(Abstract for 9th International Workshop on Water Waves and Floating Bodies, April 1994)

THE PLANING SPLASH

by

E. O. Tuck
The University of Adelaide, Australia

Summary

Linearised planing surface theory was pioneered by Maruo (1951). The effect of linearisation (justified by smallness of draft or angle of attack) is that the problem is analogous to thin airfoil theory. This analogy becomes an equivalence at high Froude number, i.e. in the absence of gravity, the pressure distribution on a planing surface of given wetted length being then identical to that on the lower surface of an equivalent thin airfoil of the same chord. Maruo (1951) developed numerical tools for incorporating gravity, and others such as Squire (1957), Cumberbatch (1958), and Oertel (1975) have made various modifications and improvements on the computations; see Tuck (1990) for a review.

One aspect of this analogy that does not carry over exactly concerns the inverse square root leading-edge singularity in the pressure distribution. In thin airfoil theory, this singularity simply models the high-velocity flow around the nose of the airfoil, toward the upper surface of the foil, and can be eliminated easily by incorporating local thickness near the nose (Tuck 1991, Lighthill 1951).

For planing surfaces, no such equivalence can hold; there is no upper surface! Instead, the pressure singularity (which formally yields a free surface that meets the body vertically in the linearised theory) must model the (nonlinear) formation of a bow splash. This was already appreciated by Wagner (1932), who used exact nonlinear but gravity-free planing flows to illustrate the phenomenon. The conclusion of Wagner (1932; see also Cumberbatch 1958 and Wehausen and Laitone 1960, p.588) is that the thickness of the jet is proportional to the square of the draft or angle of attack of the body, and in particular proportional to the square of the strength of the leading-edge pressure singularity in the linearised solution.

It is the purpose of this note to expand upon this relationship, and to demonstrate a universal local flow near the leading edge.

A local flow

Consider the two-dimensional irrotational flow whose streamlines are depicted in Figure 1, with complex potential $f(z)$ defined implicitly by the pair of equations

$$\begin{aligned} f &= \zeta - \log \zeta \\ z &= \zeta + 4\sqrt{\zeta} + \log \zeta \end{aligned} \tag{1}$$

The fluid velocity is given by

$$u - iv = \frac{\sqrt{\zeta} - 1}{\sqrt{\zeta} + 1} \tag{2}$$

which has unit magnitude when ζ is real and negative. Thus the negative real ζ axis is a free surface of constant pressure in the absence of gravity, and it is also the streamline $\psi = \pi$. The shape of this free surface in the physical (x, y) plane is given by the equation

$$x = -r + \log r; \quad r = (\pi + y)^2/16 \tag{3}$$

The free surface is asymptotically the parabola $y = -4\sqrt{-x}$ far upstream, becomes vertical at $(-1, -\pi - 4)$, and then turns back, finally approaching the plane $y = -\pi$ as $x \to -\infty$.

On the other hand the positive real ζ axis is the streamline $\psi = 0$ on which $y = 0$, and thus represents a horizontal plane wall. There is a stagnation point on that wall at $(5, 0)$ at which a stagnation streamline $\psi = 0$ bifurcates. Figure 1 shows sample streamlines including the free surface, wall, and stagnation streamlines.

This flow thus begins as a right-flowing unit-magnitude uniform stream at infinity. The top layer of thickness π is then diverted backward into a left-flowing stream, the lower surface of which is free, while the upper surface is attached to the horizontal wall. This left-flowing stream is the local representation of the bow splash or jet. Its free surface is gravity-free since the effects of gravity are negligible on the small length scales of the local bow flow, where fluid accelerations far exceed gravitational accelerations.

The above flow can easily be re-scaled to give a general character to its dimensions. Thus the flow with complex potential $U(\delta/\pi)f(z\pi/\delta)$ has a uniform stream U and a jet of thickness δ. The fluid velocity at infinity then behaves as

$$u - iv \to U - 2U\sqrt{\delta/\pi}\, z^{-1/2} \tag{4}$$

Then if p denotes the excess of pressure over the free-stream value, this implies that on the wall $y = 0$ as $x \to +\infty$,

$$p \to 2\rho U^2 \sqrt{\frac{\delta}{\pi x}} \tag{5}$$

exhibiting an inverse square root behaviour with respect to x in the outer expansion of this inner solution. This can be matched with the inner (near leading edge) expansion of an outer flow, described by linearised planing surface theory, with an inverse square root singularity in the pressure distribution as $x \to 0$. Explicit matching then relates the strength of this pressure singularity to the jet thickness and ultimately the splash drag.

In carrying out this matching by the method of matched asymptotic expansions (Van Dyke 1975), it is important to note that the inner scale is of order $\delta = \epsilon^2$, where ϵ measures the size of the draft or angle of attack, and hence measures the over-all magnitude of the outer disturbance flow. The formal inner region is defined by order unity values of inner coordinates (X, Y) defined in terms of outer coordinates (x, y) by

$$\begin{aligned} x &= x_{\text{bow}} + \epsilon^2 X \\ y &= y_{\text{bow}} + \epsilon^2 Y \end{aligned} \tag{6}$$

Here $(x_{\text{bow}}, y_{\text{bow}})$ defines the "bow" or leading edge, which is the apparent point of contact between the free surface (for $x < x_{\text{bow}}$) and the wetted body (for $x > x_{\text{bow}}$), at least as seen in the outer domain.

There is of course no such well-defined point of contact in the general case of arbitrary ϵ, since splash formation occurs near $(x_{\text{bow}}, y_{\text{bow}})$, and in principle the body is wetted by the splash far upstream as well as downstream of that point, at least until gravity re-asserts its influence and the splash falls back on the oncoming stream. However, there is a unique wetted length in the linearised outer flow, and the values of $x_{\text{bow}} = O(1)$ and $y_{\text{bow}} = O(\epsilon)$ are determined by that linearised solution, for any given body shape. These can be thought of as outer approximations to the location of the stagnation point in Figure 1, since the inner corrections to this location are $O(\epsilon^2)$. It is also because of this $O(\epsilon^2)$ stretching in (6) that the local representation of the body surface is a horizontal plane, irrespective of any $O(\epsilon)$ slopes or curvatures of the body; Figure 1 is a universal local bow flow valid for all planing surfaces.

REFERENCES

Cumberbatch, E., "Two-dimensional planing at high Froude number", *J.Fluid Mech.* **4** (1958) 466-478.

Lighthill, M.J., "A new approach to thin aerofoil theory", *Aeronaut. Quart.* **3** (1951) 193-210.

Maruo, H., "Two-dimensional theory of the hydroplane", Proc. 1st Japan Nat. Congr. for Appl. Mech., Tokyo, 1951.

Oertel, R.P., "The steady motion of a flat ship, including an investigation of the local flow near the bow", Ph.D. thesis, The University of Adelaide, 1975.

Squire, H.B., "The motion of a simple wedge along the water surface", *Proc. Roy. Soc. Lond. Ser. A* **243** (1957) 48-64.

Tuck E.O., "A criterion for leading-edge separation", *J.Fluid Mech.* **222** (1991) 33-37.

Tuck E.O., "Planing surfaces", Chapter 7 (pp 363-372) of *Numerical solution of integral equations*, ed. M. Golberg, Plenum Press, New York, 1990.

Van Dyke, M., *Perturbation Methods in Fluid Mechanics*, Parabolic Press, 1975.

Wagner, H., *Zeit. Angew. Math. Mech.* **12** (1932) 193.

Wehausen, J.V. and Laitone, E.V., "Surface Waves", in *Handbuch der Physik*, Vol 9, ed. S. Flugge, Springer-Verlag, 1960.

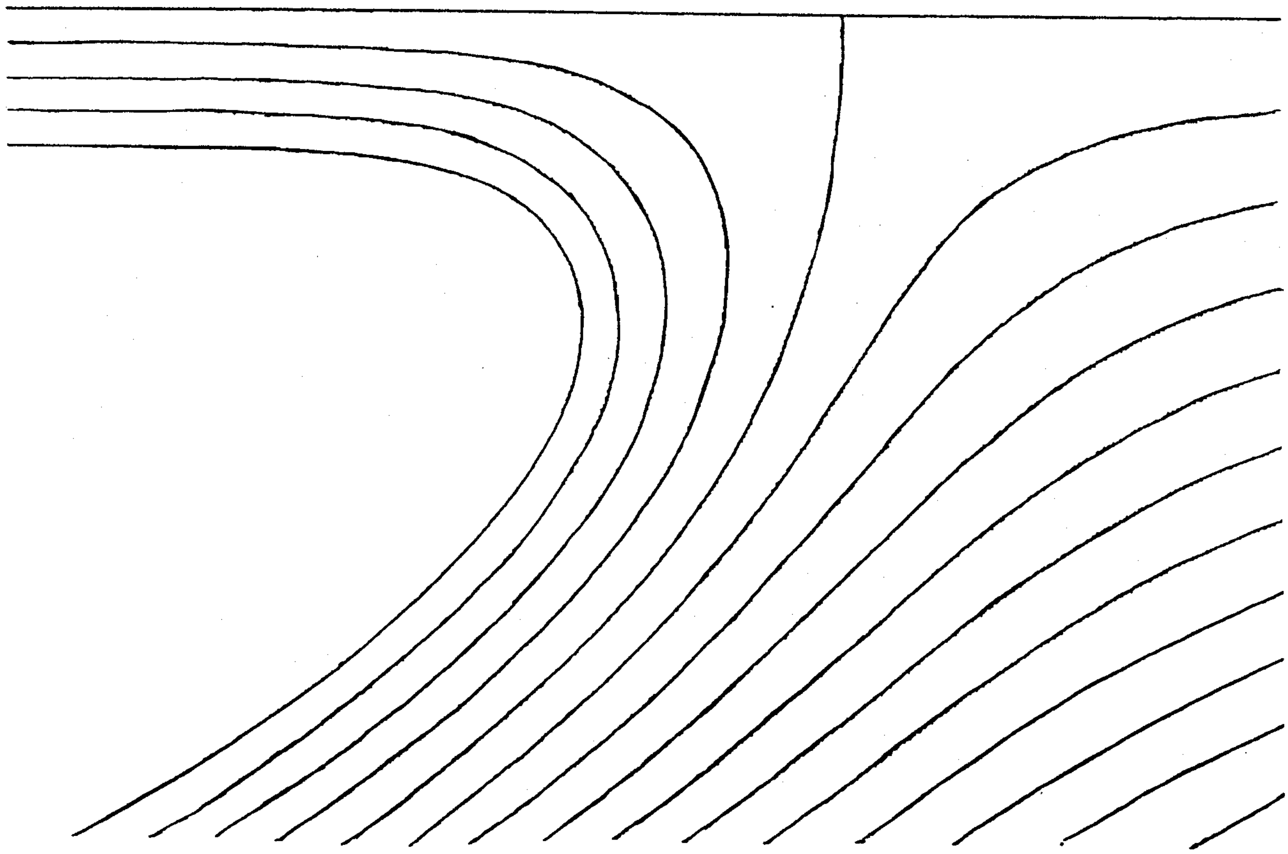

FIGURE 1.

DISCUSSION

Kajitani H.: Though my question seems to be a little outside from your present work, I consider that the inclusion of viscosity is important around quick turn of the flow hitting the inclined flat plate. We appreciate if you give us fundamental features of viscous effect on determining the physical structure of the planing splash.

Tuck E.O.: The spirit in which I have discussed the spray-root (inner) region for planing flows is that there are large curvatures of streamlines there, so that inertial accelerations of fluid particles are large, and therefore viscous forces are (locally) negligible. Viscous drag then arises not from the inner region, but rather in the outer region, and is proportional to the wetted length, to a reasonable approximation. However, I do understand that there may be indirect viscous effects in the inner region, especially if the spray jet collapse on the incoming flow, so creating a "bow vortex" with big energy dissipation. However, if α is small, the jet size $\delta = O(\alpha^2)$ is so small that this is assumed to be unimportant. (But it is very important for bluff bows!)

Palm E.: The resistance is due to two effects: the wave resistance and the splash. If you consider a body (planing) with curvature, you can minimize the splash-resistance. Would that mean that the total resistance becomes smaller?

Tuck E.O.: The quick answer is "Yes"! In fact, at any Froude number, it is possible to curve the body so as to make the splash resistance vanish – by eliminating the splash entirely. (The curvature is not too practical, being concave downwards.) It is interesting that this is quite like "shock-free entry" for aerodynamics, i.e. the choice of camber for an airfoil so as to eliminate the leading-edge singularity. In the present paper, I have only discussed the flat plate case, but indeed it would be very interesting to discuss curved bodies, and the relationship between δ and body curvature for such bodies.

Korobkin A.: What do you think about a possibility to construct higher-order approximations of the solution as $\alpha \rightarrow 0$?

Tuck E.O.: In general this would be very difficult, and perhaps better option is a full numerical solution for arbitray α. However, one aspect which would be difficult for such a numerical program, but where a second-order expansion might be useful, concerns the "ultimate fate" of the splash jet. Here I have assumed that, because its thickness is $O(\alpha^2)$, we do not need to worry about this matter. But maybe it should be studied.

Tulin M.: Since your splash model is in the gravity free limit, what is its region of applicability for finite Froude numbers?

Tuck E.O.: The theory is a small-α but arbitrary $F_\ell = U/\sqrt{g\ell}$ theory, where ℓ is the wetted length. The theory says that the inner region is of size $O(\alpha^2 l) = \delta$, so the local Froude number there is $F_\delta = U/\sqrt{g\delta} = F_\ell/\alpha$ in order, i.e. large F_δ; hence neglect of g in the inner zone. But you are really thinking how small can F_ℓ be. Obviously not as small as α, since if $F_\ell = O(\alpha)$, $F_\delta = O(1)$. In practice, I can compute down to zero speed, but there is a practical limit (discussed by Squire) below which the flow fails to separate cleanly from the trailing edge, i.e. the "Kutta" condition is not satisfied.

Newman J.N.: Can your inner solution be used in the three-dimensional problem?

Tuck E.O: The aim of such an extension would be to study flow which is approximately two-dimensional in a plane which is oblique to the free-stream direction, but normal to the hull. This extension is very challenging. It perhaps interpolates between the strictly two-dimensional (in a plane containing the free stream) theory of the present paper, and the slender-body theory of Tulin (yesterday's session) where is approximately two-dimensional in a cross-flow plane (normal to the free stream). I recommend this as a good extension to consider by others interested in this class of problems.