Hydroelastic study of wetdeck slamming by a Timoshenko beam model

Jan Kvålsvold and Odd Faltinsen
Department of Marine Hydrodynamics, Norwegian Institute of Technology

1 Introduction

Slamming against the wetdeck of a catamaran in head sea waves is studied. By wetdeck we are referring to the structural part connecting the two sidehulls of the catamaran. The local hydroelastic effects of wetdeck slamming are considered. Slamming has been widely studied in the literature, but very few studies include the hydroelastic effect. In this study the work by Kvålsvold and Faltinsen [1993] is extended to account for the interaction effects between the local hydrodynamic impact forces and the heave and the pitch motions of the catamaran. In addition, the two dimensional structural formulation is improved by accounting for the shear deformations as well as the rotatory inertia effects of the wetdeck.

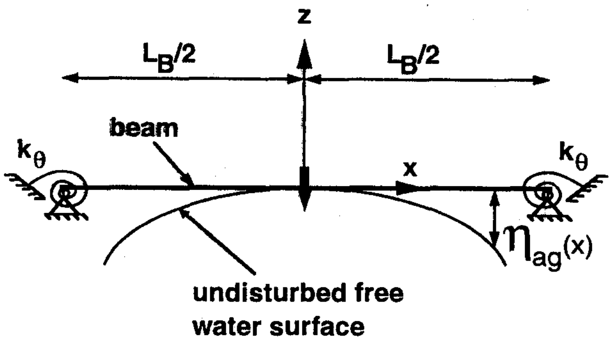

Figure 1: The definition of the local coordinate system.

2 Theory

The local xyz-coordinate system which moves with the forward speed U of the catamaran will now be defined. The x-axis is parallell to the longitudinal stiffeners in the wetdeck and is pointing towards the stern of the catamaran, the y-axis is parallel to the transverse stiffeners and is pointing towards the starboard side. The z-axis is pointing upwards. The origin of the local coordinate system is located at the line of initial impact between the free surface of the waves and the wetdeck structure. The study is limited by that the crest of a regular wave system hits the wetdeck mid between two of the transverse stiffeners.

The analysed part of the wetdeck between two of the transverse stiffeners is modelled as a Timoshenko beam with length L_B corresponding to the distance between the stiffeners. This means that the deflections of the beam are totally dominated by those of the longitudinal stiffeners. Local deformations of the plate field between two of the longitudinal stiffeners is a three dimensional effect which is not covered by this two-dimensional analysis. The transverse stiffeners are assumed to be much stiffer than the longitudinal stiffeners, so that the vertical deflections at the beam ends can be neglected. Rotatory springs are introduced at the beam ends in order to account for the restoring moment of the part of the wetdeck outside the modelled beam. k_θ is the rotatory spring coefficient. One should note that this approach does not properly account for the inertia effects of the wetdeck outside the modelled beam. It is also assumed that the beam is horizontal at the moment of initial water impact, so that both the beam deflections and the fluid flow are symmetric about the vertical z-axis. The local coordinate system together with the definition of the beam are shown Figure 1. $\eta_{ag}(x)$ is the airgap between the undisturbed free surface of the waves and the wetdeck.

If force and moment equilibrium of a beam element are required, the following governing coupled beam equations of motions arise:

$$M_B\ddot{w} + GA_s\left(\frac{\partial\beta}{\partial x} - \frac{\partial^2 w}{\partial x^2}\right) + M_B\dot{V}_g(t) = p(x,w,t) \quad (1)$$

$$M_B r^2\ddot{\beta} + GA_s\left(\beta - \frac{\partial w}{\partial x}\right) - EI\frac{\partial^2\beta}{\partial x^2} = 0 \quad (2)$$

M_B is the mass per unit length of the longitudinal stiffener together with the flange, divided by the width of the flange and $w(x,t)$ is the beam deflection which is to be interpreted as the difference between the actual vertical position of the wetdeck and the vertical position due to the rigid ship motions. G is the shear modulus, A_s is the shear area divided by the width of the flange and r is the mass radius of gyration of the beam cross section. Further, E is the Youngs modulus and I is the area moment of inertia of the beam cross section and divided by the width of the effective flange. $\dot{V}_g(t)$ is the local vertical accelerations due to heave and pitch motions of the ship. $p(x, w, t)$ is the hydrodynamic impact pressure which is a function of both time and space but due to the hydroelastic effects also a function of the beam deflections. Dot stands for the time derivative and t is the time variable. $\beta(x,t)$ is the slope of the deflection curve when the shear deformations are neglected. Then the total rotation angle $\frac{\partial w(x,t)}{\partial x}$ of a beam element is related to the shear angle $\gamma_s(x,t)$ as well as to $\beta(x,t)$ through the relation $\frac{\partial w(x,t)}{\partial x} = \gamma_s(x,t) + \beta(x,t)$. The solutions of $w(x,t)$ and $\beta(x,t)$ are expressed in terms of the beam's "dry" normal modes:

$$w(x,t) = \sum_{n=1}^{\infty} a_n(t)\psi_n(x) \quad (3)$$

$$\beta(x,t) = \sum_{n=1}^{\infty} a_n(t)\phi_n(x) \quad (4)$$

where $a_n(t)$ is the principal coordinate of vibration mode n and $\psi_n(x)$ is the eigenfunction of vibration mode n. $\phi_n(x)$ is related to $\psi_n(x)$ through the coupled equations of motions (1) and (2) with $\dot{V}_g(t)$ and $p(x, w, t)$ set equal to zero. Four beam end boundary conditions are needed in order to determine the eigenfunction $\psi_n(x)$ together with the eigenfrequency ω_n of vibration mode n. Equations (3) and (4) are substituted into equations (1) and (2) which are multiplied by $\psi_m(x)$ and $\phi_m(x)$; respectively. The arising equations are integrated over the length of the beam and added together. By using the orthogonality conditions of the beam it can be shown that the following governing modal beam equation of motions arises:

$$M_{mm}\left[\ddot{a}_m(t) + \omega_m^2 a_m(t)\right] + M_B \dot{V}_g(t) \int_{-\frac{L_B}{2}}^{\frac{L_B}{2}} \psi_m(x)dx = \int_{-c(t)}^{c(t)} p(x,w,t)\psi_m(x)dx \quad (5)$$

where

$$M_{mm} = M_B \int_{-\frac{L_B}{2}}^{\frac{L_B}{2}} \left[\psi_m^2(x) + r^2\phi_m^2(x)\right] dx \quad (6)$$

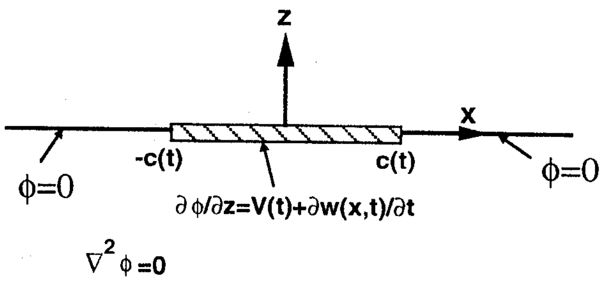

Figure 2: A simplified hydrodynamic slamming model.

In order to derive an expression of the hydrodynamic impact pressure p one needs to solve a hydrodynamic boundary value problem (HBVP). In this study a simplified two-dimensional HBVP (see Figure (2)) is set up to express the fluid flow in the impact region. It is implicitly assumed that the fluid accelerations are much greater than the acceleration of gravity g. The influence of the sidehulls on the surface elevation is not accounted for in the local hydroelastic analysis.

Assuming incompressible fluid and irrotational flow, there exists a velocity potential ϕ which satisfies the Laplace equation in the fluid domain. The boundary conditions are $\frac{\partial \phi}{\partial z} = V(t) + \dot{w}(x,t)$ on the wetted length of the beam and $\phi = 0$ on the free water surface. V is the relative velocity between the wetdeck and the waves in the impact region. $2c(t)$ is an approximation of the wetted length of the beam. One should note that $c(t)$ is unknown and part of the solution of the impact problem. The boundary conditions are transferred to $z = 0$. The hydrodynamic pressure p can be approximated by the "$-\rho\frac{\partial \phi}{\partial t}$" term in Bernoulli's equation.

First a local hydroelastic slamming analysis is carried out. This means we are considering an elastic beam which is forced with constant vertical velocity V through the crest of a regular wave system. Then $\dot{V}_g(t)$ in equation (1) is zero. The velocity V is due to the global heave and pitch motions of the catamaran and will in principle be influenced by the local slamming analysis. This will be discussed later in the text.

The fluid flow in the impact region is expressed in terms of a vortex distribution $\gamma(x,t)$ on the wetted length of the beam. The vortex density is solved from an integral equation (see Newman [1977], page 180). The vertical velocities on the wetted length of the beam are rewritten in terms of a Fourier cosine series. 20 Fourier components are used in this study. The total modal hydrodynamic force described by the right hand side of equation (5) is rewritten into three terms as:

$$\int_{-c(t)}^{c(t)} p(x,w,t)\psi_m(x)dx = - \sum_{n=1}^{\infty} B_{mn}(t)\dot{a}_n(t) - \sum_{n=1}^{\infty} A_{mn}(t)\ddot{a}_n(t) + F_{exc,m}(t) \qquad (7)$$

This expression needs some explanation. First, all the terms which are explicitly dependent on $\dot{a}_n(t)$ are collected. The force term is denoted as the modal hydrodynamic damping force and the coefficient is the modal hydrodynamic damping coefficient. Next, all the terms which are explicitly dependent on $\ddot{a}_n(t)$ are collected. The force term is denoted as the modal hydrodynamic added mass force and the coefficient is the modal hydrodynamic added mass coefficient. The remaining terms are denoted as the modal excitation force $F_{exc,m}(t)$. We know that our definition of the hydrodynamic forces may be misleading, since $A_{mn}(t)$, $B_{mn}(t)$ and $F_{exc,m}(t)$ are implicitly functions of $a_n(t)$ through their dependency on $c(t)$. The only intention for doing this, is to be able to move as much as possible of the total modal hydrodynamic force to the left hand side of equation (5). This improves the accuracy of the hydrodynamic loading as well as the stability of the time integration of the differential equations.

How to obtain $c(t)$ in the outer solution will now be discussed. In this study $c(t)$ is solved by generalizing the integral equation Wagner used in his theory. In a slamming problem where the local angles between the undisturbed free water surface and the wetdeck are small, it is important to account for the effect of the pileup water when estimating the wetted length of the beam. This is properly taken care of in this study.

3 Results

Results from the local hydroelastic slamming analysis will be presented. This means that $\dot{V}_g(t)$ in equation (1) is zero. Aluminium is selected as the wetdeck material in this study. The input data for the simulations are shown in Table 1. In the initial phase of the water impact an approximate acoustic formulation is followed by requiring that the total modal hydrodynamic force has to be below the total modal force based on the one-dimensional acoustic pressure.

In Figure 3 the total hydrodynamic force acting on the beam is shown as a function of time. The simulations are continued until the whole beam is wet. The rotatory spring stiffness is $k_\theta = \frac{3.5EI}{L_B}$ (see Figure 1). Results from both a rigid wetdeck and an elastic wetdeck are reproduced. For the elastic case, results from two different structural formulations are shown. The first is due to the elementary beam formulation (no shear deformations and rotatory inertia) and the second is due to the Timoshenko beam formulation. Following the rigid wetdeck approach, the total hydrodynamic force on the wetdeck is large and almost constant in time. By taking into account the flexibility of the wetdeck, the total hydrodynamic forces are significantly reduced. Initially, the total hydrodynamic force is equal to zero and it rapidly increases to a local maximum value. In that time range a simplified acoustic formulation is followed. Later, the impact force is reduced again. The negative force in certain time intervals indicates the possibility of cavitation. It is also clear from the figure that the shear and the rotatory inertia effects influence the hydrodynamic loading in a hydroelastic wetdeck slamming analysis.

Description	Unit	Value
Area inertia I	$[m^4/m]$	0.000011
Shear area A_s	$[m^2/m]$	0.0012
Mass M_B	$[kg/m^2]$	36.6
Beam length L_B	$[m]$	1.5
Eigenmodes N_{eig}	[-]	40
Time step dt	[s]	0.000002
Wave period T_w	$[s]$	5.7
Wave amplitude ζ_a	$[m]$	2.0
Fall velocity V	$[m/s]$	-6.0

Table 1: Input data.

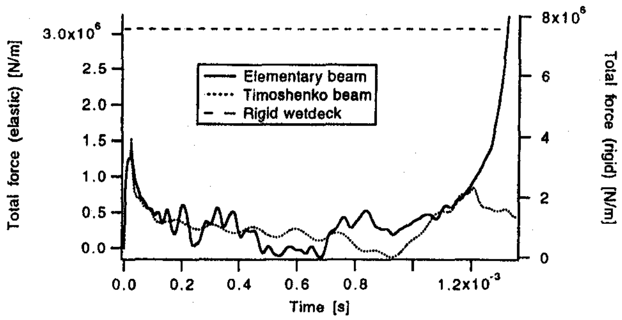

Figure 3: The total hydrodynamic forces on the beam as a function of time.

As the spray root of the jet moves towards the beam ends the total hydrodynamic force due to the elementary beam formulation increases. This part of the force curve is believed to be unrealistic since the inertia effects of the wetdeck outside the modelled beam will then become important. Such effects are not properly accounted for in this formulation and will be studied in the near future. Overall, Figure 3 shows that the total hydrodynamic forces due to wetdeck slamming is significantly reduced when the flexibility of the wetdeck is accounted for.

The interaction effects between the local hydrodynamic impact loads and the heave and the pitch motions of the catamaran have been investigated. As a basis for the coupled hydroelastic analysis we are considering the catamaran Faltinsen et al. [1992] used in their numerical and experimental study of global wave induced loads on high-speed catamarans. By Froude scaling the catamaran with the length scale ratio 22.5, this means we are considering an 85 [m] and 3000 [tons] catamaran advancing at forward speed $U = 14.1$ [m/s]. For more details about the catamaran, references are made to Faltinsen et al. [1992]. The hydrodynamic loads on the side hulls are described by a linear time domain theory based on convolution integrals and the frequency domain solution by Faltinsen and Zhao [1991]. Based on our limited numerical analysis the trend is that the larger the local hydroelastic effects are, the smaller are the interaction effects between the local slamming forces and the rigid ship motions. This is explained by two physical phenomena. First, the force impulse on the catamaran is small due to the force level and its time duration. The second reason is due to the fact that parts of the local hydrodynamic impact forces are balanced by local inertia forces.

4 Discussion

This study is based upon a two dimensional approach, both when it comes to the structural and the hydrodynamic formulations. It is expected that the three-dimensional effects will influence the results and probably reduce the total hydrodynamic forces in the initial stage of water impact. Anyhow, it is not possible to say anything how the three-dimensionalities will influence the stresses in the material. In principle, we know how to generalize this two-dimensional formulation into three dimensions. What we do not know is how to formulate the problem in order to ensure the accuracy of the solution as well as the stability of the time integration of the differential equations.

In the initial stage of water impact an approximate acoustic formulation is followed. Limited numerical studies have shown that the response of the beam is not very sensitive to the initial acoustic formulation.

In this study, we do not allow an air pocket to be trapped between the free surface of the waves and the wetdeck structure. If we for a moment consider a wedge of small deadrise angle moving toward an initially calm free water surface, the creation of an airpocket is strongly connected to the large pressure gradients in the airflow near the knuckle of the wedge. If we are located away from the bow end of the catamaran, the wetdeck structure does not have such sharp corners. This is an indication that an airpocket does not necessarily have to be trapped, but this will be studied in the future. It should be noted that some part of the wetdeck between the stiffeners may be buckled and cause an aircushion to be created.

For the longest waves, the line of first contact between the free surface of the waves and the wetdeck is located at the bow end of the catamaran. Then the HBVP in Figure 2 cannot properly describe the fluid flow in the water impact region. Anyway, around the heave and pitch resonance frequencies where the largest relative velocities occur, the line of first contact between the waves and the wetdeck is located away from the bow end of the catamaran.

This study is limited to the case when the crest of a regular wave system hits mid between two of the transverse stiffeners of the wetdeck. In addition, the inertia effects of the wetdeck outside the modelled beam is not properly accounted for. This formulation is now being generalized in order to account for an arbitrary relative position between the wetdeck and the undisturbed free water surface at the moment of first water impact. This is done by modelling the wetdeck as a system of Timoshenko beams. In that way also the inertia effects of the wetdeck are more properly accounted for. The HBVP expressing fluid flow in the impact region will differ from what is presented here, but can by a linear coordinate transformation be transformed to the HBVP in Figure 2. In addition, when the wetted length is larger than the length of the beam, it is not convenient to express the vertical velocities on the wetted length of the beam in terms of a Fourier series.

5 References

Faltinsen O., Hoff J.R., Kvålsvold J. and Zhao, R. [1992], *Global loads on high-speed catamarans.* Proc. 5th Int. Symp. on the Practical Design of Ships and Mobile Units (PRADS), vol. 1, pp. 1.360-1.373.

Faltinsen O. and Zhao R. [1991], *Numerical prediction of ship motions at high forward speed.* Phil. Trans. R. Soc. Lond. A 334, pp. 241-252.

Kvålsvold and Faltinsen [1993], *Hydroelastic modelling of slamming against wetdecks.* 8th Workshop on Water Waves and Floating Bodies, May, St.Johns, Newfoundland, Canada.

Newman, J.N. [1977], *Marine Hydrodynamics,* The MIT Press, Cambridge, Mass., USA.

Wagner, H. [1932], *Über Stoss und Gleitvergänge an der Oberfläche von Flüssigkeiten,* Zeitschr. f. Angewendte Mathematik und Mechanik, Vol. 12, Part 4, pp. 192-235.

Zhao, R., Faltinsen, O. [1992], *Slamming loads on high-speed vessels,* Proc. of the 19th ONR Conference, Korea, August.

DISCUSSION

Grue J.: How important is finite curvature of the wave profile to the problem?

Faltinsen O.: The finite curvature of the wave profile is not important for the hydroelastic response in terms of the elastic stresses in the wetdeck. It is found that the absolute maximum stress is nearly independent of the curvature of the waves and proportional to the rigid body fall velocity V as long as V is not too small. However, the curvature of the waves are important for the details about the pressure distribution along the wetted length. One should note that a large local hydrodynamic pressure does not necessarily mean large stresses in the structure.

Choi H.S.: Since the moment is to be reditributed over the structure, the deformation and the stress of a beam depends strongly on the structual boundary condition. I am interested in the boundary condition you imposed.

Faltinsen O.: The beam end boundary conditions are introduced in order to model the restoring forces of the part of the wetdeck structure outside the modelled beam. Consistent with the two-dimensional approximation, the wetdeck should ideally be modelled as several beams where the length of each beam is equal to the distance between the transverse stiffeners in the wetdeck. A single beam is used in the analysis in the presented abstract. The effect of the other beams on the restoring forces of the modelled single beam is replaced by rotatory springs at the beam end. If the other beam has been simply supported at the other end, $k_\theta = \frac{3EI}{L_B}$ (see paper for notation). If the other beam was fully clamped, $k_\theta = \frac{4EI}{L_B}$. Additionally, the resoring due to torsion of the transverse stiffener is to be accounted for by k_θ

Ando S.: Is there any element of arbitrariness at all in your determination of the forcing function $p(x, \omega, t)$?

Faltinsen O.: The behaviour of the hydrodynamic pressure $p(x, \omega, t)$ in space and time is dependent on the normal relative rigid body velocity in the impact region, the curvature of the waves and the wetdeck in the impact region as well as the hydroelastic effects. The curvature of the wetdeck and the relative normal rigid body velocity in the impact region are dependent on where the waves hit along the wetdeck. There is no arbitrariness in the determination of the forcing function $p(x, \omega, t)$. This is found as the result of a boundary value problem for each time step.

Tulin M.: Does the fall of the force to zero indicate that cavitation may occur on the beam? During the entry, the contribution of the quadratic term $(\nabla\phi)^2$ in the calculation of the pressure increases as time increases and substantially reduces the impact force. See for example R. Cointe's results on the impact of a cylinder on a flat water surface. Couldn't this effect substantially reduce your maximum force, which occurs later when $(\nabla\phi)^2/\phi_t$ may be substantial.

Faltinsen O.: A crude estimate by dividing the total hydrodynamic force by the wetted length, indicates that cavitation may occur some time after the initial impact. Cavitation is not considered. This means that parts of the total hydrodynamic force curves may be unrealistic. One could also question the correctness of the Wagner theory with a jet flow when cavitation has occured. The quadratic term in the Bernoulli equation is not considered in this slamming problem. At the spray root of the jet flow, the quadratic term will become important. However, our opinion is that the details of the pressure distribution are not important for the hydroelastic response in terms of stresses in the wetdeck. What is important is the load on the part of the wetdeck between two transverse stiffeners and the nature of the elastic deformations of the wetdeck in space and time. It is worth investigating the importance of the $(\nabla\phi)^2$ term when time increases and the effect of the ϕ_t term vanishes.