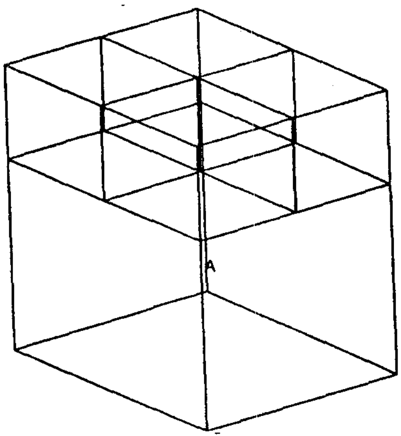

Figure 1 Face neighbours

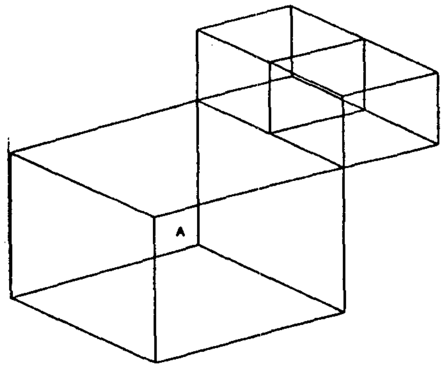

Figure 2 Edge neighbours

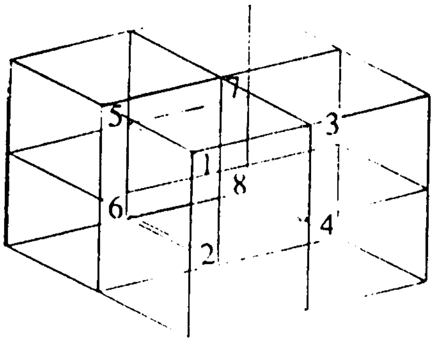

Figure 3 Octree children

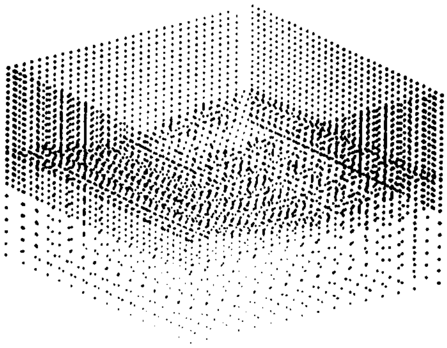

Figure 4 Seeding points

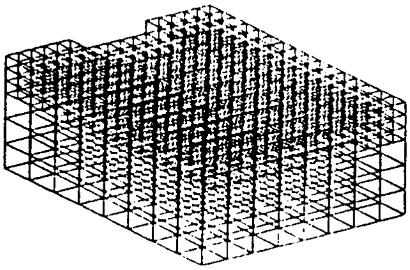

Figure 5 Octree grid

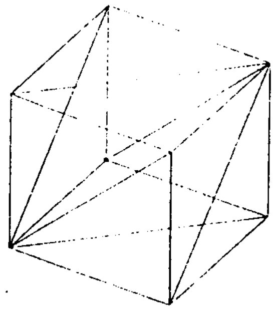

Figure 6 Tetrahedral elements

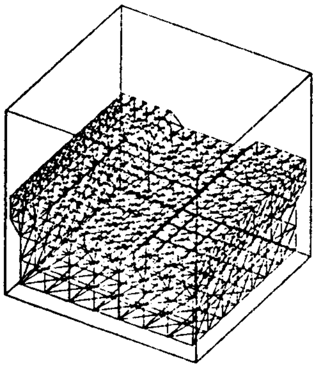

Figure 7 Initial mesh

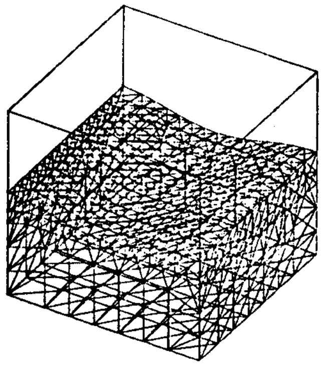

Figure 8 Final mesh

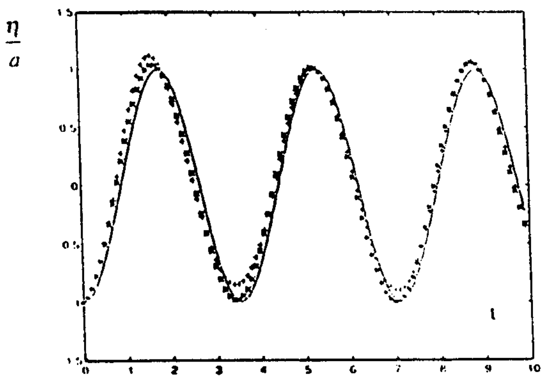

Figure 9 Wave elevation history at centre of tank (solid line: analytical; *: a/d=0.01; +: a/d=0.05)

DISCUSSION

Armenio: It is well established that FEM are well suited for solution of viscous flows on fully unstructured grids. At present you solve potential problems. Could your work be considered as preliminary to viscous free-surface unstructured computations?

Greaves et al.: Yes. The method will be used to solve potential flow problems on the unstructured octree-based finite element meshes and in the future will be extended to solve viscous flow problems.

Schultz: How suitable is this approach for the addition or subtraction of nodes for adaptivity?

Greaves et al.: The method is well suited to adaptive remeshing as cells can be added to and subtracted from the octree grid without disrupting the tree structure, once a suitable adaption indicator has been selected.