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SECOND ORDER RADIATION WAVES AT FORWARD SPEED

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We are interested in second order hydrodynamics forces on a ship running in waves at forward speed, because the better understanding of their mechanism will be essential for the accurate prediction of seakeeping at rough seas. No information is ever available either experimentally or theoretically on second order oscillatory forces on a ship in waves at forwad speed, while we have relatively much on floating bodies at zero forward speed.

Correct measurement of the second order hydrodynamic pressures is not so easy task for its smallness. So I first tried to measure the second order wave field generated by radiatin and diffraction when a ship has forward speed. Measured wave elevation is generally consistent on the repeated experiment and it is not so hard to achieve the accuracy. This attempt is described here only for the radiation waves, though we are planning to extend the method to more difficult but important problem of the diffraction waves in the near future.

Method to measure the second order radiation waves generated by the sinusoidal oscillation of a ship at forward speed is not so different from what I developed for the linear radiation waves. Place several wave probes fixed to the water tank and on a line parallel to the ship model's track, with an identical spacing Δx between each probe. Each probe comes at different time instants to an identical location in the reference frame moving with the ship's forward motion. This means that the wave probes record the wave elevation on that location at several different time instants whose interval is $\Delta x/U$. U is the ship speed.

Temporal and spatial variation of the radiation waves will be give by

$$\zeta(x,y,t) = \eta_0(x,y) + \eta_1(x,y)e^{i\omega t} + \eta_2(x,y)e^{i2\omega t} + \text{3rd order terms} \tag{1}$$

where the coordinates are defined in the right-hand system. The x axis coincides with the centerline of the ship on the equilibrium water surface and directs forward of the ship with the origin at FP. The z axis is vertically upward.

The first term on the right of equation (1) is the steady part corresponding to the Kelvin wave pattern. Naturally it might include the second order steady component. The second is the linear oscillatory part and the third the second order oscillatory part we are concerned here.

Obtain the wave elevation records $\zeta(x,y,t)$ at a location (x,y) at several different time instants. If the interval between those instants at which the waves are recorded is short enough and the number of the time instants is sufficiently large, we fit those records with the equation (1) to determine η_0, η_1 and η_2 simultaneously at this location. Fig.1 is an example of this fitting of the data. We used twelve wave probes. As seen from this figure introduction of the second order term into the equation (1) was clearly appropriate.

Those wave probes record the wave elevation everywhere on y =constant and a run of the experiment can dtermine the distribution of $\eta_{0,1,2}$ on this line. Fig.2 gives the distribution of the absolute value of η_1 on two different ys for a series 60 ship model of $C_B = 0.8$ at ballast condition. The model was forced to heave at the period of 0.6seconds and with the amplitude of $0.025L$. The ship model length L is $2m$. Some fluctuations of the curves in those figures are caused by one probe picking up AC current noise of 60 Herz, but does not affect the general accuracy of the mesurement. However this must be improved in next chance of the experiment.

Fig.3 shows the measured η_2. It is 20 to 30 percent of η_1 and will not be ignored. One reason is relatively large amplitude of heaving. No significant differnce is observed between η_1 and η_2 in the attenuation rate with the increase of y. There is no difference too between the steady part $\eta_0(x, y)$ obtained at this experiment, which I do not show here, and that at much smaller amplitude of heaving.

Maybe next step of the research will be to know implication of this rather large second order radiation waves in the context of hydrodynamic forces on the ship.

Free wave component of $\eta_1(x, y)$ which will be dominant at away from the hull surface is expressed by

$$\eta_1(x,y) = \int_{-\infty}^{\infty} F(\lambda)e^{-i(\lambda x+\mu y)}d\lambda \tag{2}$$

where λ and μ have to be determined such that a linear free surface condition is satsfied. A range of λ to which μ does not exist is excluded from the integral, because we are concerned with the free wave component.

A technique is already established of obtaining experimentally the function $F(\lambda)$ with the Fourier transform of the measured radiation waves η_1 on a line of y =constant. Practically an extrapolation of the wave records to x larger than some value is necessary in this process. Some results were presented at the 7th Workshop (A little different expression of equation (2) was used there). We are well aware that hydrodynamic forces such as added resistance and wave damping derived from the measured $F(\lambda)$ are informative to investigate accuracy of their theoretical predictions.

When the linear solution $F(\lambda)$ is known, the second order term $\eta_2(x, y)$ will be given by

$$\begin{aligned}\eta_2(x,y) = &\int_{-\infty}^{\infty} G(\nu)e^{-i(\nu x+\tau y)}d\nu \\ &+ \int_{-\infty}^{\infty} d\lambda_1 \int_{-\infty}^{\infty} d\lambda_2 H(\lambda_1,\lambda_2)F(\lambda_1)F(\lambda_2)e^{-i[(\lambda_1+\lambda_2)x+(\mu_1+\mu_2)y]}\end{aligned} \tag{3}$$

The first term is a part satisfying the homogeneous free surface condition at the frequency 2ω and the pair (ν, τ) is determined to satisfy this free surface condition. The second comes from a particular solution satisfying the free surface condition of the frequency 2ω with the quadratic forcing term on the right hand side. The relation of λ_i and $\mu_i (i = 1, 2)$ is given by the linear free surface condition. The weight function H is a algebraic function which is not written here for the sake of brevity.

The Fourier transform of equation (3) with respect to x is straightforward as follows.

$$\int_{-\infty}^{\infty} \eta_2(x,y) e^{isx} dx = -2\pi G(s) e^{-i\tau y}$$
$$-2\pi \int_{-\infty}^{\infty} H(\frac{s+\xi}{2}, \frac{s-\xi}{2}) F(\frac{s+\xi}{2}) F(\frac{s-\xi}{2}) e^{-i(\mu_1+\mu_2)y} d\xi \qquad (4)$$

Substitute the functions F given with the analysis of the measured η_1 as described previously into equation (4), then the Fourier transform of the measured $\eta_2(x,y)$ with respect to x will give the function $G(\nu)$. The result is that we are able to know experimentally the velocity potential of the second order radiation waves of the frequency of 2ω at away from the ship model.

Further evaluation will be presented of the magnitude, relative to the first order, of the heave damping determined by the empirical second order radiation waves thus obtained

Reference

Ohkusu, M., 1992 Radiation and Diffraction Waves at High Froude Number, 7th International Workshop of Water Waves and Floating Bodies, Val de Reuil

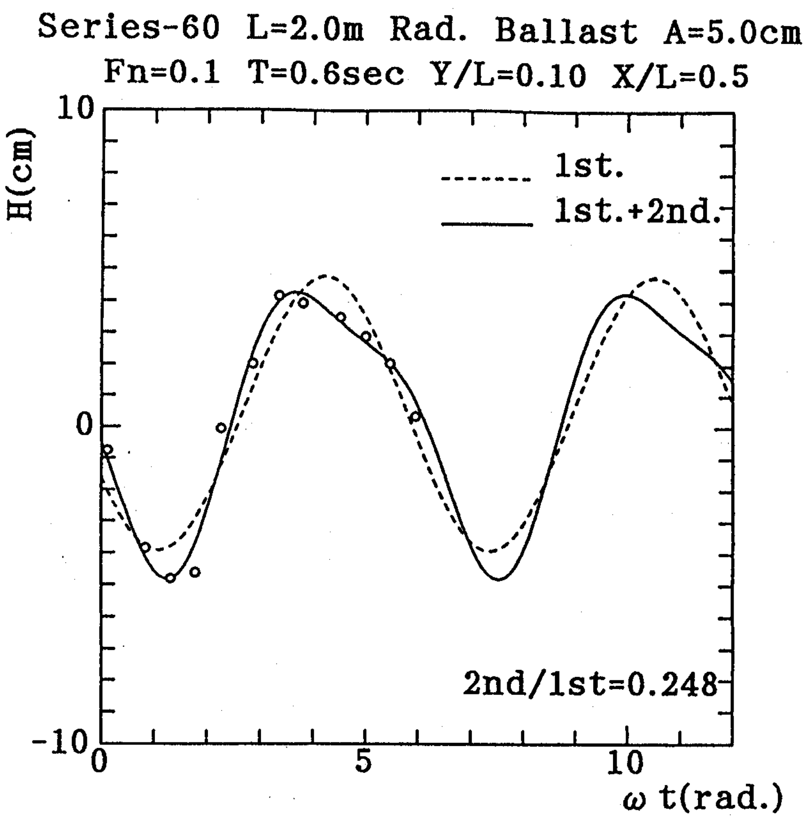

Fig.1 Time Series Fitting of Radiation Waves at a Spot

Ballast

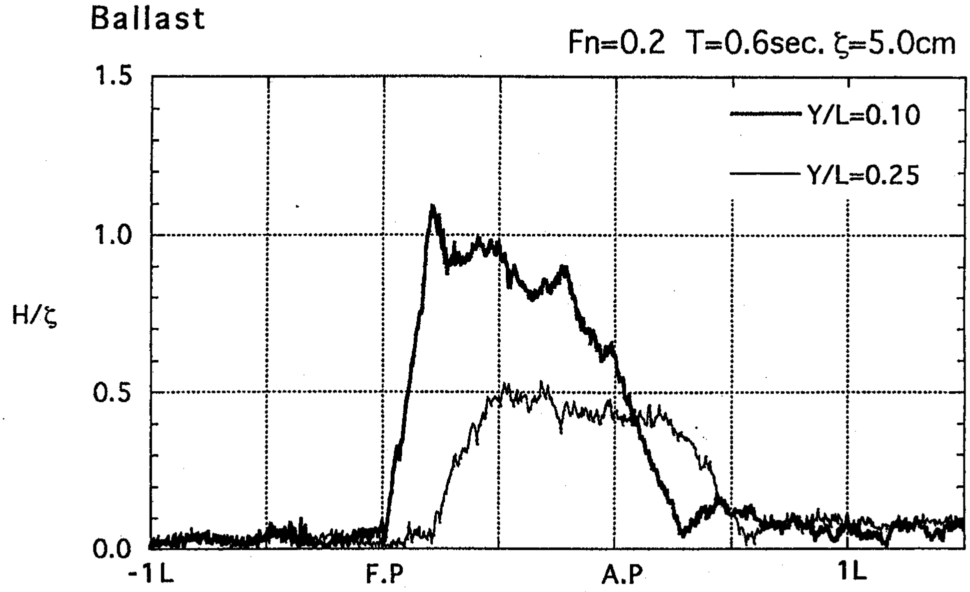

Fig2. Amplitude of First Order Radiation Waves

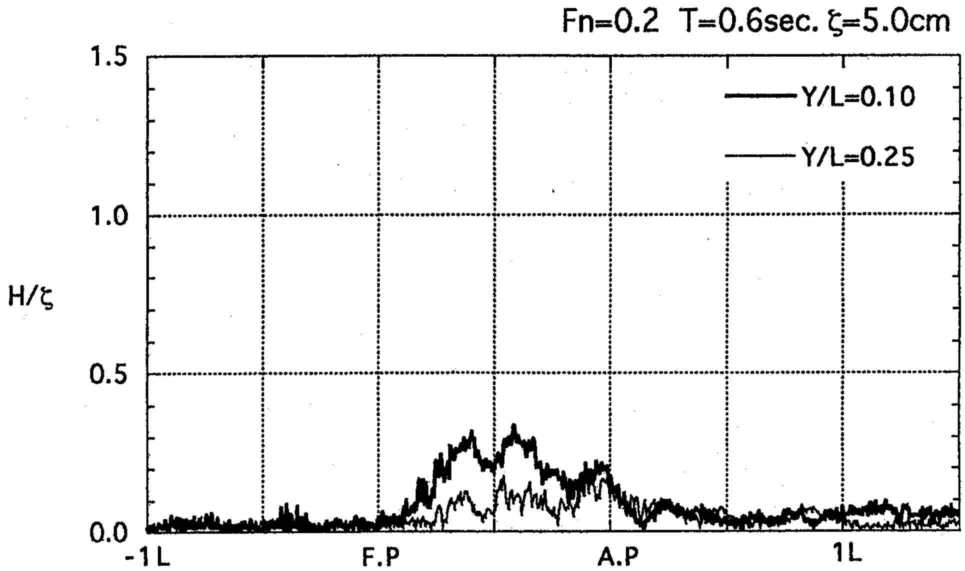

Fig3. Amplitude of Second Order Radiation Waves

DISCUSSION

Newman J.N.: What is the connection between the second-harmonic wave amplitude and the added resistance?

Ohkusu M.: I have two alternative ideas to respond to the question. But at moment I do not know which is correct. More investigation will be required to know it. (1) If 2nd order higher harmonics of waves are restricted within the region very close to the hull surface, then they are standing waves and no contribution to the added resistance. (2) If they propagate further, they transmit the wave energy and the added resistance will increase.