The Translating Pulsating Green's Function For Free Surface Computations

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1.Introduction

At the beginning of the 80's, theoretical study has been done on the translating pulsating Green's function, see for example Bougis[1], Guevel and Bougis[2] or Inglis and Price[3], but no satisfactory computations have been achieved due to the complexity of this function and the high computational time needed. More recently, new works have been done, Wu and Eatock Taylor[4], Squires and Wilson[5] or Bougis and Coudray[6]. Following the scheme of [1,2], a fast method of computations not only of this function but also of its first and second derivatives is presented here. So the method is usable on a simple personal computer with a very short CPU time. We present here the first results obtained with this method by comparison with known results. A further method of checking the computed results is to compute the remaining real and imaginary values of the terms of the linearized free surface boundary condition. Finally the method is used to compute wave patterns for a submerged pulsating source with frequency ω translating with a forward speed U at various values of the Brard parameter $\nu=\omega U/g$(g, gravitational acceleration).

2. The Green's function

The flow of an almost perfect incompressible fluid (with small artificial viscosity ε) under the free surface of an ocean with infinite limit is assumed. Amplitude and mean camber of the free surface are assumed to be weak so the free surface boundary conditions can be linearized under the classical form of the Neumann-Kelvin condition on the undisturbed free surface z=0:

$$\left. U^2\frac{\partial^2\Phi}{\partial x^2}+2i\omega U\frac{\partial\Phi}{\partial x}+g\frac{\partial\Phi}{\partial z}-\omega^2\Phi\right|_{z=0}=0 \quad (1)$$

where Φ is the unsteady perturbation velocity potential. The previous equation is written in a moving frame of reference. Computations are given using the form of [1] and will not given here for lack of space. The potential is searched in the moving frame under the following form: $\Phi=\Phi^*\cos\omega t+\Phi^{**}\sin\omega t$. With this assumptions, 2 Green's functions G_c and G_s are needed; they are related by $G_s=-\frac{1}{\omega}\frac{\partial G_c}{\partial t}$, so only the function $G=G_c$ will be computed. This function is given by $G=G_0+G_1+G_2$. The expressions for G_0 and G_1 are given in [1] and present no difficulties. G_2 is more complex and can be written as:

$$\pi L G_2=\Re\left(e^{-i\tilde{\omega}\tilde{t}}\left\{\int_0^{\theta'_c}F_1 d\theta+\int_{\theta'_c}^{\theta_c-\alpha_c}F_2 d\theta+F_3+\int_{\theta_c+\alpha_c}^{\pi/2}F_4 d\theta\right\}\right) \quad (2)$$

where $\tilde{\omega}=\omega\sqrt{L/g}$ et $\tilde{t}=t\sqrt{g/L}$ are non-dimensional frequency and time; L is a characteristic length. The limit of integration varies with the value of ν:

if $\nu<1/4$: $\theta_c=\theta'_c=\alpha_c=0$; if $1/2>\nu>1/4$: $\theta_c=\arccos(\frac{1}{4\nu})$; $\theta'_c=0$, α_c infinitly small;

if $\nu>1/2$: $\theta_c=\arccos(\frac{1}{4\nu})$; $\theta'_c=\arccos(\frac{1}{2\nu})$, α_c infinitly small;

F_1, F_2 and F_4 are complex functions involving the complex exponential integral. Once the Green's function is known, free surface elevation at point M located on z=0 due to a submerged source located at(0,0, z'<0) can be computing using:

$$h(M,M',t) = -\frac{1}{g}\frac{\partial G}{\partial t}\bigg|_{z=0} + \frac{U}{g}\frac{\partial G}{\partial x}\bigg|_{z=0}$$

3. Numerical computations

The high computational time required is due to the computations of the integrals involving the complex exponential integral. To reduce the computational time, an elaborate Runge Kutta of fourth order, written to solve differential equations and modified to compute integrals, is used. The integral is computed with a predetermined accuracy using an adaptive stepsize control so, for a given precision, the time needed for the integration increased when the source tends towards the free surface (it is twice the CPU time when the depth of immersion varies from -1m to .01m). Due to truncation errors, the kernel of the integral of G_2 gives erroneous values close to $\theta=\pi/2$. The correct value must be computed using an equivalent of the kernel close to $\theta=\pi/2$. Computations have been done on a 486-33Mhz PC in .5s instead of 45s using a trapezoidal rule (.03s for the steady Green's function). The full method is developed in Urbin[7].

4. Results of computations

To compare with the results available, the Green's function is presented versus $\sigma=\omega^2/g$ on Fig. 1 following [4], with real part on the left and imaginary part on the right. The Froude number is .6, the source point is located at (0,0,-2) and the field point is at ($r\cos\phi$, $r\sin\phi$,0). This figures shows that results are in very good agreement with those of [4]. Table I shows respectively real and imaginary parts of left hand side of equation (1), representing the error on the free surface boundary condition. Table I gives also the percentage error, the absolute values of the terms of (1) related to the square root of the mean of each term of this equation as presented in [5]. Correct satisfying of (1) with a high order of accuracy, less than 0.01%, can be seen.

σr	Real part R	Imaginary part I	per cent error R	per cent error I
.1	9.4652E-05	-3.5763E-05	8.2549E-03	1.9817E-03
.2	2.5272E-05	-2.9564E-05	6.5960E-04	1.0461E-03
.3	7.0095E-05	-9.2030E-05	2.4975E-03	6.1451E-03
.4	6.4373E-05	4.1366E-05	2.5170E-03	3.1684E-03
.5	-2.3842E-06	1.0848E-05	9.7471E-05	8.5377E-04
.6	2.3842E-07	1.3590E-05	9.9899E-06	1.0615E-03
.7	2.8610E-06	1.6212E-05	1.2160E-04	1.2435E-03
.8	8.1062E-06	1.8358E-05	3.4737E-04	1.3801E-03
.9	-1.1444E-05	2.7776E-05	4.9250E-04	2.0496E-03
1.0	-8.3447E-06	3.6120E-05	3.5969E-04	2.6231E-03

Table I: Free surface boundary condition errors (source (0,0,-2), field (.707,.707,0), F=.6)

On Fig. 2 free surface elevations are represented for the 2 limit cases of the diffraction-radiation problem ($U\to 0$) and of the steady wave resistance problem ($\omega\to 0$). Theses shapes are similar to computations done with these two particular theories. Finally, on Fig. 3, the shape of the free surface is plotted for Froude number F= 0.64 for various values of the parameter ν. The source is located at (0,0,-1) and the time is t=0s. For ν slightly less than .25, lateral waves can be observed, perpendicular to the direction of the forward speed, but downstream of the source and circular waves appear upstream of the source. For ν slightly greater than .25, the lateral waves come upstream giving with the circular waves an almost flat free surface. For ν greater than .3, the circular waves appear only in a downstream sector, the angle of which decreasing when ν increases and tending towards the Kelvin dihedral. Those shapes are in good agreement with the shape given by Chang[8].

4. Conclusion

If the use of the Green's function is very interesting, its computations present many numerical difficulties. In this paper, a fast method of computations (time has been divided by 120 compared with classical integration) of the harmonic forward speed Green's function usable on a simple personal computer has been presented. This method uses a fourth order Runge Kutta scheme and is careful of problems close to $\theta=\alpha_c$ and $\theta=\pi/2$. The results obtained have been checked by comparison with available results on the function and its first derivatives. For the second derivatives, results have been checked looking at the exact satisfactory of the free surface boundary condition. Finally evolution of the elevation of the free surface with the Brard parameter, computed with a reasonable computing time, is given. This fast method enables the use of this complete form of the Green's function and its derivatives in larger code of diffraction-radiation with forward speed to compute unsteady flow around a surface-piercing hull.

Acknowledgments

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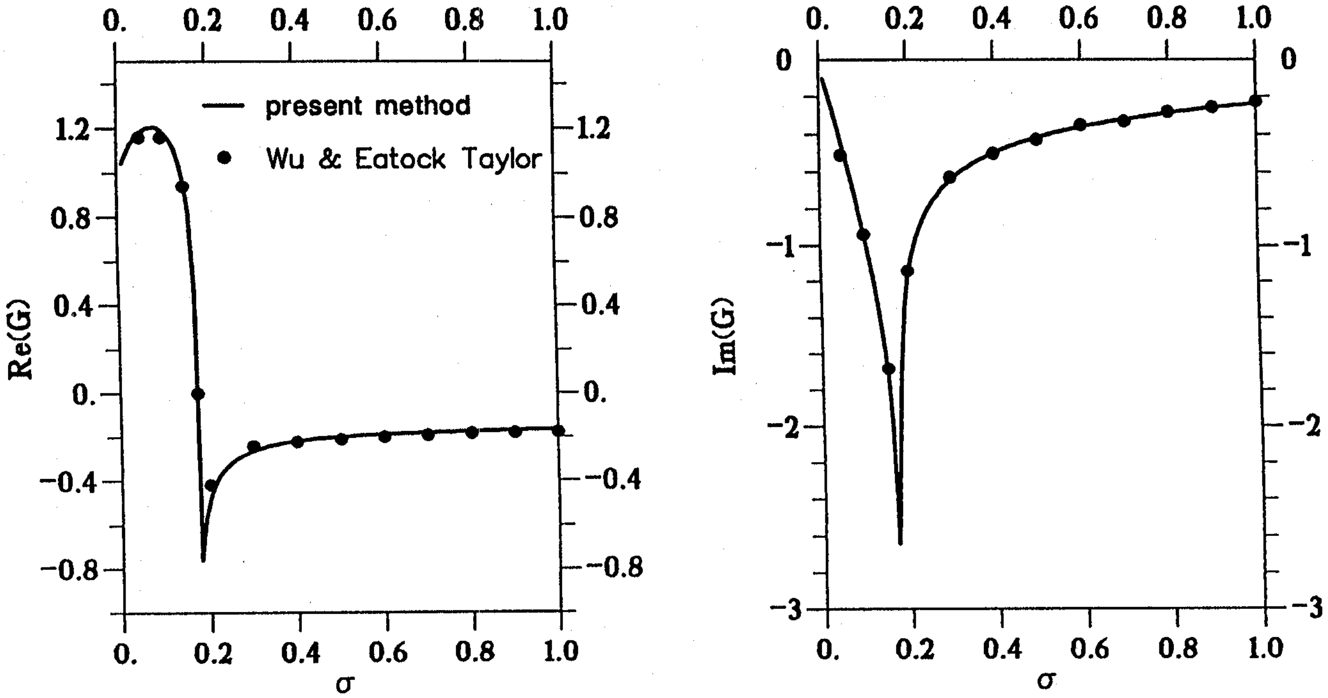


Fig. 1 Computation of harmonic forward-speed Green's fonction at $F = U/\sqrt{gr} = 0.6$, $r = 1.0$, $\sigma = \omega^2/g$, source = (0.,0.,−2.), field = $(r\cos\phi, r\sin\phi, 0.)$, $\phi = \pi/4$

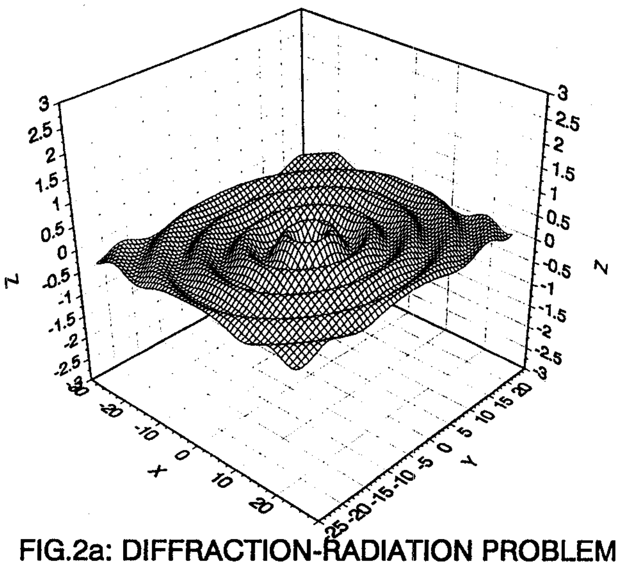


FIG.2a: DIFFRACTION-RADIATION PROBLEM
W=3rads/s;Z=-1m;U=0.03m/s

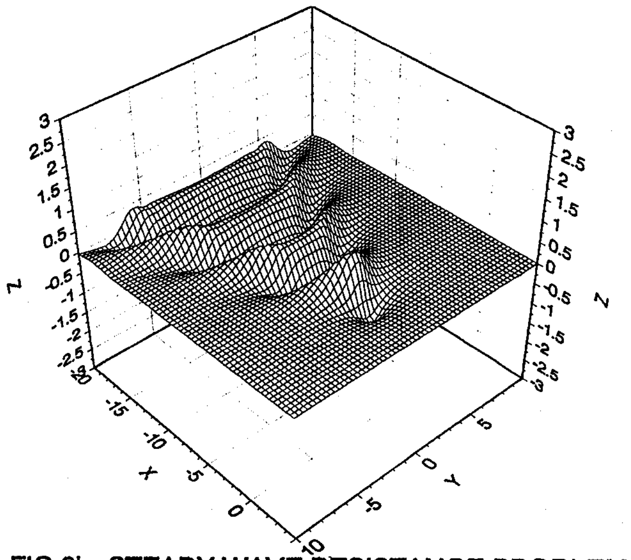
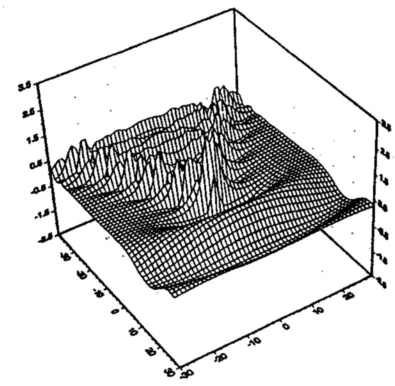
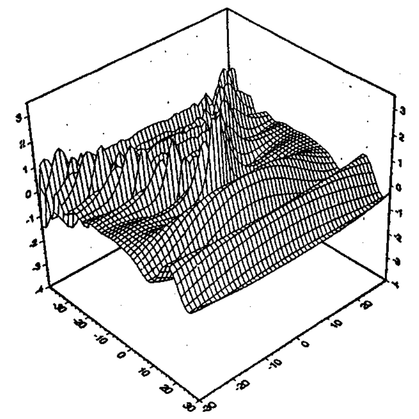


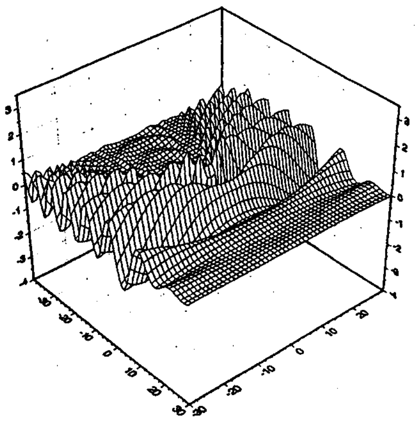
FIG.2b: STEADY WAVE RESISTANCE PROBLEM
F=0.96;Z=-1m;W=0.05rad/s



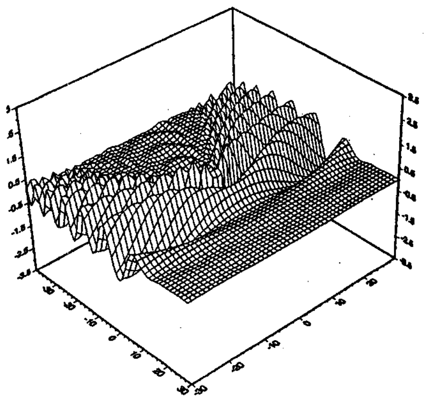
W=1.13 ; Nu=0.23



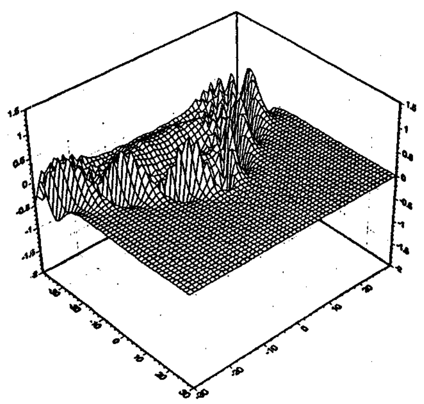
W=1.2 ; Nu=0.245



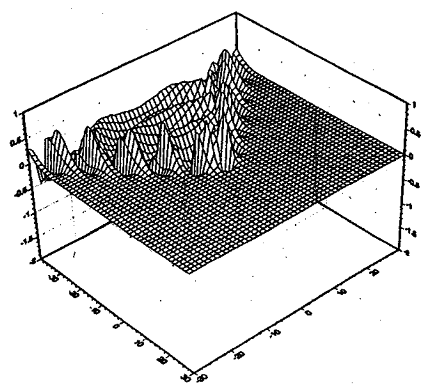
W=1.25 ; Nu=0.255



W=1.275 ; Nu=0.26



W=2.45 ; Nu=0.5



W=3.38 ; Nu=0.7

FIG3: DIFFRACTION-RADIATION WITH SPEED
F=0.64 ; Z=-1m ; T=0s

DISCUSSION

Yeung R.W. : In your abstract, on Page 10, the bottom figure for the case of $\omega = 2.45, Nu = 0.5$, a lack of symmetry about the centerplane exists. Can you explain the source of this difficulty?

Ba M. : The figures in the abstract were computed with less accuracy. You can see on the figures presented during the talk, that there is no oscillation due to a lack of precision like in the abstract.

Kashiwagi M.: 1) Have you computed for the case in which both of z and z' are zero?
2) Could you tell us CPU time as z and z' approach the free surface?

Ba M.: 1) No. But it will be done.
2) For a field point located at $(x, y, 0)$, the CPU time is multiplied by 2 when the source point moves from (0,0,-1) to (0,0,-0.01) under the free-surface. (CPU time: 0.5sec to compute one value of the Green function when the source point is located at (0,0,-1) and the field point is on the free surface.)