A HIGHER-ORDER PANEL METHOD FOR NONLINEAR 2-D FREE-SURFACE FLOWS ABOUT HYDROFOILS

Gerhard Thiart, *Dept. of Mech. Eng., Univ. of Stellenbosch, South Africa*
Volker Bertram, *IfS, Univ. of Hamburg, Germany*
Gerhard Jensen, *Hamburg Ship Model Basin, Germany*

Current methods for the calculation of fully nonlinear potential free-surface flow almost exclusively employ Rankine singularities distributed on or above the free surface. Most methods employ an upstream finite differencing technique to enforce the radiation condition, i.e. that surface waves are not allowed to travel upstream of the disturbance causing them. An exception in this respect is the technique of Jensen et al. (1986) which employs discrete sources above the free surface, staggered with respect to the collocation point on the free surface to enforce the radiation condition. Bertram (1990) applied this technique to time-harmonic free-surface flows (seakeeping). The main advantages compared to the numerical differencing scheme is that the resultant computational scheme is simpler, that no damping at the downstream boundary or modifications at the upstream boundary are necessary, and that no numerical damping of the wave height occurs downstream of the disturbance. Fig.1 demonstrates this for a case where the grid covered 50 wave lengths. Only the forward and final part of the free-surface grid are shown. The wave height shows absolutely no damping.

There is one disadvantage of the discrete source/staggered grid technique: there appears to be a limit (depending on the Froude number and the type of disturbance) on the minimum spacing of the sources. The optimum height of the sources above the free surface is approximately twice the source spacing, which is not always possible since all the sources must necessarily be above the free surface. If the source height/grid spacing ratio becomes too large, the method tends to diverge. It is therefore not always possible to check results for grid independence. Another feature of the method is that the source strength tends to oscillate severely at the downstream boundary, Fig.2. Although this is not really a problem (the resulting wave profile and velocity distributions are always smooth), it is nevertheless bothersome.

We will demonstrate that the grid refinement problem may be overcome by employing higher-order source panels on the free surface instead of discrete sources above the free surface, and that the resultant source strength distributions are smooth. For the sake of simplicity we consider here only the two-dimensional problem.

The test problem is the uncavitated steady two-dimensional flow of an ideal fluid past a hydrofoil moving with constant speed U_∞ beneath a free surface. Thiart and Bertram (1994) give details of the mathematical model which are omitted here for brevity. The free-surface boundary condition has to be satisfied iteratively, because it is a nonlinear function of the derivatives of the potential, and because the location of the free surface is not known a priori: it has to be determined as part of the solution. This is achieved by repeated application of the boundary condition linearized with respect to an approximate potential and an approximate free-surface location. The iteration starts with uniform flow and zero elevation.

The body surface is represented by straight panels of constant source strength. Vortex panels of constant strength are superimposed on the source panels. All vortex panels have the same strength. The free surface is represented by the higher-order panels developed by Hess (1973), as extended by Thiart and Bertram (1994) for the calculation of velocity derivatives. While first-order panels were adequate to resolve the velocity components on the free surface, they could not do so for the higher derivatives appearing in the free-surface boundary condition. In particular, ϕ_{xz} oscillated at double the wave frequency, leading to divergence.

The radiation condition is enforced by the "staggered-grid" method. More precisely, one collocation point is added upstream of the first panel and one panel is added downstream of the last collocation point. This is only for equidistant grids equivalent to a staggered grid. The difference becomes noticable for local grid refinements. Here "staggering" the whole grid including the refinement leads to divergence, while adding one panel respectively collocation point at the ends of the grid gives converged solutions that agree well with solutions obtained on uniformly fine grids.

Calculations for a NACA 0012 hydrofoil were carried out for a Froude number based on chord length of 0.5677, fluid depth equal to 1.8966 times chord length, 5° angle of attack, and mid-chord depth of submergence equal to 1.0345 times chord length. These conditions are close to those for which wave breaking occurs. The hydrofoil surface was represented by 180 panels distributed according to the semi-circle method. The free surface was represented by $2 \cdot N_\lambda$ panels upstream, and $3 \cdot N_\lambda$ panels downstream of the mid-chord of the hydrofoil, where N_λ denotes the number of panels per wavelength as given by linearized wave theory, i.e. $\lambda = 2\pi U_\infty/g$. The projected length of the free surface panels on the undisturbed free surface was kept constant at $\Delta x = \lambda/N_\lambda$.

Comparative calculations involving discrete sources above the free surface were also performed. The height of these sources was kept constant at $1.8 \cdot \Delta x$, except for those locations where the free surface came closer than $0.3 \cdot \Delta x$ to the source directly above it during the iterative procedure. For these locations, the source heights were adjusted to free-surface height plus $0.3 \cdot \Delta x$. A few other strategies for adjusting the source height were also tried, but all of these strategies produced almost identical results. The test case was also computed using the method of Lalli et al. (1992) to model viscosity effects at the free surface.

Table I gives calculated values of lift and wave drag. About 12 panels per wavelength are sufficient for the determination of the lift and the wave drag when using surface source panels. If discrete sources are used, the results seem to converge towards those for the surface panels up to $N_\lambda = 24$, but then diverge again for $N_\lambda = 30$. No convergence can be obtained for $N_\lambda = 36$. The surface elevations obtained with the proposed method are compared with finite-difference results of Haussling and Coleman's (1977) and experimental values of Duncan (1983) in Fig.3a (without viscosity correction) and Fig.3b (with viscosity correction). The viscosity correction significantly improves the agreement with experiments.

Table I: Lift and wave drag coefficients for a NACA 0012 hydrofoil using discrete sources above the free surface (*) and source panels on the free surface (**)

N_λ	C_l^*	C_w^*	C_l^{**}	C_w^{**}
12	0.738	0.00820	0.732	0.00736
18	0.733	0.00779	0.731	0.00734
24	0.732	0.00749	0.730	0.00733
30	0.740	0.00753	0.730	0.00733
36	-	-	0.729	0.00733

Higher-order panels on the free surface instead of the previously used discrete sources above the free surface are more efficient, because the required number of panels per wave length can be significantly less than the number of sources per wave length of previously used method. Future research will extend the method to three dimensional applications.

Acknowledgement

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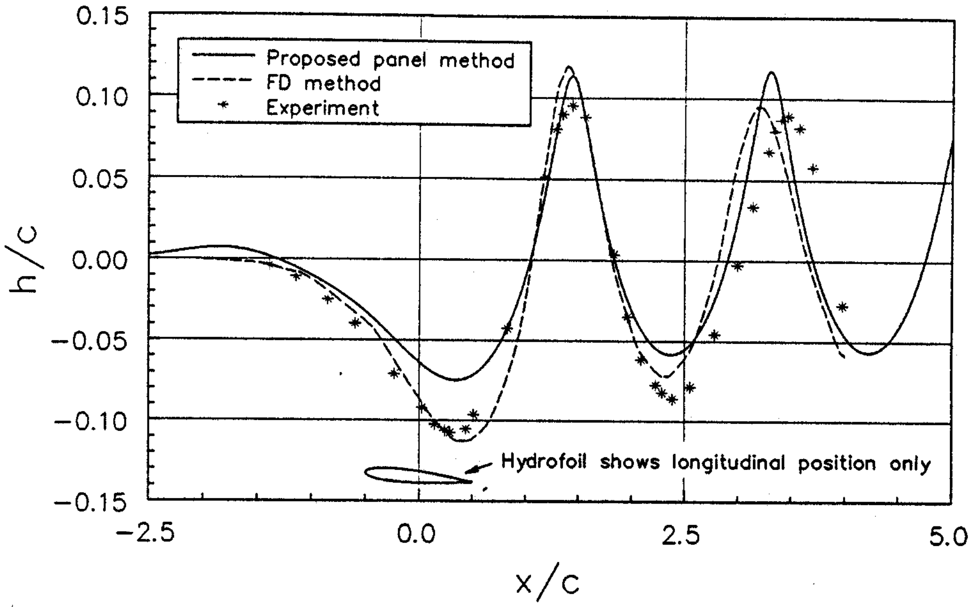

Fig.3a: Surface elevation computed by panel method; comparison with FD method of Haussling and Coleman (1977) and experiments of Duncan (1983)

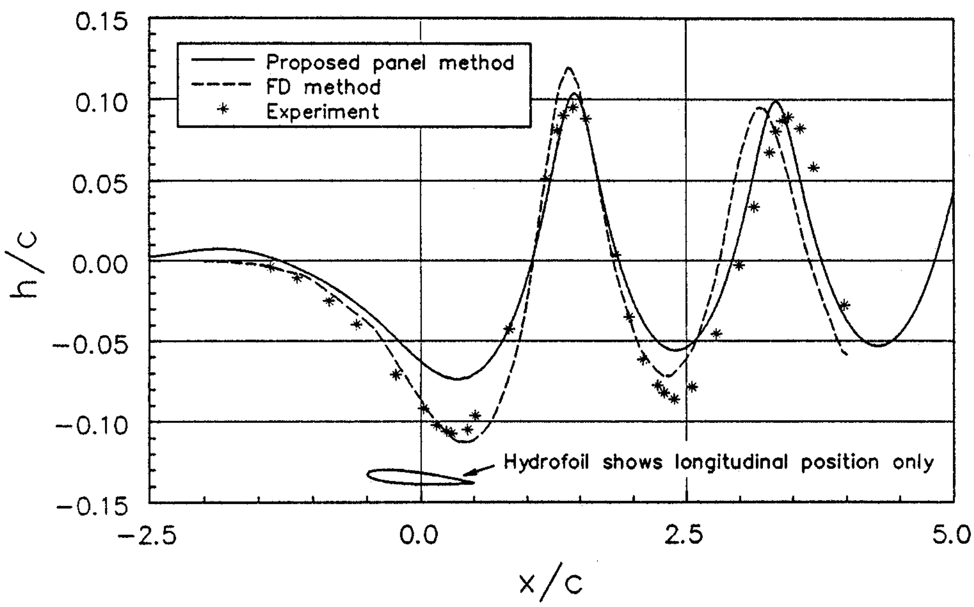

Fig.3b: as Fig.3a but panel method with viscosity correction

References

Bertram, V. (1990), *Fulfilling open-boundary and radiation condition in free-surface problems using Rankine sources*, Ship Technology Research 37

Duncan, J.H. (1983), *The breaking and non-breaking waves behind a two-dimensional hydrofoil*, 16th Symp. on Naval Hydrodynamics, Berkeley

Haussling, H.J.; Coleman, R.M. (1977) *Finite-difference computations using boundary-coordinates for free-surface potential flows generated by submerged bodies*, 2nd Int. Conf. on Numerical Ship Hydrodynamics, Berkeley

Hess, J.L. (1973), *Higher order numerical solution of the integral equation for the two-dimansional Neumann problem*, Computer Methods in Applied Mechanics and Eng. 2

Jensen, G.; Söding, H.; Mi, Z.X. (1986), *Rankine source methods for numerical solutions of the steady wave resistance problem*, 16th Symp. on Naval Hydrodynamics, Berkeley

Lalli, F.; Campana, E.; Bulgarelli, U. (1992), *Numerical simulation of fully non-linear steady free-surface flow*, Int. J. Numerical Methods in Fluids 14

Thiart, G.; Bertram, V. (1994), *Calculation of nonlinear 2-d free-surface flow using a staggered-grid panel method*, submitted to Int. J. Num. Methods in Fluids, September 1994

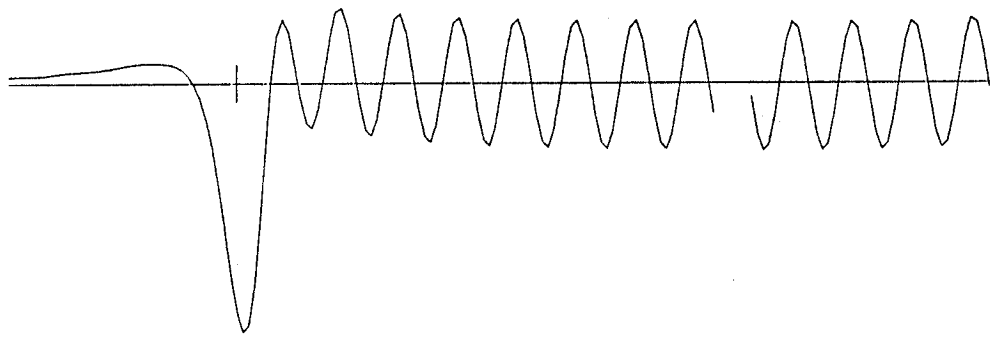

Fig.1: Wave elevation (linear computation) for case with 50 wave lengths (only upstream and downstream ends are plotted)

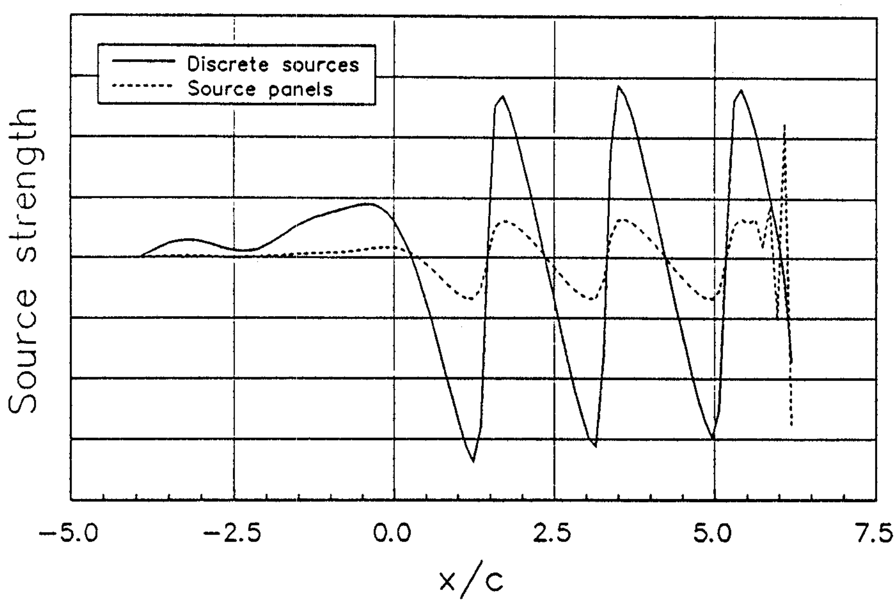

Fig.2: Source strength shows no downstream oscillations for panel method

DISCUSSION

Raven, H. C.: I am somewhat surprised by the divergence of the raised point source method for fine discretisations. For my own raised panel method I have not observed this, even with 70 panels per transverse wave length for instance, albeit for other cases. Do you think that the more strongly singular nature of point sources is the cause of numerical trouble? Or the particular near-field formulation used? Could not keeping the sources at a roughly constant distance from the free surface cure the difficulties?

[NOTE: Fig. 2 is confusing because it suggests that it is the source PANEL method that gives oscillations at the downstream boundary, which is in contradiction with the text. Have the legends been interchanged?]

Thiart, G., Bertram, V. & Jensen, G.: We agree that the more strongly singular nature of point sources can be the cause of trouble. To our knowledge there is no difference between a "near-field" and "far-field" formulation for a point source, because a point source is already the basic singularity. We did do calculations with the sources at a constant height above the free surface, but that did not alleviate the problem. Thank you for drawing our attention to the contradiction in Fig.2, the legends have indeed been interchanged inadvertently.

Zou, Z. J.: It is not quite clear to me how you satisfy the Kutta condition. Could you explain it?

Thiart, G., Bertram, V. & Jensen, G.: The Kutta condition is satisfied by requiring the pressure at the two panels adjacent to the trailing edge to be equal. The method is exactly as proposed by J. Hess and A.M.O. Smith in "*Calculation of Potential Flow about Arbitrary Bodies*", Progress in Aeronautical Sciences, Vol. 8, pp.1-138