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NUMERICAL PREDICTION OF SECOND—ORDER WAVE FORCES ON A TWIN—CYLINDER ARRAY

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1. Introduction

The existing approaches available for evaluating the second—order wave forces on offshore structures are identified into two categories, i. e.,

1. to solve the second—order boundary—value problem of wave potential including the inhomogeneous boundary condition at the undisturbed free surface, and eventually calculate the resultant second—order wave force by integration of the second—order wave potential over the wetted surface of the structure.

2. to calculate directly the second—order wave force, without solving the second—order boundary—value problem of the second—order wave potential, from a fictitious radiated wave potential of double frequency and the first—order scattered wave potential with the aid of the extended Haskind relation.

Applying the approaches from category 1, a variety of solutions of the second—order wave force on specific offshore structures have been published. However, one can find by a careful review, that even for the diffraction problem of wave propagating against a single vertical cylinder, the second—order wave forces obtainde by different authors reveal apparent discrepances, which are mostly attributed to different boundary conditions at infinity supplemented to the mathematical formulations of the boundary—value problem of the second—order wave potential.

In a previous paper the authors investigated the boundary condition at infinity incorporated to the second—order wave theory a proper mathematical expression based on fully analysis in physical sense has been developed. The newly derived inhomogeneous boundary condition at infinity makes the second—order wave theory complete in mathematical respect. Based on the new model of the second order wave diffraction theory, the wave diffraction against two—cylinder array is investigated in this paper.

2. Theoretical Description

Assume the fluid is inviscid and incompressible, the flow irrotational. Since the flow can be expressed by a velocity potential which is governed by the Laplace equation, and is subject to certain boundary conditions. Applying perturbation expansion, the velocity potential is expanded into a power series of first, second, and even higher—order potentials.

$$\Phi(x, y, z, t) = \varepsilon\varphi_1 e^{-i\omega t} + \varepsilon^2\varphi_2 e^{-2i\omega t} + \cdots\cdots$$

$$\eta(x, z, t) = \varepsilon\eta_1 + \varepsilon^2\eta_2 + \cdots\cdots$$

φ_1, φ_2—the first and second—order velocity potential, being defined as

$$\varphi_1 = \varphi_1^I + \varphi_1^S \qquad \varphi_2 = \varphi_2^I + \varphi_2^S$$

φ_1^I, φ_2^I—the incident wave potential, φ_1^S, φ_2^S—the scattered wave potential.

We obtained the following mathematical model

Second—order boundary value problem

$$\nabla^2\varphi_2^S = 0 \qquad in \qquad \Omega \tag{1}$$

$$\frac{\partial\varphi_2^S}{\partial Z} - \frac{4\omega^2}{g}qn = F^{IS} + F^{SS}, \ on \quad Z = 0 \tag{2}$$

$$\frac{\partial\varphi_2^S}{\partial n} = 0 \qquad on \quad Z = -d \tag{3}$$

$$\frac{\partial\varphi_2^S}{\partial n} = -\frac{\partial\varphi_S^I}{\partial n}, \qquad on\ body\ surface \tag{4}$$

$$boundary\ condition\ at\ infinity \tag{5}$$

where

$$F^{IS} = -\frac{i\omega}{2g}\Phi_1^I\frac{\partial}{\partial z}\left(\frac{\partial\Phi_1^S}{\partial z} - \frac{\omega^2}{g}\Phi_1^S\right) - -\frac{i\omega}{2g}\Phi_1^S\frac{\partial}{\partial z}\left(\frac{\partial\Phi_1^I}{\partial z} - \frac{\omega^2}{g}\Phi_1^I\right)$$
$$+ (2i\omega/g)(\nabla\Phi_1^I \cdot \nabla\Phi_1^S)$$

$$F^{SS} = -\frac{i\omega}{2g}\Phi_1^S\frac{\partial}{\partial z}\left(\frac{\partial\Phi_1^S}{\partial z} - \frac{\omega^2}{g}\Phi_1^S\right) + \frac{i\omega}{g}(\nabla\Phi_1^S)^2$$

the condition at infinity is

$$\lim_{r\to\infty} r^{\frac{1}{2}}\left(\frac{\partial\Phi_2^S}{\partial r} - i\lambda\Phi_2^S\right) = F^*(z,\theta)\cdot e^{ikr}(1+cos\theta) \tag{6}$$

and

$$\lambda th(\lambda d) = 4\omega^2/g$$

$$F^* = \{\{i[R(1+cos\theta) - \lambda](i\omega/g)ch^3(kd)k^2[3th^2(kd) - 1 - 2cos\theta].$$
$$A^2B_0(\theta)ch\lambda^*(z+d)\}/[\lambda^* sh(\lambda^* d) - (4\omega^2/g)ch(\lambda^* d)]\}$$

$$\lambda^* = k(2 + 2cos\theta)^{1/2}$$

The condition at infinity (6)makes the second—order problem well—posed with an unique solution.

3. Infinite Element/Finite Element Method

Equs. (1)to(6)were solved numerically by an infinite/finite element method. The solution domain was devided into an exterior region and an interior region. The boundary—value problem referring to the interior region was solved by means of FEM. Both the functionals defined over the interior and exterior domain are written as

$$\pi_{2I}(\Phi_2^S) = (1/2)\iiint_{\Omega_I}(\nabla\Phi_2^S)^2 dv = \iint_{S_{FI}}[(2\omega^2/g]\Phi_2^S$$
$$+ F^{IS} + F^{SS}]\Phi_2^S ds + \iint_{S_B}(\partial\Phi_2^I/\partial n)\Phi_2^S ds \tag{7}$$

$$\pi_{2E}(\Phi_2^S) = (1/2)\iiint_{\Omega_E}(\nabla\Phi_2^S)^2 dv = \iint_{S_{FE}}[(2\omega^2/g]\Phi_2^S$$
$$+ F^{IS} + F^{SS}]\Phi_2^S ds - \iint_{S_\infty}[(i\lambda/2)\Phi_2^S + F^* r^{-1/2}]\Phi_2^S ds \tag{8}$$

Since the asymptotic solution at infinity Φ_2^S is composed of $\Phi_2^{S''}$(free wave)$+\Phi_2^{S'}$(forced response wave), rewrite the functional defined over the exterior domain in the form of

$$\pi_{2E}(\Phi_2^{S''},\Phi_2^{S'}) = (1/2)\iiint_{\Omega_E}(\nabla\Phi\Phi_2^{S'})^2 dv - (2\omega^2/g)\iint_{S_{FE}}(\Phi_2^{S'})^2 ds$$

$$- (i\lambda/2)\iint_{S_\infty}(\Phi_2^{s''})^2 ds - \iint_{S_j}(\partial\Phi_2^{s''}/\partial r)\Phi_2^{s''} ds \tag{9}$$

20—node hexahedron isoparameteric element is adopted for the spatial discretization in the interior domain, whereas 8—node hexahedron isoparametric infinite element is used in the exterior domain. The form along the radius direction of the interpolation function, suppose that a cylindrical coordinate system is chosen with the Z—axis is set up at the mid—point between the two cylinders, can be defined in accordance with the asymptotic solution at far field of the problem. Assume the interpolation function along r

$$\Phi_2^{s''} = e^{i\lambda(r-b)}\{\Phi_b(\theta,z)(r/b)^{-3/2} + D(\theta,z)[(r/b)^{-1/2} - (r/b)^{-3/2}]\} \tag{10}$$

At the common boundary which interconnecting the exterior and interior domain, the form in terms of θ and Z of the interpolation function defined in exterior domain degenarates to that of interpolation function defined in the interior domain. Consider the interpolation along θ adn Z on boundary S_j for the undefined functions $\Phi_b(\theta,Z)$and $D(\theta,Z)$. From(10), $[\Phi_2^{s''}]_{r=b} = \Phi_b(\theta,Z)$

it implies, $\Phi_b(\theta,Z)$is the free wave potential $\Phi_2^{s''}$on S_j, therefore, we write $\Phi_b(\theta,Z) = [\Phi_2^{s}]_{r=b}$

$-[\Phi_2^{s'}]_{r=b}$, then, we have
$$\Phi_b(\theta,Z) = \sum_{i=1} M_i\Phi_{2i}^{s} - [\Phi_2^{s'}]_{r=b} \tag{11}$$

where shape function M_i takes the degenerating form on S_j of shape function N_i of FE available over the interior domain. It assures the second—order wave potential being continuous and consistent at the common boundary.

Once the second—order wave potential wave solved, the wave forces applied on each of the two cylinders are calculated by surface integration. In addition, the wave runup over the cylinders can be computed at any instant. The effect of the incident wave angle upon the wave forces applied on the front and rear cylinder is important in this study, so that the numerical analysis has to be conducted in symmetrical and asymmetrical cases.

4. Results and Discussions

Figure 3 and Figure 4 show, respectively the wave forces applied on the front and rear cylinders in the case where the incident wave angle is zero. Obviously, the magnitude of wave force for the front cylinder is larger than that of the rear cylinder.

When the incident wave angle increases to some extent, it turns into the asymmetrical case, the effects of incident angle upon the wave forces are evidently shown. As the incident angle approaches to 90°, the wave forces on either cylinder vary to a common magnitude.

The distance between the two cylinders is another facetor that affects the magnitude of the wave forces applied on the two cylinders. Numerical data has shown that as the distance increases to the extent of about six radius of the cylinder, the wave forces applied on the two—cylinder array become the same as the wave force on a single cylinder in the process of wave propagation.

Comparisons between the computed results obtained herewith and the data calculated by Masuda[2] indicate that agreement is good and satisfactory.

References

Li, B. Y. and Lu, Y. L. 1991. The condition at infinity for second—order wave diffraction, Int, Journal of Offshore and Polar Engineering. Vol. 1, No. 1

Masuda, K. et al. 1991. Nonlinear wave forces on a pair of vertical cylinders, Journal of OMAE, Vol. 113/1.

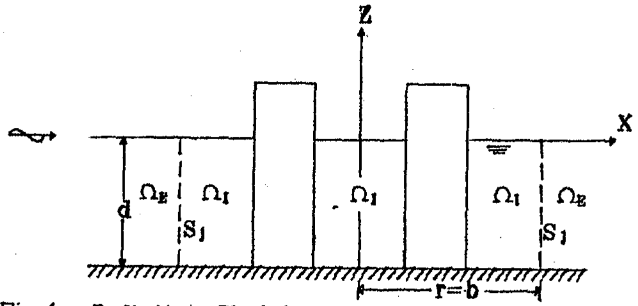

Fig.1 Defintion Sketch

Ω_I Ω_E—inner and outer—domain

S_I—fictitious boundary connecting Ω_I and Ω_E

d—water depth below SWL

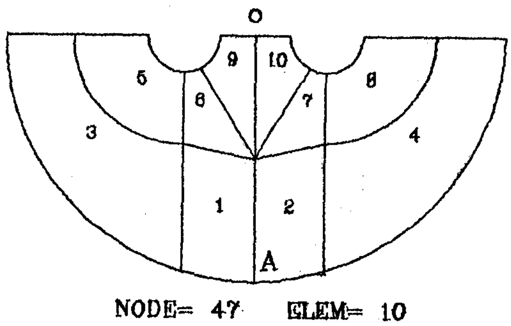

Fig.2 Elements layout in the interior domain

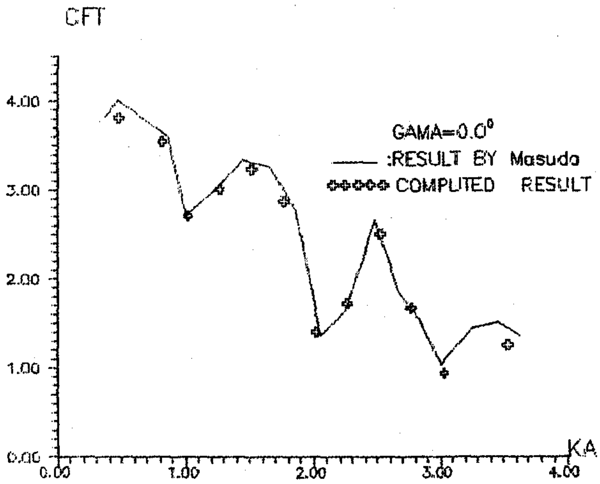

Fig.3 Maximum horizontal wave force on no.1 cylinder

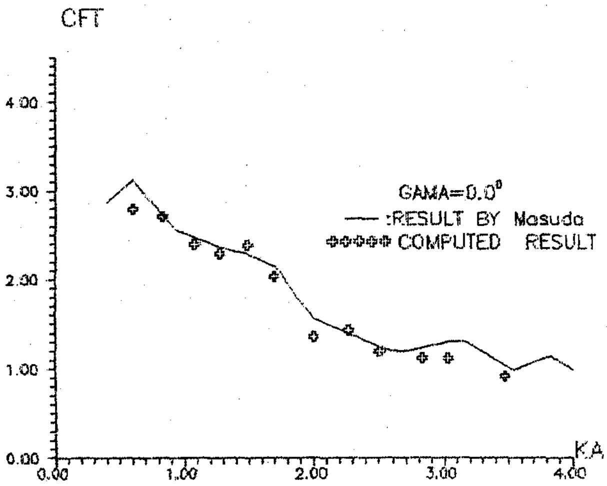

Fig.4 Maximum horizontal wave force on no.2 cylinder

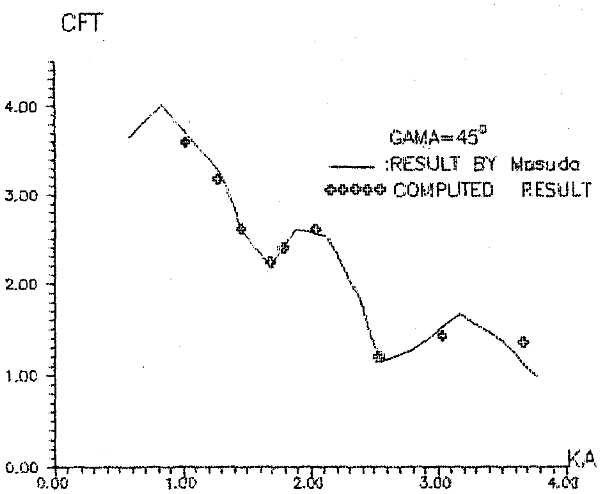

Fig.5 Maximum horizontal wave force on no.1 cylinder

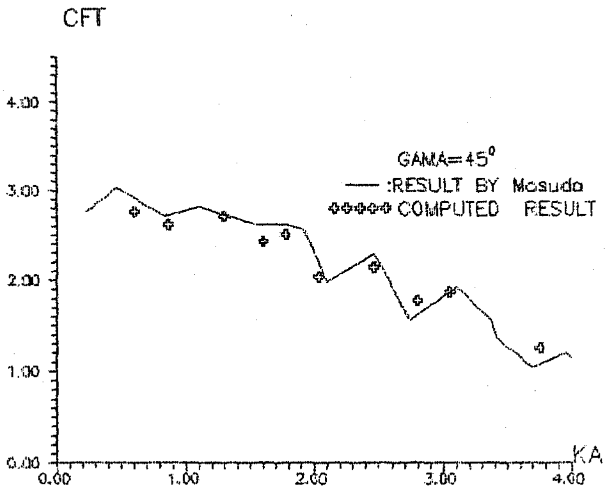

Fig.6 Maximum horizontal wave force on no.2 cylinder

DISCUSSION

Wang B.: Infinite element can deal with the radiation condition if we consider the free surface condition imposed on the even free surface. Is it possible to use it to impose free surface condition on the real wave surface?

Lu Y.: In this paper, we impose on the even free surface.