INTERACTION OF FREELY OSCILLATING VERTICAL CYLINDERS WITH WAVES AND SLOW CURRENT

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1.FORMULATION OF THE PROBLEM

The problem to be considered here is that an assemble of cylinders is floating in a water of depth h. The radius of each cylinder is a and the draught is d. In addition to a uniform current of velocity U, there comes a plane wave with a frequency of ω_0 and an incident angle of β.The body is restrained from the drift motions but is free to linear oscillation at encountering frequency ω which is defined as $\omega = \omega_0 + Uk_0 \cos\beta$ with k_0 as the wave number of the incident wave.As an extension of the previous work (Bao & Kinoshita 1993) in which only the diffraction problem was considered, the radiation problem will be included in the present work. A right-handed coordinate system is adopted. The plane z=0 coincides with the still water free surface and z-axis is positive upwards. The x-axis points along with the uniform flow so that the current is moving in the positive x direction. The viscous effect is neglected and the flow is assumed to be irrotational. There exists a velocity potential Φ_T which is decomposed into a uniform flow Ux, a steady disturbance potential $\bar{\phi}$ and an unsteady potential. The unsteady potential is in turn separated into radiation potentials ϕ_j (j = 1 - 6) of six motion modes and a scattering potential ϕ_s, i. e.

$$\Phi_T(\mathbf{x},t) = Ux+U\bar{\phi}(\mathbf{x})+Re\left\{\left[-i\omega\sum_{j=1}^{6}\xi_j\phi_j(\mathbf{x}) + \frac{\zeta_0 g}{i\omega_0}\phi_s(\mathbf{x})\right]e^{-i\omega t}\right\} \qquad (1\text{-}1)$$

The scattering potential ϕ_s includes an incident wave potential ϕ_0 and a diffraction potential ϕ_7.Then each mode of the unsteady potentials is expanded into power series of $\tau=\omega U/g$,.i e.

$$\phi_j = \phi_j^{(0)} + \tau\phi_j^{(1)} + O(\tau^2) \qquad (j=1\text{ - }7) \qquad (1\text{-}2)$$

The zeroth order potential satisfies a boundary problem same as the linear wave problem without current. The current effect enters in the first order problem whose free surface condition is no longer homogeneous and the boundary condition satisfied by the radiation potentials on the body surface involves the second order derivatives of the steady potential. Detail discussion is referred to our previous work(Bao & Kinoshita 1994).

2. SOLUTION TO THE INTERACTION PROBLEM

The interaction among cylinders is treated as some additional waves emitting from the adjacent cylinders towards the cylinder under consideration. The zeroth order problem is solved by a way similar to Kagemoto & Yue's(1986) which can also be extended to solve the first order problem.

To solve the first order problem, a particular solution which satisfies the inhomogeneous free surface condition is firstly sought. This can be done with a derivative operator or with a Green function suitable for the problem of pressure distribution over the free surface(see Wehausen & LaitonE,1960). Then a general solution which takes account of the interaction among cylinders is added. The general solution can be solved in a similar way as in the zeroth order problem if the particular solution is regarded as a kind of incident waves. The detail discussion is referred to our previous work(Bao & Kinoshita, 1993).

3. HYDRODYNAMIC FORCES

Once the potentials are solved, the hydrodynamic forces can be evaluated by integrating the hydrodynamic pressure over the wetted surface of the bodies. Alternatively, by using the Green theorem, the hydrodynamic force can be calculated without solving the first order problem, i.e.

$$F_{ij}= \rho i\omega \int_{S_0} \phi_j^{(0)} n_i ds + \tau \rho i\omega [\frac{1}{iV}\int_{S_0} \left(-\phi_i^{(0)} m_j - \mathbf{W}\cdot\nabla\phi_j^{(0)} n_i\right) ds$$

$$+i\int_{S_F} [\,\phi_i^{(0)}\mathbf{W}\cdot\nabla_2\phi_j^{(0)} - \phi_j^{(0)}\mathbf{W}\cdot\nabla_2\phi_i^{(0)}]\, ds\ ] + O(\tau^2) \qquad (3\text{-}1)$$

where m_j has the same definition as done by Newman(1979). By the Tuck theorem(Ogilvie & Tuck 1969), the integral involving m_j, which contains the second order derivatives of the steady potential, can be evaluated by the integral of $\mathbf{W}\cdot\nabla\phi^{(0)}$. Thus the hydrodynamic force may be written as

$$F_{ij}= \rho i\omega \int_{S_0} \phi_j^{(0)} n_i ds + \tau \rho i\omega [\frac{1}{iV}\int_{S_0} \left(\mathbf{W}\cdot\nabla\phi_i^{(0)} n_j - \mathbf{W}\cdot\nabla\phi_j^{(0)} n_i\right) ds$$

$$+i\int_{S_F} [\,\phi_i^{(0)}\mathbf{W}\cdot\nabla_2\phi_j^{(0)} - \phi_j^{(0)}\mathbf{W}\cdot\nabla_2\phi_i^{(0)}]\, ds\ ] + O(\tau^2) \qquad (3\text{-}2)$$

The first term is the same as the result without current except that the frequency is shifted to the encountering frequency. The second term accounts for the the current effect which consists of the contribution from the first order potential and the interaction of the steady potential with the zeroth order potential.

The wave drifting force is calculated by integrating the second order terms in hydrodynamic pressure over the wetted body surface.The wave drifting force can also be expanded into a power series of τ, i. e. $\overline{F}_x = \overline{F}_x^{(0)} + \tau\overline{F}_x^{(1)}$. The first term $\overline{F}_x^{(0)}$ is the wave drift force without current and is the same as usual. The second term $\overline{F}_x^{(1)}$ takes account of the the current effects, which is given by:

$$\overline{F}_x^{(1)} = Re\Big\{ \frac{\rho}{2}\int_{S0} [\nabla\phi^{(0)}\cdot\nabla\phi^{(1)*} - i\omega_0\big(\nabla\phi^{(1)}\cdot(\xi^{(0)*} + \Theta^{(0)*}\times x)$$

$$+\nabla\phi^{(0)}\cdot(\xi^{(1)*} + \Theta^{(1)*}\times x) + \frac{k_0 cos\,\beta}{V_0}\nabla\phi^{(0)}\cdot(\xi^{(0)*} + \Theta^{(0)*}\times x)\big)]\cos\theta\, ds$$

$$-\frac{\rho g}{2}\int_{C_0} \zeta_R^{(0)}\zeta_R^{(1)*}\cos\theta dl - \frac{\omega_0^2 M}{2}\left(\xi_3^{(0)}\xi_5^{(1)*} + \xi_3^{(1)}\xi_5^{(0)*} + \frac{2k_0 cos\,\beta}{V_0}\xi_3^{(0)}\xi_5^{(0)*}\right)\Big\} \qquad (3\text{-}3a)$$

where the amplitudes of oscillation have also been expanded into power series of τ, i.e. $\xi_j = \xi_j^{(0)} + \tau\xi_j^{(1)}$ The total zeroth order potential $\phi^{(0)}$ and relative wave elevation $\zeta_R^{(0)}$ are defined as usual while the first order ones are given by

$$\phi^{(1)} = -i\omega_0 \sum_{j=1}^{6}\left(\xi_j^{(1)}\phi_j^{(0)} + \xi_j^{(0)}\phi_j^{(1)} + \frac{k_0 cos\,\beta}{V_0}\xi_j^{(0)}\phi_j^{(0)}\right) + \frac{\zeta_0 g}{i\omega_0}\phi_s^{(1)} \qquad (3\text{-}3b)$$

$$\zeta_R^{(1)} = \frac{i\omega_0}{g}\left(\phi^{(1)} + \frac{k_0 cos\,\beta}{V_0}\phi^{(0)}\right) - \frac{1}{\omega_0}\mathbf{W}\cdot\nabla\phi^{(0)} - \left(\xi_3^{(1)} + \xi_4^{(1)}y - \xi_5^{(1)}x\right) \qquad (3\text{-}3c)$$

4. EVALUATION OF FREE-SURFACE INTEGRAL

To evaluate hydrodynamic forces or to obtain the particular solution for the first order potential, it involves integrals over the free surface which extends to infinity. Some of these integrals are critical in convergence. Therefore, it should be cautious to carry out these integrals. The integral takes a form like

$$I = \int_0^{2\pi} d\theta \int_a^{\infty} \left[\phi_i^{(0)}\mathbf{W} \cdot \nabla_2 \phi_j^{(0)} - \phi_i^{(0)}\mathbf{W} \cdot \nabla_2 \phi_i^{(0)}\right] r dr \tag{4-1}$$

The integral with respect to θ can be done easily. We will concentrate our discussion on the integral with respect to r. The integrand is a product of three potentials or their derivatives which are expanded into eigenfunction series. Multiplying terms by terms, the integrand is sorted into two groups. A close inspection shows that the first term in the expansion for unsteady potential $\phi_i^{(0)}$ and $\phi_j^{(0)}$ is a Hankel or Bessel function, while it is a power function of r or $1/r$ for the steady potential represented by $\mathbf{W}$. The product of these terms can be integrated analytically by means of the integral formulae of Bessel functions. Two typical integrals are shown here

$$\int Z_n(k_0r)B_{n+2}(k_0r)\frac{dr}{r} = \frac{k_0r}{4(n+1)}\left[Z'_n(k_0r)B_{n+2}(k_0r) - Z_n(k_0r)B'_{n+2}(k_0r)\right] \tag{4-2a}$$

$$\int \left[Z_n(k_0r)B_n(k_0r) + Z_{n+1}(k_0r)B_{n+1}(k_0r)\right] r dr$$

$$= \frac{k_0^2r^2}{2}\left[Z_n(k_0r)B'_{n+1}(k_0r) - Z'_n(k_0r)B_{n+1}(k_0r) + Z'_{n+1}(k_0r)B_n(k_0r) - Z_{n+1}(k_0r)B'_n(k_0r)\right] \tag{4-2b}$$

where Z and B represent one kind of Bessel(Hankel) function and the prime indicates its derivative with respect to the argument. The remaining terms of these potentials are all the modified Bessel functions of second kind which decay exponentially as the argument tends to infinity. The product of these terms can be integrated accurately by numerical method.

5. THE CORNER EFFECTS

In this section we are going to discuss the singularity around the bottom corner of the cylinder. To make it easier to understand, let us first examine the evaluation of the force without solving the first order potential. Difficulty arises from the integral of m_j term in equation (3-1), i. e.

$$I = -\int_{S_0} \phi_i^{(0)} m_j ds \tag{5-1}$$

where m_j is the second order derivative of the steady potential. Since m_j is singular at the bottom corner of the cylinder, this integral does not converge. It needs interpretation. It should also be noted that the Tuck theorem can not be applied directly to the integral due to the discontinuity of m_j at the corner. However, it is reasonable to assume that the hydrodynamic forces remain finite despite the singularity of the potential.

Physically, the singular behavior of the potential generates a concentrate pressure at the corner. When the hydrodynamic forces are calculated, an additional term should be inserted into equation (3-1) to take this concentrate pressure into account. Therefore the value of I should be replaced by

$$I_0 = -\int_{S_0} \phi_i^{(0)} m_j \, ds + P \tag{5-2}$$

where P represents the contribution from the concentrate pressure.

To determine the value of P, the Tuck theorem is applied to the bottom surface S_b and the side wall S_w of the cylinder separately.

$$\int_{S_b} W \cdot \nabla \phi_i^{(0)} n_j \, ds = -\int_{S_b} \phi_i^{(0)} m_j ds + \int_{C_b} \phi_i^{(0)} (W^{(in)} \times dl)_j \tag{5-3a}$$

$$\int_{S_w} W \cdot \nabla \phi_i^{(0)} n_j \, ds = -\int_{S_w} \phi_i^{(0)} m_j ds - \int_{C_b} \phi_i^{(0)} (W^{(out)} \times dl)_j \tag{5-3b}$$

where C_b is the corner line, i. e. the intersection of S_b and S_w. The superscription 'in' (or 'out') indicates that the value of inner (or outer) region is taken.

It is noted that the sum of S_b and S_w gives the body surface S_0. By summing two sides of these two equations respectively and comparing with (5-2) it can be found that the contribution of the concentrate pressure is given by two line integrals

$$P = \int_{C_b} \phi_i^{(0)} (W^{(in)} \times dl)_j - \int_{C_w} \phi_i^{(0)} (W^{(out)} \times dl)_j \tag{5-4}$$

For smooth body surface, steady velocity is continuous over any line on the body surface. Hence, these two line integrals are equal and cancel with each other. However, for a body with sharp corner like a truncated cylinder, due to the discontinuity of the steady velocity W over the corner, these two line integrals are generally not equal and can not be canceled with each other.

Replacing the integral of m_j term in (3-1) by the value of I_0 in (5-2), it can be found that equation (3-2) still holds and Timman-Newman reciprocal relation is satisfied.

When the first order radiation problems are solved, the same difficulty arises at the corner of the cylinder. Since it also involves surface integrals of m_j terms, the same treatment discussed above can be applied in solving these problems.

6. EXPERIMENT

Experiments with an assembly of four cylinders were carried out in a towing tank. Oscillating motions as well as wave drifting force are measured. Calculated results are in good agreement with the experimental ones, which confirms the validation of the present approach.

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