Linear analyses of a 2-D floating and liquid filled membrane structure in waves

Rong Zhao
MARINTEK, NORWAY

ABSTRACT: A complete linear theory to analyse a two-dimensional floating and liquid filled membrane structure in waves has been presented. An approximate solution of the dynamic tension is developed for checking the complete linear solution. Numerical results have been presented for the dynamic tension and motions.

1 INTRODUCTION

The motivation for studying the problem of a two-dimensional floating and liquid filled membrane structure in waves is to further investigate the hydroelastic and hydrodynamic effects of a long flexible barge(tube) which can be used to carry and transport oil and liquids lighter than water. The shape of a two-dimensional floating and liquid filled membrane structure in calm water is shown in fig.1. The hydroelastic effect is important for the hoop tension of the tube in waves. An approximate solution for a long flexible barge in waves has been presented by Zhao and Triantafyllou(1994).

In this paper we are going to present a completely linear theory to predict tension, motions and hydroelastic deformations of a two-dimensional floating and liquid filled membrane structure in beam sea waves. Before we carry out the dynamic analysis, the static shape and tensions of a flexible tube have been evaluated by a numerical iteration scheme which is developed by Zhao and Triantafyllou(1994). In our analyses, the motions, hydroelastic deformations and tension of a floating membrane structure in waves are found by a linear perturbation of the static solutions. The hydroelastic deformations have been taken care of in the body boundary condition. The Greens second identity has been applied for the velocity potentials which describe the fluid motions inside and outside the membrane. For solving the total problem, the dynamic equations has been used for the motions of the membrane. To verify the theory, an approximate solution is developed to predict the dynamic tensions in beam sea waves. The numerical results have been tested and checked with different physical relations and an approximately solution for the dynamic tensions.

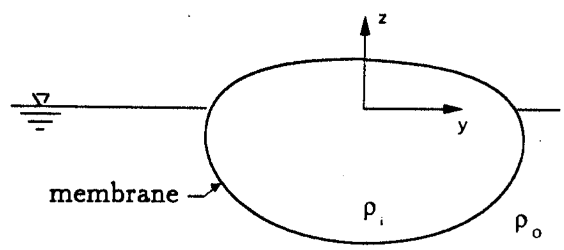

Figure 1: The shape of a two-dimensional floating and liquid filled membrane structure in calm water and a y-z coordinate system are shown. The densities of fluid inside and outside the membrane are ρ_i and ρ_o ($\rho_i < \rho_o$).

2 THEORETICAL APPROACHES

2.1 Theoretical formulation for a complete linear solution

A complete linear solution to predict dynamic tension and motions of a two-dimensional floating and liquid filled membrane structure in beam sea waves is presented here. Fig.1 shows a typical two-dimensional floating membrane structure in calm water. The fluid densities inside and outside the membrane are ρ_i and ρ_o. The filling ratio γ is defined as $\gamma = A_0/A_{max}$, where A_0 is the area inside the two-dimensional membrane structure and A_{max} is the area for the maximum filling. One assumes that ρ_o is larger than ρ_i, the area inside the membrane is completely filled by the fluid, the thickness of the membrane(skin of the tube) is infinitely thin and one neglects the mass of the membrane. Further one assumes that the amplitude of the incident wave amplitude is small compared with the characteristic dimension of the membrane. The problem is solved in the frequency domain. Since the viscus effects are neglected here, the problem may be solved by using a potential theory. First we introduce two velocity potential $\Phi_I = Re(\phi_I e^{i\omega t})$ and $\Phi_O = Re(\phi_O e^{i\omega t})$, where Φ_I is the velocity potential which describe the fluid motion inside the membrane and Φ_O outside the membrane. Re denotes the real part. ϕ_I and ϕ_O are complex. Both Φ_I and Φ_O satisfy the Laplace equation

$$\nabla^2\Phi_I = 0, \quad \nabla^2\Phi_O = 0 \tag{1}$$

in the fluid domains

The velocity potential Φ_O is divided into

$$\Phi_O = \Phi_1 + \Phi_2 + \Phi_3 \tag{2}$$

where Φ_1 is the velocity potential of the linear regular incident wave which can be written as

$$\Phi_1(y,z,t) = Re(\frac{g}{\omega}\zeta_a e^{i\omega t - iky + kz}) \tag{3}$$

here g,i, ζ_a, ω, t and k are the acceleration of gravity, the complex unit, the incident wave amplitude, the circular frequency of oscillation, the time variable and the wave number; Φ_2 is the diffraction potential when the motion of the membrane is ignored and Φ_3 is the velocity potential due to the motion of the membrane. ω and k satisfies the following dispersion relation

$$k = \frac{\omega^2}{g} \tag{4}$$

Φ_O satisfies the following linear free-surface condition

$$-\omega^2\Phi_O + g\frac{\partial\Phi_O}{\partial z} = 0 \quad on \quad z = 0 \tag{5}$$

In our analysis each component of Φ_O in eq.(2) satisfies the eq.(5).

The body boundary conditions for Φ_2 and Φ_3 are

$$\frac{\partial\Phi_2}{\partial n} = -\frac{\partial\Phi_1}{\partial n} \quad on \quad S_{B1} \tag{6}$$

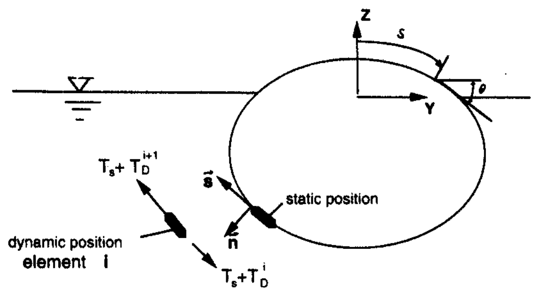

Figure 2: The coordinate (s,n) and symbols used in the eq.(10) and (11) are illustrated

and

$$\frac{\partial\Phi_3}{\partial n} = \vec{V}\vec{n} \quad on \quad S_{B1} \tag{7}$$

where $\vec{V}$, $\vec{n}$ and S_{B1} are the velocity for each point on the membrane structure, the normal vector with the positive direction into the fluid(which is outside the membrane) and the mean wetted body surface.

The similar boundary condition is satisfied of Φ_I, namely

$$\frac{\partial\Phi_I}{\partial n_1} = \vec{V}\vec{n_1} \quad on \quad S_B \tag{8}$$

where S_B and $\vec{n_1}$ are the mean surface of the membrane and the normal vector with positive direction into the fluid(which is inside the membrane).

In addition Φ_2 and Φ_3 satisfy the radiation condition. That means that the body can only generate waves which propagate away from the body. The velocity potential Φ_2 is solved by the numerical method developed by Zhao and Faltinsen(1988).

Due to the hydroelastic deformation of the membrane structure we need linear dynamic equations for the motions of each element of the membrane in the $\vec{n}$ and $\vec{s}$ directions(see fig.2). When one neglects the mass of the membrane and assume that thickness of the membrane is going to zero, the following linear dynamic equations are obtained(see for instance Triantafyllou (1990)).

$$-T_D\frac{d\theta_s}{ds} - T_S\frac{\partial\theta_D}{\partial s} + P_D = 0 \tag{9}$$

and

$$\frac{\partial T_D}{\partial s} = 0 \qquad (10)$$

where s, $\theta(\theta = \theta_s + \theta_D)$, T_D, T_s,θ_s, θ_D and P_D are the length defined in fig.2, the angle defined in fig.2, the dynamic tension, the static tension, the time independent part of θ, the time dependent part of θ and the dynamic pressure. Further P_D can be written as

$$P_D = P_{ID} - P_{OD} - F_{ex} \qquad (11)$$

where

$$P_{ID} = -\rho_I \frac{\partial \Phi_I}{\partial t} - \rho_I g z^1 \quad on \quad S_B \qquad (12)$$

which is the dynamic pressure inside the membrane due to the motions of the membrane. Here z^1 is the vertical motion of each element on the membrane. P_{OD} can be written as

$$P_{OD} = -\rho_O \frac{\partial \Phi_3}{\partial t} - \rho_O g z^1 \quad on \quad S_{B1} \qquad (13)$$

which is the dynamic pressure outside the membrane due to the motions of the membrane.

And

$$F_{ex} = -\rho_O \frac{\partial(\Phi_1 + \Phi_2)}{\partial t} \quad on \quad S_{B1} \qquad (14)$$

is the excitation force due to the incident wave.

Based on the equations(from (1) to (14)) and the assumptions we mentioned above, one may solve the problem by a numerical method.

2.2 An approximate solution for predicting the dynamic tension

To verify the numerical solution that we have presented in the previous section, an approximate solution is formulated here for predicting the dynamic tension. The method is the extension of the method developed by Zhao and Triantafyllou(1994). The hydroelastic deformation is neglected when we calculate the dynamic pressure inside and outside the membrane.

We may assume the dynamic tension T_D can be written as

$$T_D = \frac{\partial T_D}{\partial \eta_3}\eta_3 + \frac{\partial T_D}{\partial \dot{\eta_3}}\dot{\eta_3} + \frac{\partial T_D}{\partial \ddot{\eta_3}}\ddot{\eta_3} + \frac{\partial T_D}{\partial F_{ex}}F_{ex} \qquad (15)$$

which is dependent on the heave motion η_3,velocity $\dot{\eta_3}$, acceleration $\ddot{\eta_3}$ and the excitation forces F_{ex}.

The coefficient $\frac{\partial T_D}{\partial \eta_3}$, $\frac{\partial T_D}{\partial \dot{\eta_3}}$, $\frac{\partial T_D}{\partial \ddot{\eta_3}}$ and $\frac{\partial T_D}{\partial F_{ex}}$ are predicted by a quasi static analysis which will be further expressed here.

Since the hydroelstic deformation of the membrane is neglected here for a two-dimensional membrane structure, the heave motion and the dynamic pressure outside the membrane can be calculated by the method developed by Zhao and Faltinsen(1988) which assumes that the body is rigid. Based on the dynamic pressure components, the corresponding dynamic tensions can be predicted by a similar method as Zhao and Triantafyllou(1994) which is used to estimate the static tension. The only differences are the pressure outside the membrane is the static pressure plus the dynamic pressure component, and the gravity of acceleration of the fluid inside the tube will be g plus the acceleration due to the dynamic force component. Details about the approach can be found in Zhao and Triantafyllou(1994).

3 NUMERICAL RESULTS AND VERIFICATION

A computer program to analyse a two-dimensional floating and liquid filled membrane structure in waves has been developed based on the theoretical procedure presented in the previous section. The numerical cord has been tested at different level. The details are ignored here.

The motions and tensions for the complete linear solution have been compared with the results of an approximated solution. Fig.3 shown an example of dynamic tensions and heave motions which are predicted by the exact and approximate solutions. It seems there are good agreements between the results except in the frequency domain near the natural frequency of the heave motion. This is due to the hydroelastic deformations which has not been included in the approximate solution.

The important part of this work is to predict the dynamic tension of a floating membrane structure. The dynamic hoop tensions are dependent

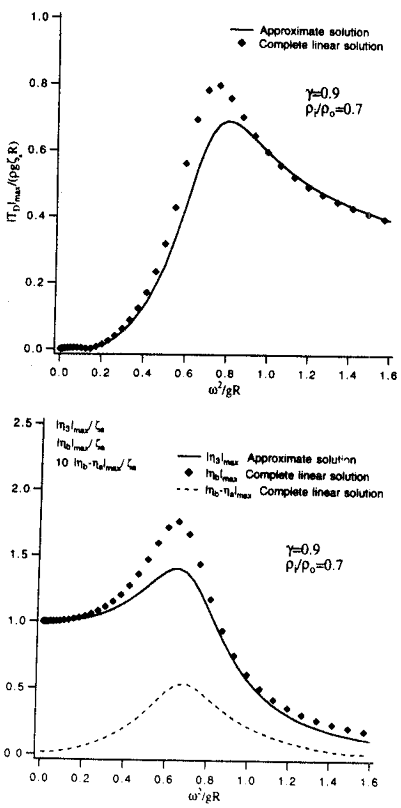

Figure 3: The dynamic tensions T_D, and the vertical motions ζ_3, ζ_a and the relative motion $|\zeta_a - \zeta_b|$ have been presented by the exact and approximate solutions. ζ_3 is the heave motion of the rigid body, ζ_a is the heave motion of the highest point on the membrane and ζ_b is the heave motion of the lowest point on the membrane. Here R is the radius of the membrane structure when $\gamma = 1.0$, ω is the frequency of oscillation, g is the acceleration of gravity, ρ_o is the fluid density outside the membrane, ρ_i is the fluid density inside the membrane and ζ_a is incident wave amplitude.

on the incident wave length, fillings ratio and the relative ratio between the fluid inside and outside the membrane. Fig.4 show the dynamic tension as function of incident wave length for different fillings ratio and relative density of fluid. It seems that the max. dynamic tension increase when fillings ratio increases or the relative density ρ_i/ρ_o increases.

4 CONCLUTION

A complete linear theory to predict the dynamic tension and motions of a two-dimensional floating and liquid filled membrane structure in waves is presented. Numerical code and results has been carefully verified and presented for different filling ratios and fluid densities inside the membrane. An approximate solution of the dynamic tension is developed and good agreement between the exact and approximate solutions is obtained.

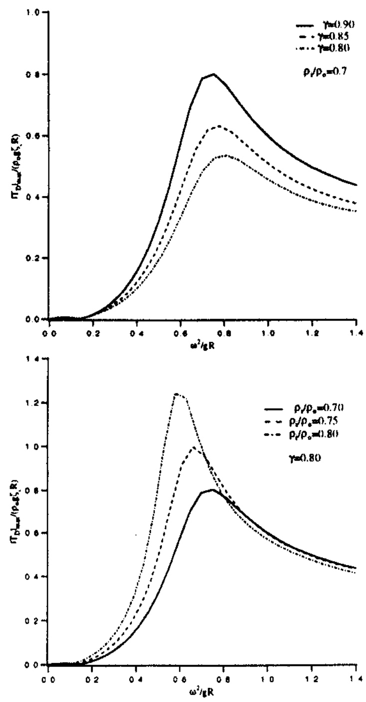

Figure 4: The dynamic tensions T_D have been presented for different fillings ratios and fluid densities. The definition of symbols can be found in fig.3

5 REFERENCES

Triantafyllou, M.S., Cable mechanics with marine applications, Lecture note, Department of Ocean Engineering, MIT, 1990.

Zhao, R. and Faltinsen, O.M., "Interaction between waves and current on a two-dimensional body in the free surface," Applied Ocean Res., Vol.10, No.2 , pp.87-99, 1988.

Zhao,R. and Triantafyllou, M.S., "Hydroelastic analyses of a long flexible tube in waves," Proceedings of International Conference on Hydroelasticity in Marine Technology, Trondheim, 1994.