NUMERICAL SIMULATION IN VISCOUS FLUID OF A TWO-DIMENSIONAL FORCED HEAVING CYLINDER ON A FREE SURFACE

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INTRODUCTION

A numerical method is proposed here to solve the problem of forced sinusoidal heaving motion of a free surface piercing cylinder in viscous and incompressible fluid.
The flow is supposed unsteady and laminar. Two dimensional Navier-Stokes equations are discretised by a finite-difference method. Free surface elevation and coupled velocity-pressure system are computed independently. The original aspects of this method are both consideration of the free surface boundary conditions and regridding of the moving physical domain at each iteration.
Hydrodynamic forces and coefficients are calculated and compared with experimental results.
The interest of this simulation in viscous fluid is the numerical calculation of viscous effects, which at this time can only be obtained by experiments or empirical formulas.

EQUATIONS

Navier-Stokes equations are written using an orthogonal system (x^1,x^2). The origin of the system is chosen on the undisturbed free surface and x^2 is upward positive oriented. Independant unknowns are the cartesian components (u^1,u^2) of the total velocity and total pressure P. Gravity forces are introduced using the change of variable $p = P + \rho g x^2$. ν is the kinematic viscosity
The physical domain is transformed into calculation domain using a system of curvilinear coordinates $(\varepsilon^1,\varepsilon^2)$. In this computational domain free surface and other boundaries are described by coordinate lines. With this method the computational domain is a rectangular box which remains unmodified during the simulation whereas the physical domain evolves.
The partial transformation consists in using the Cartesian components of velocity and pressure according to the curvilinear system $(\varepsilon^1,\varepsilon^2)$. Navier-Stokes equations thus become :

$$\frac{\partial u^\alpha}{\partial t} - \frac{v_g^j}{J} b_j^k \frac{\partial u^\alpha}{\partial \varepsilon^k} + \frac{u^j}{J} b_j^k \frac{\partial u^\alpha}{\partial \varepsilon^k} = -\frac{1}{\rho J} b_\alpha^k \frac{\partial p}{\partial \varepsilon^k} + \nu\left(g^{kl}\frac{\partial^2 u^\alpha}{\partial \varepsilon^k \partial \varepsilon^l} + f^k \frac{\partial u^\alpha}{\partial \varepsilon^k}\right) \quad (1) \qquad \frac{1}{J} b_j^i \frac{\partial u^j}{\partial \varepsilon^i} = 0 \quad (2)$$

Free surface boundary conditions consist of one kinematic condition on the free surface elevation h and two dynamic conditions expressing the continuity of normal and tangential stresses on the free surface [1]. The implicit discrete form of kinematic equation is used to calculate free surface elevation at each time step. The two dynamic conditions allow to obtain full linear systems (3).

DISCRETISATION AND NUMERICAL SOLUTION

The structured monoblock mesh (fig. 1) is computed using a transfinite interpolation method [3].
Transport equation (1) is first linearized. Discretisation is achieved using a multiexponential scheme based on a decomposition of the transport equation according to each curvilinear direction. A collocated node-centered disposition is retained for the independant unknowns of the problem. The continuity equation and pressure gradient are finally discretized using classical first order non-centered schemes [1]. These discretisations lead to the two linear systems hereunder :

$$(E - A)U + GP = f \quad , \quad DU = g \qquad (3)$$

Vectors U and P respectively depict velocity and pressure components for each node.
Matrices E and A are issued from equation (1), G and D are the discrete forms of gradient and divergence operators. Direct resolution is impossible because of storing problems for solving full linear systems. So we use the SIMPLER iterative algorithm [7]. In SIMPLER the matrices are inverted by a ILU preconditionned bi-CGSTAB method [9].

RESULTS

The sinusoidal forced-heaving motion of the free surface piercing body is defined by its amplitude A and its frequency ω.
Nondimensional hydrodynamic forces are first calculated for a heaving circular cylinder of radius r. Computations are performed for two motion amplitudes (A=0.2r and 0.4r) and several frequencies. Added mass (fig. 2), damping coefficient (fig. 3), 2nd-harmonic (fig. 4) and 3rd-harmonic (fig. 5) force amplitudes are obtained by a Fourier analysis of the only pressure forces. Results are in quite good agreement with experimental results [8] [10] and numerical calculations obtained with potential-flow theory [6]. Particularly the added masses increase with the motion amplitude and the damping coefficients evolve with a reverse trend which is confirmed by experimental results. Moreover values of the damping coefficients at low frequencies are over-estimated by our calculation. This is probably due to viscous effects which are not taken into account and which can become significant at low frequencies [13]. Bad calculations for A=0.4r and great frequencies are due to not enough refined meshes (fig. 4 and 5).
For the computations above-mentioned the first-order amplitude of waves generated by the cylinder is calculated with Fourier analysis of the displacement of progressive waves. Computations for two amplitudes (fig. 6) are in good agreement with experimental results [5] [8].
Free surface profiles in the vicinity of the circular heaving cylinder (fig. 7) are compared with results obtained by a fully-nonlinear potential-flow model [4]. The amplitude motion is here A=0.4r, nondimensional wave number $Kr = \omega^2 r / g$ is 2.0.

In order to compute viscous forces, meshes with an important refinement in the vicinity of the body are necessary to take into account boundary layer effects accurately. This leads to an increase of calculation time (18 CPU hours on a 40 Mflops computer to simulate 20 seconds of heave). Moreover, there are important numerical problems at the intersection between body and free surface. We use here a body with sharp edges for which viscous effects are more important.
For a cylinder having a rectangular section (A/B=0.30, B/d=1.0, $\omega^2 B / 2g$=2.0 where B is beam and d the draft) velocity-vector and vorticity-contour plots at t=3.18T (T is the period of the vertical motion) are compared with the calculation of Yeung & *al.* in viscous fluid [11] (fig. 8). Structure of the flows is quite similar. A small asymmetry in the vorticity-contour plot can be seen for Yeung & *al.* calculation (fig. 8 (b)). Moreover differences on free surface profiles exist.
For another case ($\omega B^2 / \nu$=1000, $\omega^2 B / 2g$=2.0, A/B=0.30, d/B=1.0) viscous forces are calculated (fig. 9). The shear-stress contribution is about 7 % and the normal viscous stress is about 1 % of the pressure force. These results are in quite good agreement with those of Yeung & *al.* [12].
Numerical problems encountered for meshes with important refinement arise from the mathematical singularity of the kinematic condition at the junction between body and free surface. At this contact point, no-slip and kinematic conditions are both available. The existence of a meniscus solves this singularity. This is confirmed numerically but leads to the breakdown of the calculation. These problems are solved by a limitation of the slope of this meniscus on the body. Convergence is still problematical. The only real solution seems to be direct calculation of the total coupled system in velocity-pressure-free surface elevation [2]. This method will permit to use exactly free surface boundary conditions and to solve the problem of incident waves on an oscillating surface piercing body.

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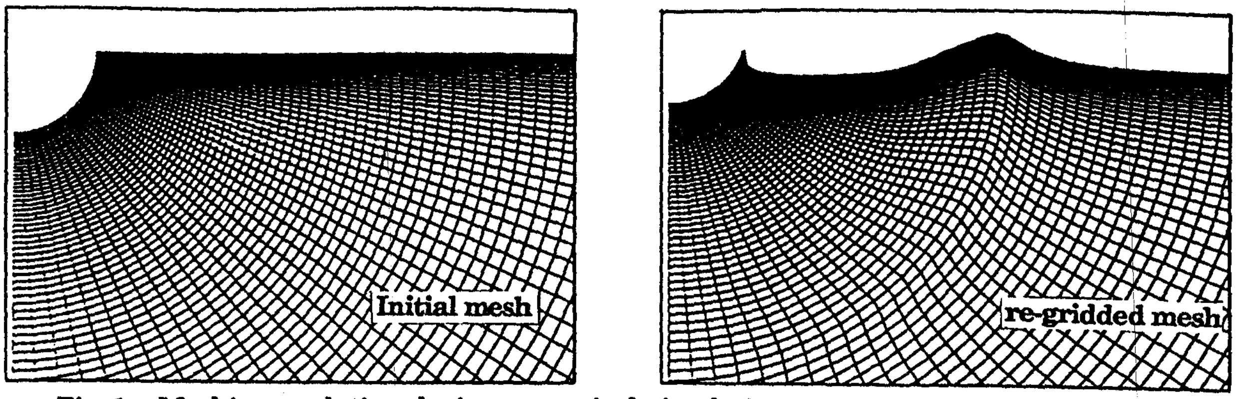

Fig 1 : Meshing evolution during numerical simulation.

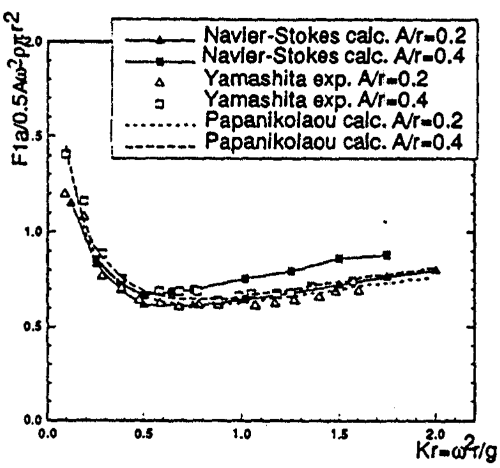

Fig. 2 : Added mass.

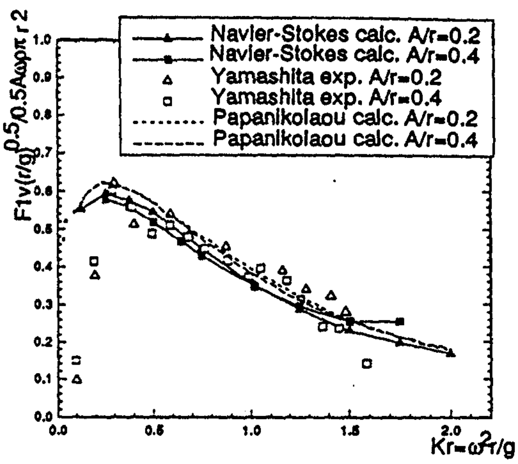

Fig. 3 : Damping coefficient.

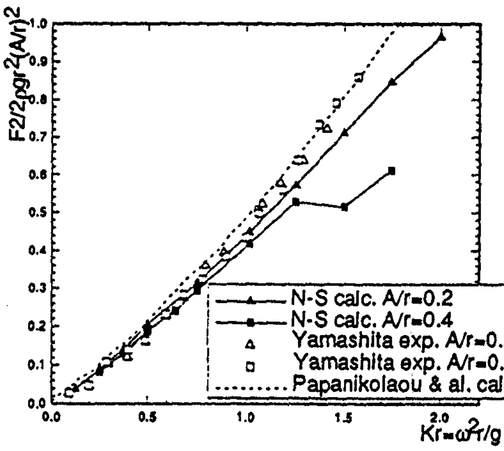

Fig. 4 : 2nd-harmonic force amplitude.

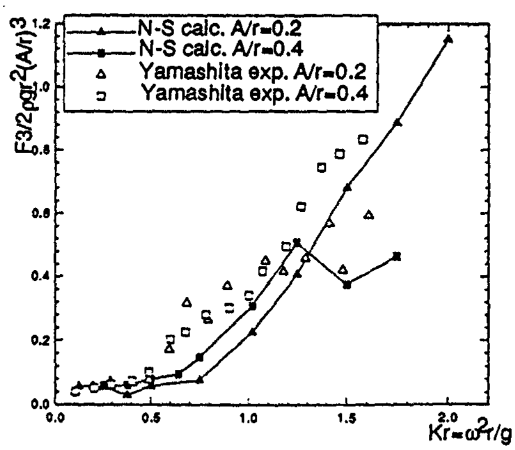

Fig. 5 : 3rd-harmonic force amplitude.

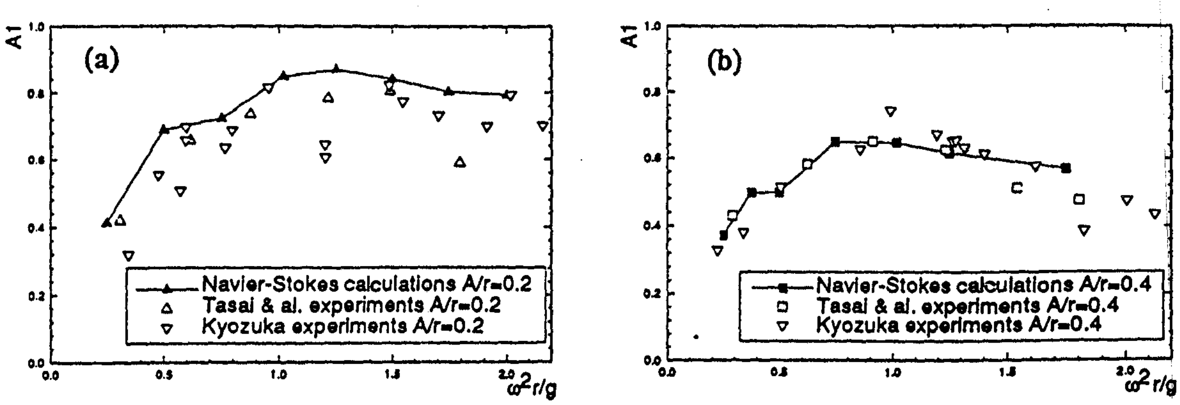

Fig. 6 : Amplitude of first order wave-circular cylinder for A/r=0.2 (a) and A/r=0.4 (b).

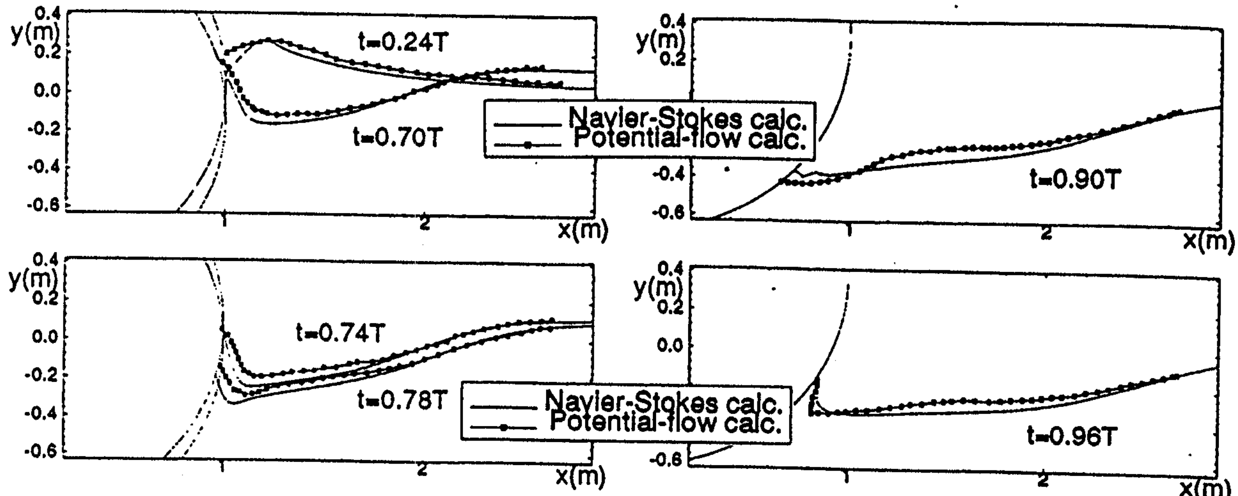

Fig. 7 : Free-surface profiles generated by a heaving circular cylinder.
Body displacement : $y_c = A\cos(\omega t)$, A=0.4r, $Kr = \omega^2 r / g$=2.0.

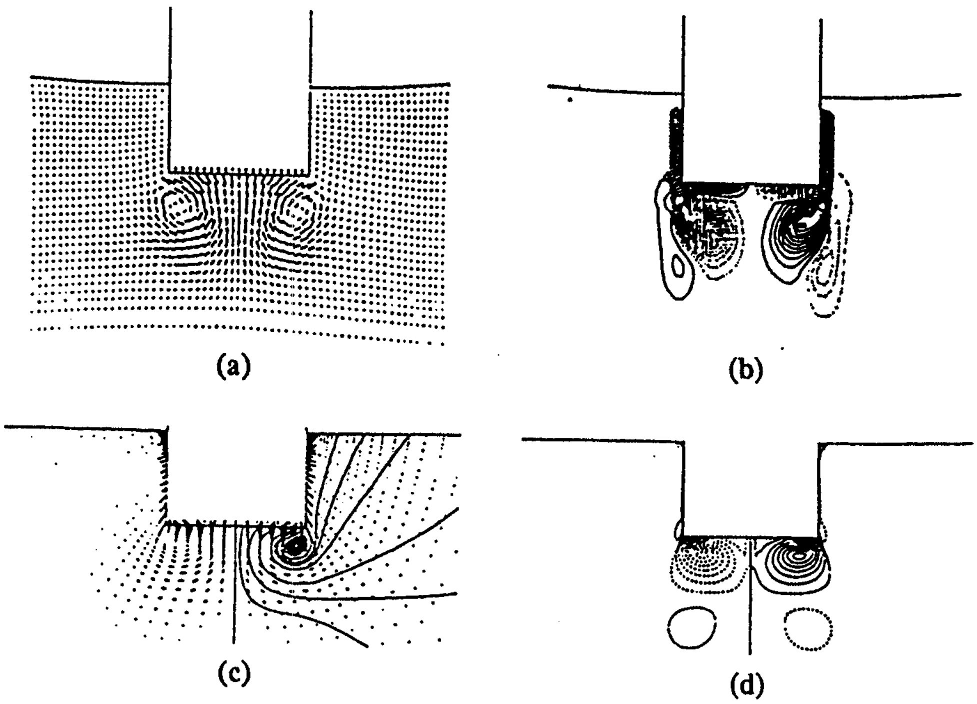

Fig. 8 : Velocity-vector and vorticity-contour plots at t=3.18T.
Method presented here (c & d), Yeung & Ananthakrishnan calculations (a & b).
A/B=0.30, B/d=1.0, $\omega^2 B/2g$=2.0.

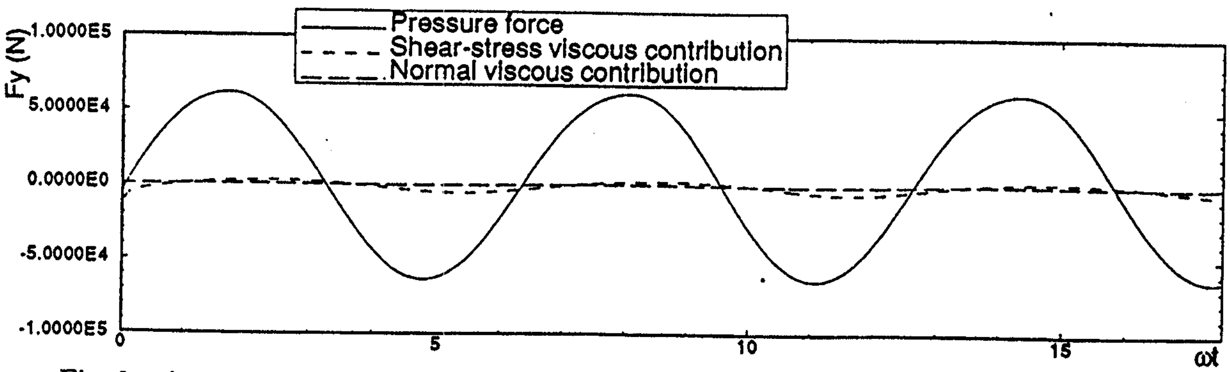

Fig. 9 : viscous-stress and pressure contributions to the heave force.
$\omega B^2 / \nu$=1000, $\omega^2 B / 2g$=2.0, A/B=0.30, d/B=1.0.

DISCUSSION

Yeung, R. W. & Ananthakrishnan, P. : This work is nicely put together. We are pleased that the authors' results such as viscous stress and pressure forces for the heaving rectangle are in agreement with our published results. We believe, however, that the method you used has been unsuccessful in capturing the vorticity structure generated along the sides of the body. The vorticity patterns shown by us are due to the interaction of the side and bottom boundary layers with the shed vortex that was generated during the upward stroke of the earlier cycle. Perhaps the authors can comment on that. Further, it appears your wave-elevation computations display little curvature. This does not seem reasonable for a motion amplitude to body draft ratio of 30%. There may have been difficulties in your implementation of the free-surface conditions. Perhaps some additional verification of the code is appropriate.

Gentaz, L., Alessandrini, B. & Delhommeau, G. : Concerning the little curvature of free surface elevations an error has been found in the implementation of the re-gridding of the physical domain. Additional computations show more important free surface elevations which seem to be comparable with Yeung's plots. Moreover these free surface elevations are in good agreement with others' computations in perfect flow given by the computer code in perfect flow of Mr. Clément (references given in the article of Clément and Domgin in this workshop). In our computations, vorticity structures are really generated along the sides of the heaving body. The exact process of vorticity formation has not yet been studied, but interactions between side and bottom boundary layers and shed vortices explained by Yeung should be found again.

Raven, H. C. : You report that the over-estimation of the damping coefficients at low frequencies may be due to viscous effects that are not taken into account. Do you mean that the resolution was insufficient on your coarser grid? If so, on a coarse grid it may be better to neglect the boundary layer altogether (by imposing a free-slip hull b.c.) rather than imposing a no-slip b.c.; the latter may cause an unphysical "boundary layer" that is too thick and pollutes your results.

Gentaz, L., Alessandrini, B. & Delhommeau, G. : With coarse grids, the boundary layer is not taken into account with accuracy. So the viscous part of the forces which are acting on the body can not be computed. However, effects of viscosity in the flow can be noted. A free-slip condition on the heaving body would probably be used for coarse grids, but these computations on coarse grids have been made only to show the ability of the method to calculate hydrodynamic coefficients in good agreement with experimental results (except for particular cases like low frequencies). Our purpose is to make computations with enough refined meshes to simulate correctly boundary layer effects and then to estimate the influence of the viscosity on the hydrodynamic coefficients.