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Time-simulations of second-order forces

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Introduction

The simulation of linear wave-forces on floating bodies in real sea-states are often performed by the summation of sinusoidal components. The second-order wave forces, which are of great importance to the design of offshore structures, are then obtained by the double summation of sinusoidal components. In order to study the statistics of the second-order forces and motions, relatively long records must be obtained of the simulated quantity. The records of the slow-drift forces and motions of floating bodies must in particular be long, due to slow convergence of the statistical parameters (Emmerhoff, 1994).

The abstract presents a method for the fast computation of second-order forces using the combination of Fast Fourier Transform (FFT) and a representation of the Quadratic Transfer Function (QTF) by summations of basis functions. The presented method requires $O(N \log N)$ operations, where N is the number of sinusoidal components, whereas an alternative straight forward summation of the sinusoidal components requires $O(N^3)$ operations.

Formulation

The second-order wave force $F(t)$ can be written as a double Fourier integral of the form,

$$F(t) \;=\; (2\pi)^{-2} \iint_{-\infty}^{+\infty} \mathrm{d}\omega_1 \mathrm{d}\omega_2 e^{-\mathrm{i}(\omega_1+\omega_2)t} Z(\omega_1) Z(\omega_2) \mathcal{F}(\omega_1, \omega_2), \tag{1}$$

where $Z(\omega_1)$ is the complex wave-amplitude with frequency ω_1 and $\mathcal{F}(\omega_1, \omega_2)$ is the QTF at the frequencies ω_1, ω_2. The expression (1) can be computed by FFT, which requires $O(N^2 \log N)$ operations. This abstract, however, presents a more efficient method, which takes advantage of the following two facts:

- The domain for which the product $Z(\omega_1)Z(\omega_2)\mathcal{F}(\omega_1, \omega_2)$ is non-zero is finite and not very large, due to the rapid decay of the wave-spectrum for large frequencies in real sea-states.

- The QTF can be represented by a sum of basis-functions for each direction ω_1 and ω_2, which allows the double-integral to be written as a sum of products of single integrals.

The B-spline basis functions were here selected for the representation of the QTF. Using the notation of (Rogers & Adams, 1990), the QTF is thus written,

$$\mathcal{F}(\omega_1, \omega_2) = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} B_{ij} N_{i,k}(\omega_1) M_{j,k}(\omega_2), \tag{2}$$

where the magnitude of n_1, n_2 depends on the nature of the QTF. The functions $N_{i,k}(\omega_1)$, $M_{j,k}(\omega_2)$ are the B-spline basis functions of order k, where $k = 2$ defines a bi-linear representation and $k = 3$ a bi-quadratic representation. The definition of the basis functions are found in (Rogers & Adams 1990), page 446. The B_{ij}'s are unknown coefficients which are determined by enforcing the equation (2) at a number of discrete frequencies $\omega_1^{(p)}$, $\omega_2^{(q)}$, and by solving the corresponding system of equations. The number of coefficients $n_1 n_2$ may be less than the number of the discrete frequencies $\omega_1^{(p)}$, $\omega_2^{(q)}$, which makes the system of equations over-determined. The system may then be solved by the method of least squares. With the B_{ij} coefficients known and using equations (1), (2), the second-order force can be written,

$$F(t) = \sum_{i=1}^{n_1} f_{i1} f_{i2}, \tag{3}$$

$$f_{i1} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega_1 e^{-i\omega_1 t} Z(\omega_1) N_i(\omega_1), \tag{4}$$

$$f_{i2} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega_2 e^{-i\omega_2 t} Z(\omega_2) \sum_{j=1}^{n_2} B_{ij} M_j(\omega_2). \tag{5}$$

The integrals in (4)-(5) can now be computed by single FFT.

Results

The surge quadratic difference frequency transfer function $\mathcal{F}(\omega_1, -\omega_2)$ for a realstic offshore platform has been computed over a rectangular region of ω_1, ω_2 in the first quadrant using the computer code SWIM. The real part of the QTF, multiplied the JONSWAP spectrum, is shown in Figure 1. The number of terms of spline functions, n_1, n_2 were selected to 20, and the QTF-matrix evaluated at 20*20=400 locations. The B-spline basis functions were here bi-quadratic ($k = 3$).

The time-history of the second-order force is shown in Figure 2, as computed by the expression (3). The time-history was compared with the results of the alternative expression (1), obtained by double FFT, and the results were identical to 5 digits.

Some more results of the efficiency of the method will be presented at the workshop.

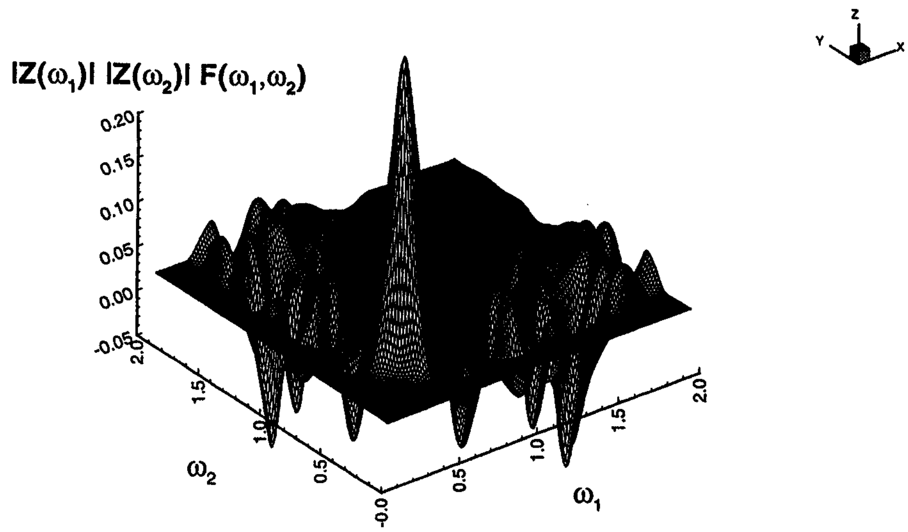

Figure 1: $|Z(\omega_1)||Z(\omega_2)|F(\omega_1, \omega_2)$ for a realistic platform.

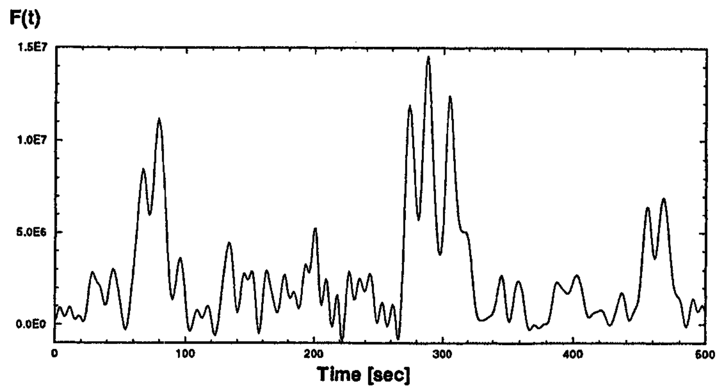

Figure 2: The second-order force $F(t)$.

Acknowledgement

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References

[1] Emmerhoff, O. J. 1994. The slow-drift motions of offshore structures. *Ph.D Thesis.*

[2] Rogers, D. F. & Adams, J. A. 1990. Mathematical Elements for Computer Graphics. Second edition. *McGraw-Hill.*

DISCUSSION

Yeung, R. W.: A crucial assumption in this development is that the QTF is separable with respect to the ω_1 and ω_2 variables. The very good agreement with the direct double FFT method perhaps suggests that it is in fact the case. Can it be established?

Emmerhof, O. J., Kim, S. & Sclavounos, P.D. : The assumption in this method is that the QTF can be represented by polynomials in each direction in a piecewise manner. So for finite and reasonably smooth QTFs, only a reasonable number of coefficients are required in the expansion (the splines). Therefore, in the limit, any QTF is separable with respect to ω_1 and ω_2.

Martin, P. A.: Isn't it true that

$$\mathscr{F}(\omega_1,\omega_2) = \mathscr{F}(\omega_2,\omega_1)?$$

If so, why don't you write

$$\mathscr{F}(\omega_1,\omega_2) = \sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}\, N_{i,k}(\omega_1)\, N_{j,k}(\omega_2),$$

with $B_{ij} = B_{ji}$ so that your representation is symmetric?

Emmerhof, O. J., Kim, S. & Sclavounos, P.D. : It is true that

$$\mathscr{F}(\omega_1,\omega_2) = \mathscr{F}(\omega_2,\omega_2)$$

for the difference frequency force. And you are right that one can use the same spline basis functions for the ω_1 direction and the ω_2 direction. That's exactly what we used in the examples. In reference [2], however, they use the notation that is shown in eq. (2).